Formelsamling i kretsteori, ellära och elektronik

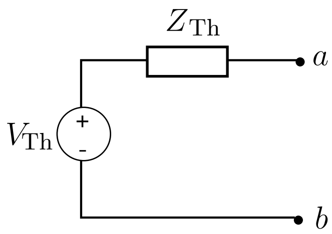
Elektro- och informationsteknik
Lunds tekniska högskola
Maj 2013

Kretsteori

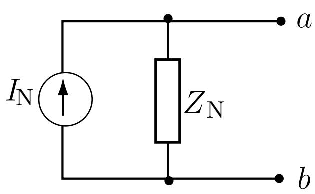
Komplexvärden

- Realdelskonvention: $v(t) = \mathrm{Re}\{V\mathrm{e}^{\mathrm{j}\omega t}\}$ och $i(t) = \mathrm{Re}\{I\mathrm{e}^{\mathrm{j}\omega t}\}$.
- Imaginärdelskonvention: $v(t) = \mathrm{Im}\{V\mathrm{e}^{\mathrm{j}\omega t}\}$ och $i(t) = \mathrm{Im}\{I\mathrm{e}^{\mathrm{j}\omega t}\}$.

Tvåpolsekvivalenter



Thévenin



Norton

Komplex effekt

$S = \frac{1}{2} V I^* = P + \mathrm{j}Q = |S|(\cos\varphi + \mathrm{j}\sin\varphi)$

S = komplex effekt [VA]

$|S|$ = skenbar effekt [VA]

$P = \mathrm{Re}\, S$ = aktiv effekt (=tidsmedelvärdet av effektförbrukningen) [W]

$Q = \mathrm{Im}\, S$ = reaktiv effekt $[\mathrm{VA_r}] = [\mathrm{VAR}]$

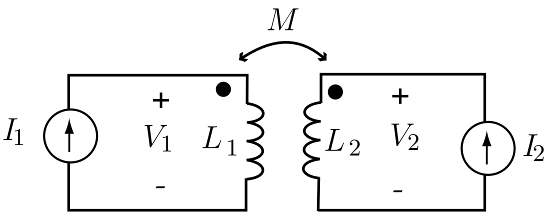
$\cos\varphi$ = effektfaktor

Effektanpassningsregeln

$$Z_{\mathrm{L}} = Z_{\mathrm{i}}^* \quad \text{och} \quad \max\{P_{\mathrm{L}}\} = \frac{|V|^2}{8R_{\mathrm{i}}}.$$

Ömsesidig induktans

$$\begin{cases} V_1 = j\omega L_1 I_1 + j\omega M I_2 \\ V_2 = j\omega L_2 I_2 + j\omega M I_1 \end{cases}$$



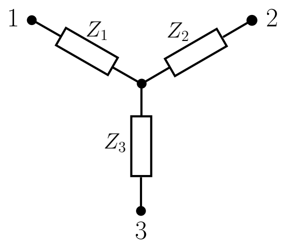
$L_1,\ L_2 =$ självinduktanser

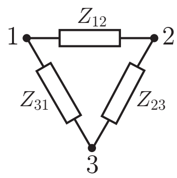
$M =$ ömsesidig induktans

$M = k\sqrt{L_1 L_2}$ där $0 \leq k \leq 1$

$k =$ kopplingsfaktorn

Nätverkstransformation





Y till △

$$Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$Z_{23} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$

$$Z_{31} = Z_3 + Z_1 + \frac{Z_3 Z_1}{Z_2}$$

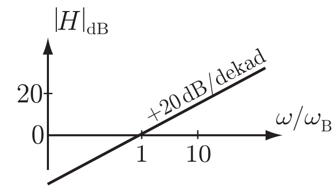
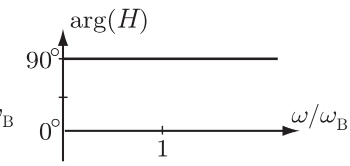
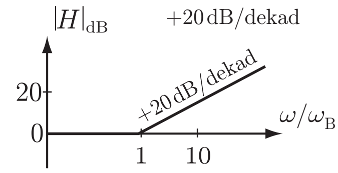
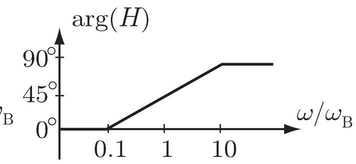
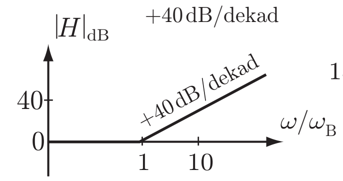
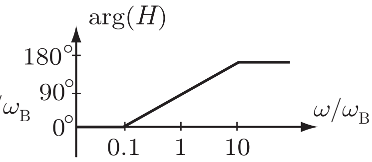
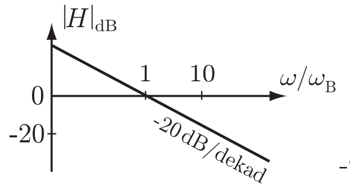
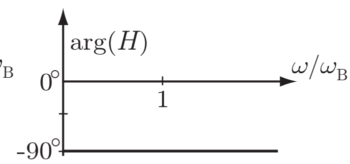
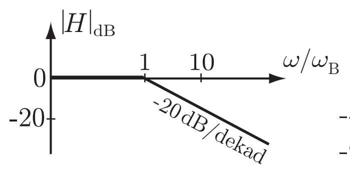
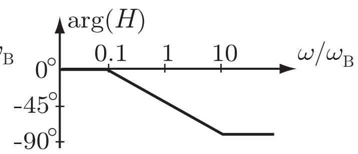
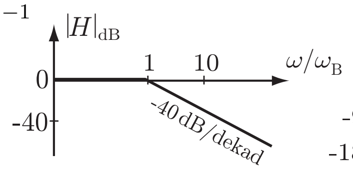
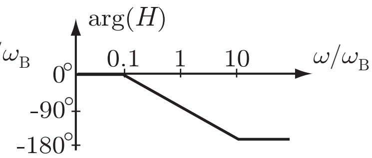
△ till Y

$$Z_1 = \frac{Z_{31} Z_{12}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_2 = \frac{Z_{12} Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_3 = \frac{Z_{23} Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

Rätlinjeapproximationer av Bodediagram

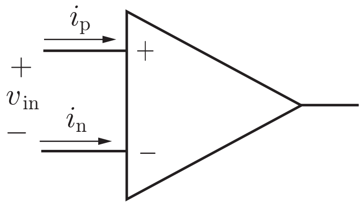
$H(s)$	Amplitud	Fas
$\dfrac{s}{\omega_B}$		
$1+\dfrac{s}{\omega_B}$		
$1+2\zeta\dfrac{s}{\omega_B}+\dfrac{s^2}{\omega_B^2}$		
$\left(\dfrac{s}{\omega_B}\right)^{-1}$		
$\left(1+\dfrac{s}{\omega_B}\right)^{-1}$		
$\left(1+2\zeta\dfrac{s}{\omega_B}+\dfrac{s^2}{\omega_B^2}\right)^{-1}$		

OBS! Det skall gälla att $|\zeta| \leq 1$.

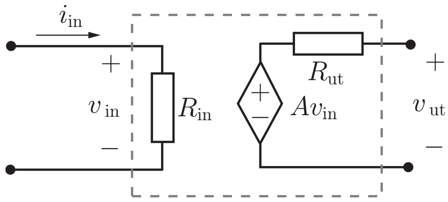
Elektronik

Ideal operationsförstärkare (OP)

För en ideal OP är $i_\mathrm{p} = i_\mathrm{n} = 0$. Vi använder vanligtvis negativ återkoppling där också $v_\mathrm{in} = 0$.



Kretsmodell av spänningsförstärkare



Dioder

Shockleyekvationen

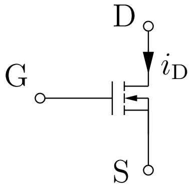
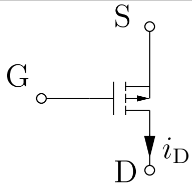
$$i_\mathrm{D} = I_\mathrm{s}\left(\mathrm{e}^{\frac{v_\mathrm{D}}{nV_\mathrm{T}}} - 1\right)$$

där $V_\mathrm{T} = \frac{kT}{q}$, $q \approx 1.6 \cdot 10^{-19}$ C och $k \approx 1.38 \cdot 10^{-23}$ J/K.

Dynamisk resistans

$$r_\mathrm{d} = \frac{1}{\left.\frac{\mathrm{d}i_\mathrm{D}}{\mathrm{d}v_\mathrm{D}}\right|_Q}$$

MOSFET

	NMOS	PMOS
Kretssymbol		
$\mu \approx$	$675\,\mathrm{cm^2V^{-1}s^{-1}}$	$240\,\mathrm{cm^2V^{-1}s^{-1}}$
$\kappa \approx$	$115\,\mu\mathrm{AV^{-2}}$	$40\,\mu\mathrm{AV^{-2}}$
$V_\mathrm{t} \approx$	$+0.5\,\mathrm{V}$	$-0.6\,\mathrm{V}$
Subtröskel (strypt område)	$v_\mathrm{GS} \leq V_\mathrm{t}$, $v_\mathrm{DS} \geq 0$, $i_\mathrm{D} = 0$	$v_\mathrm{GS} \geq V_\mathrm{t}$, $v_\mathrm{DS} \leq 0$, $i_\mathrm{D} = 0$
Linjärt område	$v_\mathrm{GS} \geq V_\mathrm{t}$, $0 \leq v_\mathrm{DS} \leq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(2(v_\mathrm{GS} - V_\mathrm{t})v_\mathrm{DS} - v_\mathrm{DS}^2)$	$v_\mathrm{GS} \leq V_\mathrm{t}$, $0 \geq v_\mathrm{DS} \geq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(2(v_\mathrm{GS} - V_\mathrm{t})v_\mathrm{DS} - v_\mathrm{DS}^2)$
Mättnads-område	$v_\mathrm{GS} \geq V_\mathrm{t}$, $v_\mathrm{DS} \geq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(v_\mathrm{GS} - V_\mathrm{t})^2$	$v_\mathrm{GS} \leq V_\mathrm{t}$, $v_\mathrm{DS} \leq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(v_\mathrm{GS} - V_\mathrm{t})^2$
v_DS, v_GS	Vanligtvis positiva	Vanligtvis negativa

$K = \frac{W}{L}\frac{\kappa}{2}$

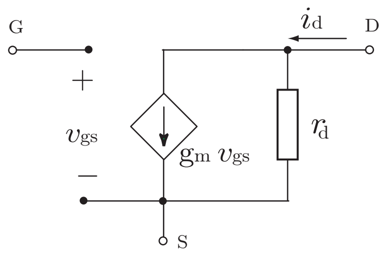
Småsignalmodell

Småsignalmodell för en FET, där

$$g_\mathrm{m} = \left.\frac{\partial i_\mathrm{D}}{\partial v_\mathrm{GS}}\right|_\mathrm{arbetspunkt}$$

och

$$\frac{1}{r_\mathrm{d}} = \left.\frac{\partial i_\mathrm{D}}{\partial v_\mathrm{DS}}\right|_\mathrm{arbetspunkt}$$



Ellära

Lorentzkraften

$$\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Elektriskt fält och potential från punktladdning

$$\boldsymbol{E} = \frac{q}{4\pi\varepsilon_0 r^2}\boldsymbol{e}_r \qquad V = \frac{q}{4\pi\varepsilon_0 r}$$

Spänning

$$U = V_1 - V_2 = \int_{P_1}^{P_2} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r}$$

Elektrisk dipol

$$\boldsymbol{p} = p\boldsymbol{e}_z \qquad p = q\ell \qquad \boldsymbol{E} = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\boldsymbol{e}_r + \sin\theta\boldsymbol{e}_\theta)$$

Polarisation $\boldsymbol{P}$ och elektrisk flödestäthet $\boldsymbol{D}$

$$\boldsymbol{D} = \varepsilon_0\boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{P} = \chi_e\varepsilon_0\boldsymbol{E}$$
$$\varepsilon_r = 1 + \chi_e$$

Strömtäthet och resistans för rak ledare

$$\boldsymbol{J} = \frac{i}{A}\boldsymbol{e}_x \qquad R = \rho\frac{\ell}{A}$$

Plattkondensator

$$C = \varepsilon_r\varepsilon_0\frac{A}{d}$$

RCL-beräkningar

Kretsparametrarna i fältuttryck:

R	C	L
$R = \dfrac{v_a - v_b}{i}$	$C = \dfrac{q}{v_a - v_b}$	$L = \dfrac{\phi}{i}$
$\int_S \boldsymbol{J} \cdot \boldsymbol{e}_n \, \mathrm{d}S = i$	$\oint_S \boldsymbol{D} \cdot \boldsymbol{e}_n \, \mathrm{d}S = q$	$\int_S \boldsymbol{B} \cdot \boldsymbol{e}_n \, \mathrm{d}S = \phi$
$\int_{P_a}^{P_b} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = v_a - v_b$	$\int_{P_a}^{P_b} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = v_a - v_b$	$\oint_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} = i$
$\boldsymbol{J} = \sigma \boldsymbol{E}$	$\boldsymbol{D} = \varepsilon_0 \varepsilon_{\mathrm{r}} \boldsymbol{E}$	$\boldsymbol{B} = \mu_0 \mu_{\mathrm{r}} \boldsymbol{H}$

Fälten uppfyller följande villkor:

$$\nabla \times \boldsymbol{E} = 0 \quad \Leftrightarrow \quad \boldsymbol{E} = -\nabla V \quad \Leftrightarrow \quad \oint_C \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = 0$$

$$\nabla \cdot \boldsymbol{J} = 0 \quad \Leftrightarrow \quad \oint_S \boldsymbol{J} \cdot \boldsymbol{e}_n \, \mathrm{d}S = 0$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad \Leftrightarrow \quad \oint_S \boldsymbol{B} \cdot \boldsymbol{e}_n \, \mathrm{d}S = 0$$

Kretsparametrarna i effekt- och energiuttryck:

	R	C	L
Krets	$p = Ri^2 = v^2/R$	$w_{\mathrm{e}} = \dfrac{1}{2}Cv^2 = \dfrac{1}{2}q^2/C$	$w_{\mathrm{m}} = \dfrac{1}{2}Li^2 = \dfrac{1}{2}\phi^2/L$
Fält	$p = \int \boldsymbol{E} \cdot \boldsymbol{J} \, \mathrm{d}V$	$w_{\mathrm{e}} = \dfrac{1}{2} \displaystyle\int \boldsymbol{D} \cdot \boldsymbol{E} \, \mathrm{d}V$	$w_{\mathrm{m}} = \dfrac{1}{2} \displaystyle\int \boldsymbol{B} \cdot \boldsymbol{H} \, \mathrm{d}V$

Transmissionsledningsgeometrier:

Parameter	**Koaxial**	**Tvåtråds**	**Platta**
R' [Ω/m]	$\dfrac{1}{2\pi\delta\sigma_{\mathrm{c}}}\left[\dfrac{1}{a} + \dfrac{1}{b}\right]$ $(\delta \ll a, t)$	$\dfrac{1}{\pi a \delta \sigma_{\mathrm{c}}}$ $(\delta \ll a)$	$\dfrac{2}{w\delta\sigma_{\mathrm{c}}}$ $(\delta \ll t)$
L' [H/m]	$\dfrac{\mu}{2\pi} \ln \dfrac{b}{a}$	$\dfrac{\mu}{\pi} \cosh^{-1} \dfrac{d}{2a}$	$\dfrac{\mu d}{w}$
G' [S/m]	$\dfrac{2\pi\sigma}{\ln \frac{b}{a}}$	$\dfrac{\pi\sigma}{\cosh^{-1} \frac{d}{2a}}$	$\dfrac{\sigma w}{d}$
C' [F/m]	$\dfrac{2\pi\varepsilon}{\ln \frac{b}{a}}$	$\dfrac{\pi\varepsilon}{\cosh^{-1} \frac{d}{2a}}$	$\dfrac{\varepsilon w}{d}$ $(w \gg d)$

Transmissionsledningar

Ledningsekvationerna, förlustfri dubbelledning

$$\begin{aligned} -\frac{\partial v}{\partial z} &= L\,\frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial z} &= C\,\frac{\partial v}{\partial t} \end{aligned}$$

Allmän lösning, förlustfri dubbelledning

$$\begin{aligned} v &= v^+(z - v_\mathrm{p}t) + v^-(z + v_\mathrm{p}t) \\ i &= \frac{1}{Z_0}v^+(z - v_\mathrm{p}t) - \frac{1}{Z_0}v^-(z + v_\mathrm{p}t) \end{aligned}$$

$$v_\mathrm{p} = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}} \qquad LC = \mu_r\mu_0\varepsilon_r\varepsilon_0$$

Ledningsekvationerna, sinusformigt tidsberoende

$$\begin{aligned} -\frac{\mathrm{d}V}{\mathrm{d}z} &= RI + \mathrm{j}\omega LI \\ -\frac{\mathrm{d}I}{\mathrm{d}z} &= GV + \mathrm{j}\omega CV \end{aligned}$$

Allmän lösning, sinusformigt tidsberoende

$$\begin{aligned} V(z) &= V_1\mathrm{e}^{-\gamma z} + V_2\mathrm{e}^{\gamma z} \\ I(z) &= \frac{1}{Z_0}(V_1\mathrm{e}^{-\gamma z} - V_2\mathrm{e}^{\gamma z}) \end{aligned}$$

Utbredningskonstant

$$\gamma = \sqrt{(R + \mathrm{j}\omega L)(G + \mathrm{j}\omega C)} = \alpha + \mathrm{j}\beta$$

Karakteristisk impedans

$$Z_0 = \sqrt{\frac{R + \mathrm{j}\omega L}{G + \mathrm{j}\omega C}}$$

Impedansen för en dubbelledning med längden l avslutad med Z_L

$$Z_\mathrm{in} = Z_0\frac{Z_L\cosh(\gamma\ell) + Z_0\sinh(\gamma\ell)}{Z_0\cosh(\gamma\ell) + Z_L\sinh(\gamma\ell)} = Z_0\frac{1 + \mathit{\Gamma}\mathrm{e}^{-2\gamma\ell}}{1 - \mathit{\Gamma}\mathrm{e}^{-2\gamma\ell}}$$

Impedansen för en förlustfri ledning med längden l avslutad med Z_L

$$Z_\mathrm{in} = Z_0\frac{Z_L\cos(\beta l) + \mathrm{j}Z_0\sin(\beta l)}{Z_0\cos(\beta l) + \mathrm{j}Z_L\sin(\beta l)} = Z_0\frac{1 + \mathit{\Gamma}\mathrm{e}^{-\mathrm{j}2\beta\ell}}{1 - \mathit{\Gamma}\mathrm{e}^{-\mathrm{j}2\beta\ell}}$$

Reflektionsfaktorn för spänning vid belastningen

$$\mathit{\Gamma} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Matematiska formler och samband

Trigonometriska formler

$$\sin\alpha = \cos(\alpha - \pi/2)$$
$$\cos\alpha = \sin(\alpha + \pi/2)$$
$$\cos^2\alpha + \sin^2\alpha = 1$$
$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$
$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$A\cos\alpha + B\sin\alpha = \sqrt{A^2+B^2}\cos(\alpha-\beta) \text{ där } \cos\beta = \frac{A}{\sqrt{A^2+B^2}},\ \sin\beta = \frac{B}{\sqrt{A^2+B^2}}$$

$$\cos\alpha = \frac{\mathrm{e}^{\mathrm{j}\alpha} + \mathrm{e}^{-\mathrm{j}\alpha}}{2}$$
$$\sin\alpha = \frac{\mathrm{e}^{\mathrm{j}\alpha} - \mathrm{e}^{-\mathrm{j}\alpha}}{2\mathrm{j}}$$

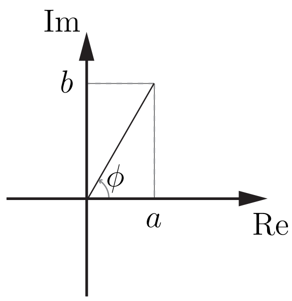
$$\mathrm{e}^{\mathrm{j}\alpha} = \cos\alpha + \mathrm{j}\sin\alpha$$

Komplexa tal

$$z = a + \mathrm{j}b = |z|\mathrm{e}^{\mathrm{j}\phi}$$

där

$$|z| = \sqrt{a^2+b^2} \quad \text{och om } a > 0 \text{ är} \quad \phi = \arctan\frac{b}{a}$$



Ekvationssystem (2×2)

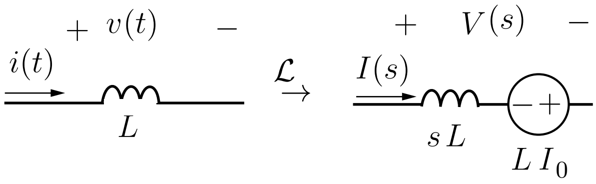
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

med lösning

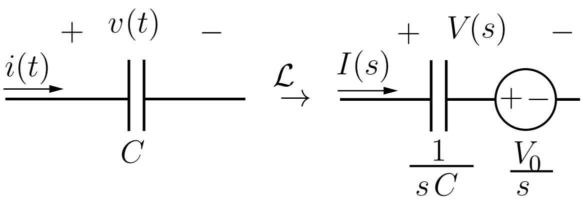
$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

Laplacetransformen

Spole med $i(0^-) = I_0$:
$V(s) = L(sI(s) - I_0)$



Kondensator med $v(0^-) = V_0$:
$I(s) = C(sV(s) - V_0)$



	$f(t)$	$F(s)$
1.	$\alpha f(t)$	$\alpha F(s)$
2.	$f_1(t) + f_2(t) + f_3(t) + \cdots$	$F_1(s) + F_2(s) + F_3(s) + \cdots$
3.	$\dfrac{\mathrm{d}f(t)}{\mathrm{d}t}$	$sF(s) - f(0^-)$
4.	$\displaystyle\int_0^t f(\tau)\,\mathrm{d}\tau$	$\dfrac{F(s)}{s}$
5.	$f(t-a)\,\mathrm{u}(t-a), \quad a > 0$	$\mathrm{e}^{-as}F(s)$
6.	$\mathrm{e}^{-at}f(t)$	$F(s+a)$
7.	$f(at), \quad a > 0$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

Begynnelsevärdessatsen $\quad \lim_{t\to 0_+} f(t) = \lim_{s\to\infty} sF(s)$

Slutvärdessatsen $\quad \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$\dfrac{\mathrm{d}^n}{\mathrm{d}t^n}\,\delta(t)$	s^n
3.	$\mathrm{u}(t)$, enhetssteget	$\dfrac{1}{s}$
4.	$\dfrac{t^n}{n!}\ \mathrm{u}(t)$	$\dfrac{1}{s^{n+1}}$
5.	$\mathrm{e}^{-at}\ \mathrm{u}(t)$	$\dfrac{1}{s+a}$
6.	$\dfrac{t^n}{n!}\,\mathrm{e}^{-at}\ \mathrm{u}(t)$	$\dfrac{1}{(s+a)^{n+1}}$
7.	$\dfrac{\mathrm{e}^{-at}-\mathrm{e}^{-bt}}{b-a}\ \mathrm{u}(t)$	$\dfrac{1}{(s+a)(s+b)}$
8.	$\dfrac{a\mathrm{e}^{-at}-b\mathrm{e}^{-bt}}{a-b}\ \mathrm{u}(t)$	$\dfrac{s}{(s+a)(s+b)}$
9.	$\sin(\omega_0 t)\ \mathrm{u}(t)$	$\dfrac{\omega_0}{s^2+\omega_0^2}$
10.	$\cos(\omega_0 t)\ \mathrm{u}(t)$	$\dfrac{s}{s^2+\omega_0^2}$
11.	$\big(\sin(\omega_0 t)-\omega_0 t\,\cos(\omega_0 t)\big)\ \mathrm{u}(t)$	$\dfrac{2\omega_0^3}{(s^2+\omega_0^2)^2}$
12.	$\omega_0 t\,\sin(\omega_0 t)\ \mathrm{u}(t)$	$\dfrac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$
13.	$\mathrm{e}^{-at}\,\sin(\omega_0 t)\ \mathrm{u}(t)$	$\dfrac{\omega_0}{(s+a)^2+\omega_0^2}$
14.	$\mathrm{e}^{-at}\,\cos(\omega_0 t)\ \mathrm{u}(t)$	$\dfrac{s+a}{(s+a)^2+\omega_0^2}$

Koordinatsystem

Kartesiska koordinater (x, y, z)

Ortsvektor $\boldsymbol{r} = x\,\boldsymbol{e}_x + y\,\boldsymbol{e}_y + z\,\boldsymbol{e}_z$

Linjeelement $\mathrm{d}\boldsymbol{l} = \mathrm{d}x\,\boldsymbol{e}_x + \mathrm{d}y\,\boldsymbol{e}_y + \mathrm{d}z\,\boldsymbol{e}_z$

Volymelement $\mathrm{d}v = \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z$

Differentialoperatorer

$$
\begin{aligned}
\boldsymbol{\nabla} V &= \boldsymbol{e}_x \frac{\partial V}{\partial x} + \boldsymbol{e}_y \frac{\partial V}{\partial y} + \boldsymbol{e}_z \frac{\partial V}{\partial z} \\
\boldsymbol{\nabla} \cdot \boldsymbol{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\boldsymbol{\nabla} \times \boldsymbol{A} &= \boldsymbol{e}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \boldsymbol{e}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \boldsymbol{e}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
\boldsymbol{\nabla}^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}
\end{aligned}
$$

Cylinderkoordinater $(r_{\mathrm{c}}, \varphi, z)$

Ortsvektor $\boldsymbol{r} = r_{\mathrm{c}}\,\boldsymbol{e}_{r_{\mathrm{c}}} + z\,\boldsymbol{e}_z$

Enhetsvektorer $\boldsymbol{e}_{r_{\mathrm{c}}} = \cos\varphi\,\boldsymbol{e}_x + \sin\varphi\,\boldsymbol{e}_y$

$\boldsymbol{e}_{\varphi} = -\sin\varphi\,\boldsymbol{e}_x + \cos\varphi\,\boldsymbol{e}_y$

Linjeelement $\mathrm{d}\boldsymbol{l} = \mathrm{d}r_{\mathrm{c}}\,\boldsymbol{e}_{r_{\mathrm{c}}} + r_{\mathrm{c}}\ \mathrm{d}\varphi\,\boldsymbol{e}_{\varphi} + \mathrm{d}z\,\boldsymbol{e}_z$

Volymelement $\mathrm{d}v = r_{\mathrm{c}}\ \mathrm{d}r_{\mathrm{c}}\ \mathrm{d}\varphi\,dz$

Differentialoperatorer

$$
\begin{aligned}
\boldsymbol{\nabla} V &= \boldsymbol{e}_{r_{\mathrm{c}}} \frac{\partial V}{\partial r_{\mathrm{c}}} + \boldsymbol{e}_{\varphi} \frac{1}{r_{\mathrm{c}}} \frac{\partial V}{\partial \varphi} + \boldsymbol{e}_z \frac{\partial V}{\partial z} \\
\boldsymbol{\nabla} \cdot \boldsymbol{A} &= \frac{1}{r_{\mathrm{c}}} \frac{\partial}{\partial r_{\mathrm{c}}} (r_{\mathrm{c}} A_{r_{\mathrm{c}}}) + \frac{1}{r_{\mathrm{c}}} \frac{\partial A_{\varphi}}{\partial \varphi} + \frac{\partial A_z}{\partial z} \\
\boldsymbol{\nabla} \times \boldsymbol{A} &= \boldsymbol{e}_{r_{\mathrm{c}}} \left(\frac{1}{r_{\mathrm{c}}} \frac{\partial A_z}{\partial \varphi} - \frac{\partial A_{\varphi}}{\partial z} \right) + \boldsymbol{e}_{\varphi} \left(\frac{\partial A_{r_{\mathrm{c}}}}{\partial z} - \frac{\partial A_z}{\partial r_{\mathrm{c}}} \right) \\
&\quad + \boldsymbol{e}_z \frac{1}{r_{\mathrm{c}}} \left[\frac{\partial}{\partial r_{\mathrm{c}}} (r_{\mathrm{c}} A_{\varphi}) - \frac{\partial A_{r_{\mathrm{c}}}}{\partial \varphi} \right] \\
\boldsymbol{\nabla}^2 V &= \frac{1}{r_{\mathrm{c}}} \frac{\partial}{\partial r_{\mathrm{c}}} \left(r_{\mathrm{c}} \frac{\partial V}{\partial r_{\mathrm{c}}} \right) + \frac{1}{r_{\mathrm{c}}^2} \frac{\partial^2 V}{\partial \varphi^2} + \frac{\partial^2 V}{\partial z^2}
\end{aligned}
$$

Sfäriska koordinater (r, θ, φ)

Ortsvektor $\boldsymbol{r} = r\,\boldsymbol{e}_r$

Enhetsvektorer $\boldsymbol{e}_r = \sin\theta\cos\varphi\,\boldsymbol{e}_x + \sin\theta\sin\varphi\,\boldsymbol{e}_y + \cos\theta\,\boldsymbol{e}_z$

$\boldsymbol{e}_\theta = \cos\theta\cos\varphi\,\boldsymbol{e}_x + \cos\theta\sin\varphi\,\boldsymbol{e}_y - \sin\theta\,\boldsymbol{e}_z$

$\boldsymbol{e}_\varphi = -\sin\varphi\,\boldsymbol{e}_x + \cos\varphi\,\boldsymbol{e}_y$

Linjeelement $\mathrm{d}\boldsymbol{l} = \mathrm{d}r\,\boldsymbol{e}_r + r\,\mathrm{d}\theta\,\boldsymbol{e}_\theta + r\sin\theta\,\mathrm{d}\varphi\,\boldsymbol{e}_\varphi$

Volymelement $\mathrm{d}v = r^2\sin\theta\;\mathrm{d}r\;\mathrm{d}\theta\;\mathrm{d}\varphi$

Differentialoperatorer

$$\boldsymbol{\nabla} V = \boldsymbol{e}_r\frac{\partial V}{\partial r} + \boldsymbol{e}_\theta\frac{1}{r}\frac{\partial V}{\partial\theta} + \boldsymbol{e}_\varphi\frac{1}{r\sin\theta}\frac{\partial V}{\partial\varphi}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{A} &= \boldsymbol{e}_r\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\varphi\sin\theta) - \frac{\partial A_\theta}{\partial\varphi}\right]\\ &+\boldsymbol{e}_\theta\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right] + \boldsymbol{e}_\varphi\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\end{aligned}$$

$$\boldsymbol{\nabla}^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\varphi^2}$$