Formelsamling i kretsteori, ellära och elektronik

Elektro- och informationsteknik
Lunds tekniska högskola
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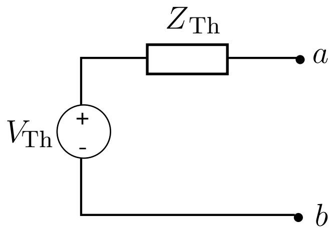
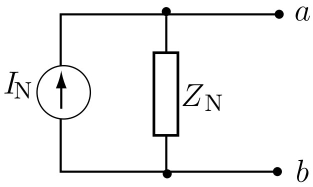
Komplexvärden

- Realdelskonvention: $v(t) = \mathrm{Re}\{V\mathrm{e}^{\mathrm{j}\omega t}\}$ och $i(t) = \mathrm{Re}\{I\mathrm{e}^{\mathrm{j}\omega t}\}$.
- Imaginärdelskonvention: $v(t) = \mathrm{Im}\{V\mathrm{e}^{\mathrm{j}\omega t}\}$ och $i(t) = \mathrm{Im}\{I\mathrm{e}^{\mathrm{j}\omega t}\}$.

Tvåpolsekvivalenter



Thévenin



Norton

Komplex effekt

$$S = \frac{1}{2}VI^* = P + \mathrm{j}Q = |S|(\cos\varphi + \mathrm{j}\sin\varphi)$$

$S =$ komplex effekt [VA]

$|S| =$ skenbar effekt [VA]

$P = \mathrm{Re}\, S =$ aktiv effekt (=tidsmedelvärdet av effektförbrukningen) [W]

$Q = \mathrm{Im}\, S =$ reaktiv effekt [$\mathrm{VA_r}$] = [VAR]

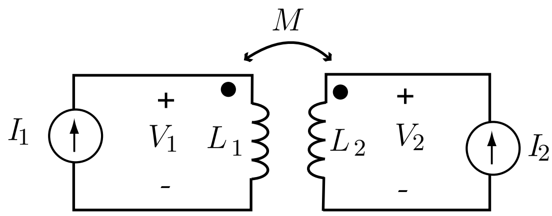
$\cos\varphi =$ effektfaktor

Effektanpassningsregeln

$$Z_{\mathrm{L}} = Z_{\mathrm{i}}^* \quad \text{och} \quad \max\{P_{\mathrm{L}}\} = \frac{|V|^2}{8R_{\mathrm{i}}}.$$

Ömsesidig induktans

$$\begin{cases} V_1 = j\omega L_1 I_1 + j\omega M I_2 \\ V_2 = j\omega L_2 I_2 + j\omega M I_1 \end{cases}$$



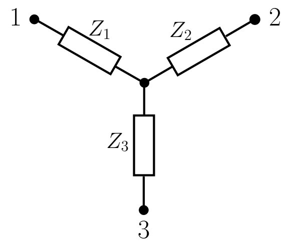
$L_1,\ L_2 =$ självinduktanser

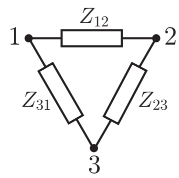
$M =$ ömsesidig induktans

$M = k\sqrt{L_1 L_2}$ där $0 \leq k \leq 1$

$k =$ kopplingsfaktorn

Nätverkstransformation





Y till △

$$Z_{12} = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$$

$$Z_{23} = Z_2 + Z_3 + \frac{Z_2 Z_3}{Z_1}$$

$$Z_{31} = Z_3 + Z_1 + \frac{Z_3 Z_1}{Z_2}$$

△ till Y

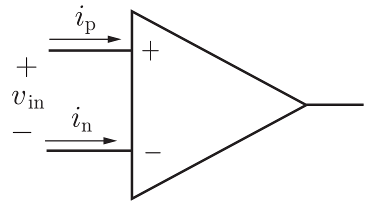
$$Z_1 = \frac{Z_{31} Z_{12}}{Z_{12} + Z_{23} + Z_{31}}$$

$$Z_2 = \frac{Z_{12} Z_{23}}{Z_{12} + Z_{23} + Z_{31}}$$

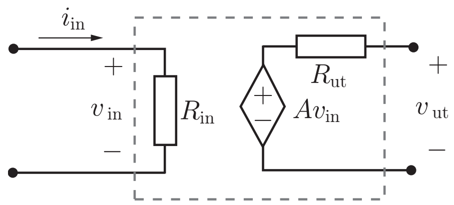
$$Z_3 = \frac{Z_{23} Z_{31}}{Z_{12} + Z_{23} + Z_{31}}$$

Ideal operationsförstärkare (OP)

För en ideal OP är $i_p = i_n = 0$. Vi använder vanligtvis negativ återkoppling där också $v_{in} = 0$.



Kretsmodell av spänningsförstärkare



Dioder

Shockleyekvationen

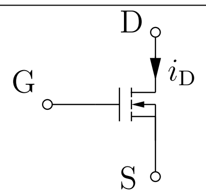
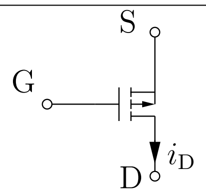
$$i_\mathrm{D} = I_\mathrm{s}\left(\mathrm{e}^{\frac{v_\mathrm{D}}{nV_\mathrm{T}}} - 1\right)$$

där $V_\mathrm{T} = \frac{kT}{q}$, $q \approx 1.6 \cdot 10^{-19}\,\mathrm{C}$ och $k \approx 1.38 \cdot 10^{-23}\,\mathrm{J/K}$.

Dynamisk resistans

$$r_\mathrm{d} = \frac{1}{\left.\frac{\mathrm{d}i_\mathrm{D}}{\mathrm{d}v_\mathrm{D}}\right|_Q}$$

MOSFET

	NMOS	PMOS
Kretssymbol		
$\mu \approx$	$675\,\mathrm{cm^2V^{-1}s^{-1}}$	$240\,\mathrm{cm^2V^{-1}s^{-1}}$
$\kappa \approx$	$115\,\mu\mathrm{AV^{-2}}$	$40\,\mu\mathrm{AV^{-2}}$
$V_\mathrm{t} \approx$	$+0.5\,\mathrm{V}$	$-0.6\,\mathrm{V}$
Subtröskel (strypt område)	$v_\mathrm{GS} \leq V_\mathrm{t}$, $v_\mathrm{DS} \geq 0$, $i_\mathrm{D} = 0$	$v_\mathrm{GS} \geq V_\mathrm{t}$, $v_\mathrm{DS} \leq 0$, $i_\mathrm{D} = 0$
Linjärt område	$v_\mathrm{GS} \geq V_\mathrm{t}$, $0 \leq v_\mathrm{DS} \leq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(2(v_\mathrm{GS} - V_\mathrm{t})v_\mathrm{DS} - v_\mathrm{DS}^2)$	$v_\mathrm{GS} \leq V_\mathrm{t}$, $0 \geq v_\mathrm{DS} \geq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(2(v_\mathrm{GS} - V_\mathrm{t})v_\mathrm{DS} - v_\mathrm{DS}^2)$
Mättnads-område	$v_\mathrm{GS} \geq V_\mathrm{t}$, $v_\mathrm{DS} \geq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(v_\mathrm{GS} - V_\mathrm{t})^2$	$v_\mathrm{GS} \leq V_\mathrm{t}$, $v_\mathrm{DS} \leq v_\mathrm{GS} - V_\mathrm{t}$, $i_\mathrm{D} = K(v_\mathrm{GS} - V_\mathrm{t})^2$
v_DS, v_GS	Vanligtvis positiva	Vanligtvis negativa

$K = \frac{W}{L}\frac{\kappa}{2}$

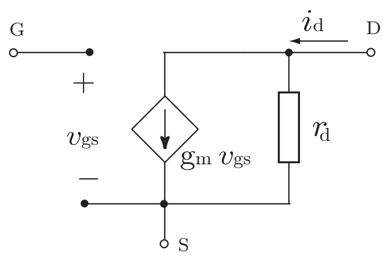
Småsignalmodell

Småsignalmodell för en FET, där

$$g_\mathrm{m} = \left.\frac{\partial i_\mathrm{D}}{\partial v_\mathrm{GS}}\right|_\mathrm{arbetspunkt}$$

och

$$\frac{1}{r_\mathrm{d}} = \left.\frac{\partial i_\mathrm{D}}{\partial v_\mathrm{DS}}\right|_\mathrm{arbetspunkt}$$



Trigonometriska formler

$$\sin\alpha = \cos(\alpha - \pi/2)$$
$$\cos\alpha = \sin(\alpha + \pi/2)$$
$$\cos^2\alpha + \sin^2\alpha = 1$$
$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$
$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$$
$$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$$
$$2\cos\alpha\cos\beta = \cos(\alpha+\beta) + \cos(\alpha-\beta)$$

$$A\cos\alpha + B\sin\alpha = \sqrt{A^2+B^2}\cos(\alpha-\beta) \text{ där } \cos\beta = \frac{A}{\sqrt{A^2+B^2}},\ \sin\beta = \frac{B}{\sqrt{A^2+B^2}}$$

$$\cos\alpha = \frac{\mathrm{e}^{\mathrm{j}\alpha} + \mathrm{e}^{-\mathrm{j}\alpha}}{2}$$
$$\sin\alpha = \frac{\mathrm{e}^{\mathrm{j}\alpha} - \mathrm{e}^{-\mathrm{j}\alpha}}{2\mathrm{j}}$$

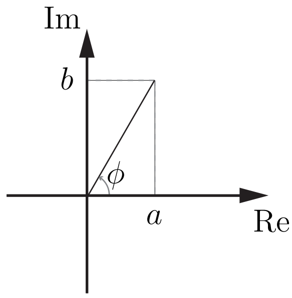
$$\mathrm{e}^{\mathrm{j}\alpha} = \cos\alpha + \mathrm{j}\sin\alpha$$

Komplexa tal

$$z = a + \mathrm{j}b = |z|\mathrm{e}^{\mathrm{j}\phi}$$

där

$$|z| = \sqrt{a^2+b^2} \quad \text{och om } a > 0 \text{ är} \quad \phi = \arctan\frac{b}{a}$$



Ekvationssystem (2×2)

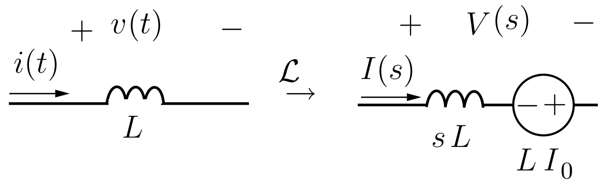
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

med lösning

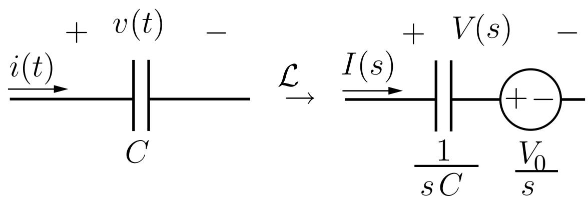
$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$$

Laplacetransformen

Spole med $i(0^-) = I_0$:
$V(s) = L(sI(s) - I_0)$



Kondensator med $v(0^-) = V_0$:
$I(s) = C(sV(s) - V_0)$



	$f(t)$	$F(s)$
1.	$\alpha f(t)$	$\alpha F(s)$
2.	$f_1(t) + f_2(t) + f_3(t) + \cdots$	$F_1(s) + F_2(s) + F_3(s) + \cdots$
3.	$\dfrac{\mathrm{d}f(t)}{\mathrm{d}t}$	$sF(s) - f(0^-)$
4.	$\displaystyle\int_0^t f(\tau)\,\mathrm{d}\tau$	$\dfrac{F(s)}{s}$
5.	$f(t-a)\,\mathrm{u}(t-a), \quad a > 0$	$\mathrm{e}^{-as}F(s)$
6.	$\mathrm{e}^{-at}f(t)$	$F(s+a)$
7.	$f(at), \quad a > 0$	$\dfrac{1}{a}F\left(\dfrac{s}{a}\right)$

Begynnelsevärdessatsen $\quad \displaystyle\lim_{t\to 0_+} f(t) = \lim_{s\to\infty} sF(s)$

Slutvärdessatsen $\quad \displaystyle\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

	$f(t)$	$F(s)$
1.	$\delta(t)$	1
2.	$\dfrac{\mathrm{d}^n}{\mathrm{d}t^n}\,\delta(t)$	s^n
3.	$\mathrm{u}(t)$, enhetssteget	$\dfrac{1}{s}$
4.	$\dfrac{t^n}{n!}\,\mathrm{u}(t)$	$\dfrac{1}{s^{n+1}}$
5.	$\mathrm{e}^{-at}\,\mathrm{u}(t)$	$\dfrac{1}{s+a}$
6.	$\dfrac{t^n}{n!}\,\mathrm{e}^{-at}\,\mathrm{u}(t)$	$\dfrac{1}{(s+a)^{n+1}}$
7.	$\dfrac{\mathrm{e}^{-at}-\mathrm{e}^{-bt}}{b-a}\,\mathrm{u}(t)$	$\dfrac{1}{(s+a)(s+b)}$
8.	$\dfrac{a\mathrm{e}^{-at}-b\mathrm{e}^{-bt}}{a-b}\,\mathrm{u}(t)$	$\dfrac{s}{(s+a)(s+b)}$
9.	$\sin(\omega_0 t)\,\mathrm{u}(t)$	$\dfrac{\omega_0}{s^2+\omega_0^2}$
10.	$\cos(\omega_0 t)\,\mathrm{u}(t)$	$\dfrac{s}{s^2+\omega_0^2}$
11.	$\big(\sin(\omega_0 t)-\omega_0 t\,\cos(\omega_0 t)\big)\,\mathrm{u}(t)$	$\dfrac{2\omega_0^3}{(s^2+\omega_0^2)^2}$
12.	$\omega_0 t\,\sin(\omega_0 t)\,\mathrm{u}(t)$	$\dfrac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$
13.	$\mathrm{e}^{-at}\,\sin(\omega_0 t)\,\mathrm{u}(t)$	$\dfrac{\omega_0}{(s+a)^2+\omega_0^2}$
14.	$\mathrm{e}^{-at}\,\cos(\omega_0 t)\,\mathrm{u}(t)$	$\dfrac{s+a}{(s+a)^2+\omega_0^2}$

TRANSMISSIONSLEDNINGAR

Ledningsekvationerna, förlustfri dubbelledning

$$\begin{aligned} -\frac{\partial v}{\partial z} &= L\,\frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial z} &= C\,\frac{\partial v}{\partial t} \end{aligned}$$

Allmän lösning, förlustfri dubbelledning

$$\begin{aligned} v &= v^+(z - v_\mathrm{p}t) + v^-(z + v_\mathrm{p}t) \\ i &= \frac{1}{Z_0}v^+(z - v_\mathrm{p}t) - \frac{1}{Z_0}v^-(z + v_\mathrm{p}t) \end{aligned}$$

$$v_\mathrm{p} = \frac{1}{\sqrt{LC}} \qquad Z_0 = \sqrt{\frac{L}{C}} \qquad LC = \mu_r\mu_0\varepsilon_r\varepsilon_0$$

Ledningsekvationerna, sinusformigt tidsberoende

$$\begin{aligned} -\frac{\mathrm{d}V}{\mathrm{d}z} &= RI + \mathrm{j}\omega LI \\ -\frac{\mathrm{d}I}{\mathrm{d}z} &= GV + \mathrm{j}\omega CV \end{aligned}$$

Allmän lösning, sinusformigt tidsberoende

$$\begin{aligned} V(z) &= V_1\mathrm{e}^{-\gamma z} + V_2\mathrm{e}^{\gamma z} \\ I(z) &= \frac{1}{Z_0}(V_1\mathrm{e}^{-\gamma z} - V_2\mathrm{e}^{\gamma z}) \end{aligned}$$

Utbredningskonstant

$$\gamma = \sqrt{(R + \mathrm{j}\omega L)(G + \mathrm{j}\omega C)} = \alpha + \mathrm{j}\beta$$

Karakteristisk impedans

$$Z_0 = \sqrt{\frac{R + \mathrm{j}\omega L}{G + \mathrm{j}\omega C}}$$

Impedansen för en dubbelledning med längden l avslutad med Z_L

$$Z_\mathrm{in} = Z_0\frac{Z_L\cosh(\gamma\ell) + Z_0\sinh(\gamma\ell)}{Z_0\cosh(\gamma\ell) + Z_L\sinh(\gamma\ell)} = Z_0\frac{1 + \Gamma\mathrm{e}^{-2\gamma\ell}}{1 - \Gamma\mathrm{e}^{-2\gamma\ell}}$$

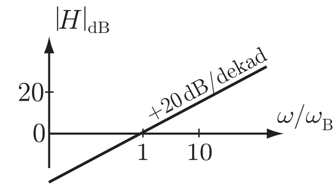
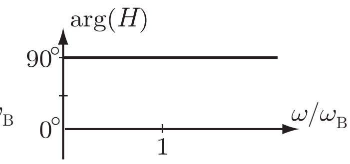
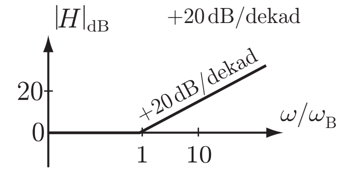
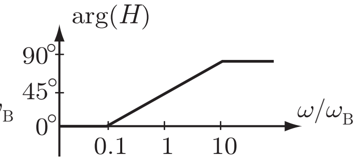
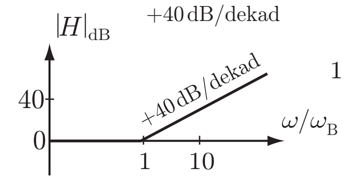
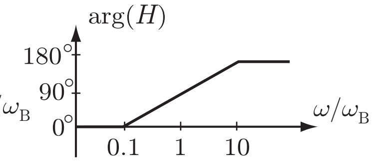
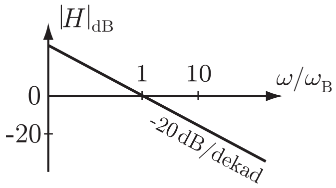
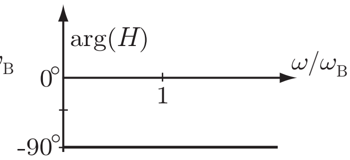
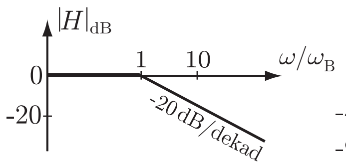
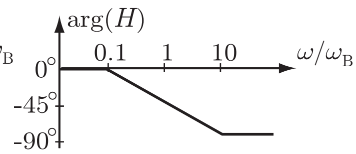
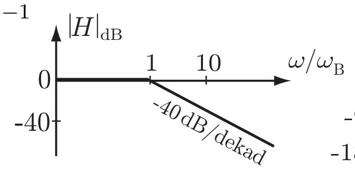
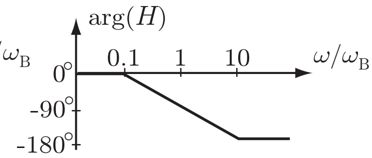
Impedansen för en förlustfri ledning med längden l avslutad med Z_L

$$Z_\mathrm{in} = Z_0\frac{Z_L\cos(\beta l) + \mathrm{j}Z_0\sin(\beta l)}{Z_0\cos(\beta l) + \mathrm{j}Z_L\sin(\beta l)} = Z_0\frac{1 + \Gamma\mathrm{e}^{-\mathrm{j}2\beta\ell}}{1 - \Gamma\mathrm{e}^{-\mathrm{j}2\beta\ell}}$$

Reflektionsfaktorn för spänning vid belastningen

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Rätlinjeapproximationer av Bodediagram

$H(s)$	Amplitud	Fas
$\dfrac{s}{\omega_B}$		
$1+\dfrac{s}{\omega_B}$		
$1+2\zeta\dfrac{s}{\omega_B}+\dfrac{s^2}{\omega_B^2}$		
$\left(\dfrac{s}{\omega_B}\right)^{-1}$		
$\left(1+\dfrac{s}{\omega_B}\right)^{-1}$		
$\left(1+2\zeta\dfrac{s}{\omega_B}+\dfrac{s^2}{\omega_B^2}\right)^{-1}$		

RCL-beräkningar

Kretsparametrarna i fältuttryck:

R	C	L
$R = \dfrac{v_a - v_b}{i}$	$C = \dfrac{q}{v_a - v_b}$	$L = \dfrac{\phi}{i}$
$\int_S \boldsymbol{J} \cdot \boldsymbol{e}_n \, \mathrm{d}S = i$	$\oint_S \boldsymbol{D} \cdot \boldsymbol{e}_n \, \mathrm{d}S = q$	$\int_S \boldsymbol{B} \cdot \boldsymbol{e}_n \, \mathrm{d}S = \phi$
$\int_{P_a}^{P_b} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = v_a - v_b$	$\int_{P_a}^{P_b} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = v_a - v_b$	$\oint_C \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{r} = i$
$\boldsymbol{J} = \sigma \boldsymbol{E}$	$\boldsymbol{D} = \varepsilon_0 \varepsilon_\mathrm{r} \boldsymbol{E}$	$\boldsymbol{B} = \mu_0 \mu_\mathrm{r} \boldsymbol{H}$

Fälten uppfyller följande villkor:

$$\nabla \times \boldsymbol{E} = 0 \quad \Leftrightarrow \quad \boldsymbol{E} = -\nabla V \quad \Leftrightarrow \quad \oint_C \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{r} = 0$$

$$\nabla \cdot \boldsymbol{J} = 0 \quad \Leftrightarrow \quad \oint_S \boldsymbol{J} \cdot \boldsymbol{e}_n \, \mathrm{d}S = 0$$

$$\nabla \cdot \boldsymbol{B} = 0 \quad \Leftrightarrow \quad \oint_S \boldsymbol{B} \cdot \boldsymbol{e}_n \, \mathrm{d}S = 0$$

Kretsparametrarna i effekt- och energiuttryck:

	R	C	L
Krets	$p = Ri^2 = v^2/R$	$w_\mathrm{e} = \dfrac{1}{2} C v^2 = \dfrac{1}{2} q^2/C$	$w_\mathrm{m} = \dfrac{1}{2} L i^2 = \dfrac{1}{2} \phi^2/L$
Fält	$p = \int \boldsymbol{E} \cdot \boldsymbol{J} \, \mathrm{d}V$	$w_\mathrm{e} = \dfrac{1}{2} \displaystyle\int \boldsymbol{D} \cdot \boldsymbol{E} \, \mathrm{d}V$	$w_\mathrm{m} = \dfrac{1}{2} \displaystyle\int \boldsymbol{B} \cdot \boldsymbol{H} \, \mathrm{d}V$