Formelsamling

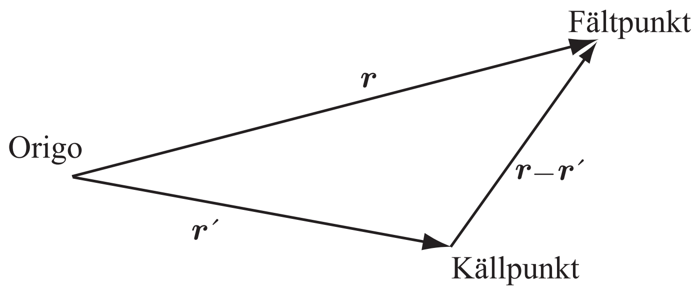
Elektromagnetisk fältteori
för F och Pi

ETE055 & ETEF01

Institutionen för elektro- och informationsteknik
Lunds tekniska högskola
Juni 2014

Innehåll

Elstatik



Coulombs lag

Kraften[1] $\boldsymbol{F}(\boldsymbol{r})$ på en punktladdning q_1 i punkten $\boldsymbol{r}$ orsakad av en punktladdning q i punkten $\boldsymbol{r}'$

$$\boldsymbol{F}(\boldsymbol{r}) = \frac{q_1 q\,(\boldsymbol{r}-\boldsymbol{r}')}{4\pi\varepsilon_0\,|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

Elektrisk fältstyrka $\boldsymbol{E}$ i vakuum

1. från punktladdning med laddning q i $\boldsymbol{r}'$

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q\,(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

2. från volymladdningstäthet ρ i volymen V

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\iiint_V \frac{\rho(\boldsymbol{r}')\,(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}v'$$

3. från ytladdningstäthet ρ_S på ytan S

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\iint_S \frac{\rho_S(\boldsymbol{r}')\,(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}S'$$

4. från linjeladdningstäthet ρ_l på kurvan C

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int_C \frac{\rho_l(\boldsymbol{r}')\,(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}\,\mathrm{d}l'$$

5. från punktdipol $\boldsymbol{p} = p\hat{\boldsymbol{z}}$ i origo

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\hat{\boldsymbol{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)$$

6. från linjeladdning ρ_l

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{\rho_l}{2\pi\varepsilon_0 r_\mathrm{c}}\hat{\boldsymbol{r}}_\mathrm{c}$$

[1] Koordinatbeteckningar, t.ex. ortsvektorn $\boldsymbol{r}$, finns i avsnittet Koordinatsystem på sidan 9.

Kraft $\boldsymbol{F}$ på punktladdning q

1. $\boldsymbol{F} = q\,\boldsymbol{E}$ (gäller i elstatiken)
2. $\boldsymbol{F} = q\,(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ (Lorentz kraftlag)

Elektrisk potential V

$$\boldsymbol{E} = -\boldsymbol{\nabla} V \qquad \text{(gäller i elstatiken)}$$

1. från punktladdning med laddning q i $\boldsymbol{r}'$

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

2. från volymladdningstäthet ρ i volymen V

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\iiint\limits_V \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}v'$$

3. från ytladdningstäthet ρ_S på ytan S

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\iint\limits_S \frac{\rho_S(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}S'$$

4. från linjeladdningstäthet ρ_l på kurvan C

$$V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int\limits_C \frac{\rho_l(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}l'$$

5. från punktdipol $\boldsymbol{p} = p\hat{\boldsymbol{z}}$ i origo

$$V(\boldsymbol{r}) = \frac{\boldsymbol{p}\cdot\boldsymbol{r}}{4\pi\varepsilon_0 r^3} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

6. från linjeladdning ρ_l

$$V(\boldsymbol{r}) = \frac{\rho_l}{2\pi\varepsilon_0}\ln\frac{1}{r_\mathrm{c}}$$

Gauss lag på differential- respektive integralform

$$\begin{cases} \boldsymbol{\nabla}\cdot\boldsymbol{E} = \rho/\varepsilon_0 \\ \displaystyle\iint \boldsymbol{E}\cdot\hat{\boldsymbol{n}}\,\mathrm{d}S = \iiint \rho\,\mathrm{d}v/\varepsilon_0 \end{cases} \quad \text{eller} \quad \begin{cases} \boldsymbol{\nabla}\cdot\boldsymbol{D} = \rho_\mathrm{f} \\ \displaystyle\iint \boldsymbol{D}\cdot\hat{\boldsymbol{n}}\,\mathrm{d}S = \iiint \rho_\mathrm{f}\,\mathrm{d}v \end{cases}$$

där $\hat{\boldsymbol{n}}$ är den från volymen utåtriktade enhetsnormalvektorn.

Polarisation $\boldsymbol{P}$

$$\boldsymbol{P} = \frac{\Delta \boldsymbol{p}}{\Delta v}$$

Samband mellan polarisation $\boldsymbol{P}$, $\boldsymbol{E}$ och $\boldsymbol{D}$

$$\begin{cases} \boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} & \text{(gäller allmänt)} \\ \boldsymbol{D} = \varepsilon_r \varepsilon_0 \boldsymbol{E} \end{cases}$$

Polarisationsladdning, även kallad bunden laddning,

$$\begin{cases} \rho_p = -\boldsymbol{\nabla} \cdot \boldsymbol{P} \text{ bunden volymladdningstäthet} \\ \rho_{pS} = \hat{\boldsymbol{n}}_1 \cdot (\boldsymbol{P}_1 - \boldsymbol{P}_2) \text{ bunden ytladdningstäthet} \end{cases}$$

där enhetsnormalvektorn $\hat{\boldsymbol{n}}_1$ är riktad från område 1 till område 2.

Randvillkor

$$\begin{cases} \boldsymbol{E}_t \text{ kontinuerlig} \\ \rho_S = \hat{\boldsymbol{n}}_2 \cdot (\boldsymbol{D}_1 - \boldsymbol{D}_2) \end{cases} \qquad \begin{cases} \boldsymbol{E}_t \text{ kontinuerlig} \\ \rho_S = \hat{\boldsymbol{n}}_2 \varepsilon_0 \cdot (\varepsilon_{r1} \boldsymbol{E}_1 - \varepsilon_{r2} \boldsymbol{E}_2) \end{cases}$$

där ρ_S är fri ytladdningstäthet, enhetsnormalvektorn $\hat{\boldsymbol{n}}_2$ är riktad från område 2 till område 1.

Elektrostatisk energi W_e

1. för system med diskreta laddningar Q_i

$$W_\mathrm{e} = \frac{1}{2} \sum_i Q_i V_i$$

2. för kontinuerlig laddningsfördelning ρ

$$W_\mathrm{e} = \frac{1}{2} \iiint \rho V \, \mathrm{d}v$$

3. beräknad ur $\boldsymbol{E}$ och $\boldsymbol{D}$

$$W_\mathrm{e} = \frac{\varepsilon_0}{2} \iiint |\boldsymbol{E}|^2 \, \mathrm{d}v \quad \text{eller} \quad W_\mathrm{e} = \frac{1}{2} \iiint \boldsymbol{E} \cdot \boldsymbol{D} \, \mathrm{d}v$$

Vridmoment $\boldsymbol{T}_e$ på elektrisk dipol $\boldsymbol{p}$

$$\boldsymbol{T}_e = \boldsymbol{p} \times \boldsymbol{E}$$

Kraft $\boldsymbol{F}$ på elektrisk dipol $\boldsymbol{p}$

$$\boldsymbol{F} = (\boldsymbol{p} \cdot \nabla) \boldsymbol{E} = \nabla (\boldsymbol{p} \cdot \boldsymbol{E})$$

Spegling

1. Spegling av punktladdning q i ledande sfär med radien a. Punktladdningen q är placerad på avståndet d från centrum av sfären. Punktladdningarna q_i och q_m tänkes placerade i spegelpunkten respektive sfärens centrum.

$$q_i = -q\frac{a}{d} \qquad d_i = \frac{a^2}{d}$$

$$Q_s = q_i + q_m \quad V_s = \frac{q_m}{4\pi\varepsilon_0 a}$$

2. Spegling av linjeladdning ρ_l i ledande cylinder med radien a och med laddning per längdenhet $-\rho_l$. Linjeladdningen ρ_l är placerad på avståndet d från cylinderaxeln.

$$\rho_i = -\rho_l \qquad d_i = \frac{a^2}{d}$$

Likström

Strömtäthet $\boldsymbol{J}$

$$I = \iint \boldsymbol{J} \cdot \hat{\boldsymbol{n}} \, \mathrm{d}S$$

Kontinuitetsekvationen på differential- respektive integralform

$$\begin{aligned} \boldsymbol{\nabla} \cdot \boldsymbol{J} + \frac{\partial \rho}{\partial t} &= 0 \\ \iint \boldsymbol{J} \cdot \hat{\boldsymbol{n}} \, \mathrm{d}S &= -\frac{dQ}{dt} \end{aligned}$$

Ohms lag

$$\boldsymbol{J} = \sigma \boldsymbol{E}$$

Effekt P

$$P = \iiint \boldsymbol{J} \cdot \boldsymbol{E} \, \mathrm{d}v$$

där σ är materialets lednigsförmåga.

Randvillkor

$$\begin{cases} \hat{\boldsymbol{n}}_2 \cdot (\boldsymbol{J}_1 - \boldsymbol{J}_2) = 0 & \text{(ingen ytström)} \\ \boldsymbol{E}_{t1} = \boldsymbol{E}_{t2} \end{cases}$$

Magnetostatik

Magnetisk flödestäthet B i vakuum

1. från punktdipol $\boldsymbol{m} = m\,\hat{\boldsymbol{z}}$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)$$

2. från strömtäthet $\boldsymbol{J}(\boldsymbol{r}')$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}\,\mathrm{d}v'$$

3. från strömbana

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{I d\boldsymbol{l}' \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3}$$

4. från cirkulär trådslinga

$$\boldsymbol{B}(x = 0, y = 0, z) = \frac{\mu_0 I}{2}\frac{b^2}{(b^2 + z^2)^{3/2}}\,\hat{\boldsymbol{z}}$$

5. från lång rak strömbana

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0 I}{2\pi r_{\mathrm{c}}}\,\hat{\boldsymbol{\phi}}$$

Vektorpotential A i vakuum

1. från strömtäthet $\boldsymbol{J}(\boldsymbol{r}')$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}\,\mathrm{d}v'$$

2. från strömbana

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int \frac{I\,d\boldsymbol{l}'}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

3. från lång rak strömbana

$$\boldsymbol{A} = \frac{\mu_0 I}{2\pi}\ln(\frac{1}{r})\hat{\boldsymbol{z}}$$

4. från punktdipol $\boldsymbol{m}$

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m} \times \boldsymbol{r}}{r^3}$$

Magnetiskt flöde Φ

$$\Phi = \iint \boldsymbol{B} \cdot \hat{\boldsymbol{n}}\,\mathrm{d}S = \oint \boldsymbol{A} \cdot d\boldsymbol{l}$$

Självinduktans L och ömsesidig induktans M

$$\begin{cases} \Phi_1 = L_1 I_1 + M I_2 \\ \Phi_2 = L_2 I_2 + M I_1 \end{cases}$$

Magnetisk fältstyrka $\boldsymbol{H}$

Samband mellan magnetisering $\boldsymbol{M}$, $\boldsymbol{B}$ och $\boldsymbol{H}$

$$\begin{cases} \boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \text{ (gäller allmänt)} \\ \boldsymbol{B} = \mu_r \mu_0 \boldsymbol{H} \end{cases}$$

Ampères lag

$$\begin{cases} \boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \\ \oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I \end{cases} \quad \text{eller} \quad \begin{cases} \boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J}_\mathrm{f} \\ \oint \boldsymbol{H} \cdot d\boldsymbol{l} = I_\mathrm{f} \end{cases}$$

Ekvivalent strömtäthet

$$\begin{aligned} \boldsymbol{J}_m &= \boldsymbol{\nabla} \times \boldsymbol{M} && \text{volymströmtäthet} \\ \boldsymbol{J}_{mS} &= \boldsymbol{M} \times \hat{\boldsymbol{n}} && \text{ytströmtäthet} \end{aligned}$$

Randvillkor

$$\begin{cases} \hat{\boldsymbol{n}}_2 \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = \boldsymbol{J}_s \\ \boldsymbol{B}_n \text{ kontinuerlig} \end{cases}$$

Magnetiska kraftlagen

$$\mathrm{d}\boldsymbol{F}_m = I \; \mathrm{d}\boldsymbol{l} \times \boldsymbol{B}$$

Magnetiskt moment $\boldsymbol{m}$ för strömslinga

$$\boldsymbol{m} = \iint I \, \hat{\boldsymbol{n}} \; \mathrm{d}S$$

Vridmoment $\boldsymbol{T}_m$ på magnetisk dipol $\boldsymbol{m}$

$$\boldsymbol{T}_m = \boldsymbol{m} \times \boldsymbol{B}$$

Kraft $\boldsymbol{F}$ på magnetisk dipol $\boldsymbol{m}$

$$\boldsymbol{F} = (\boldsymbol{m} \cdot \nabla)\boldsymbol{B} + \boldsymbol{m} \times (\nabla \times \boldsymbol{B}) = \nabla(\boldsymbol{m} \cdot \boldsymbol{B})$$

Magnetisk energi

$$W_\mathrm{m} = \frac{1}{2} \iiint \boldsymbol{J} \cdot \boldsymbol{A} \; \mathrm{d}v = \frac{1}{2} \iiint \boldsymbol{B} \cdot \boldsymbol{H} \; \mathrm{d}v = \frac{1}{2} \sum_i \sum_j L_{ij} I_i I_j$$

Magnetisk energi, två spolar

$$W_\mathrm{m} = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M I_1 I_2$$

Elektromagnetiska fält

Induktionslagen

$$R\,I = -\frac{d\Phi}{dt}$$

Inducerad emk $\mathcal{E}$

$$\begin{aligned} \mathcal{E} &= \oint (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot d\boldsymbol{l} \\ \mathcal{E} &= -\frac{d\Phi}{dt} \end{aligned}$$

Induktionslagen på differential- respektive integralform

$$\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\ \oint \boldsymbol{E} \cdot d\boldsymbol{l} &= -\iint \frac{\partial \boldsymbol{B}}{\partial t} \cdot \hat{\boldsymbol{n}}\, \mathrm{d}S \end{aligned}$$

Maxwells ekvationer

$$\begin{aligned} \boldsymbol{\nabla} \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\ \boldsymbol{\nabla} \times \boldsymbol{H} &= \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \\ \boldsymbol{\nabla} \cdot \boldsymbol{D} &= \rho \\ \boldsymbol{\nabla} \cdot \boldsymbol{B} &= 0 \end{aligned}$$

Konstanter

$$\mu_0 = 4\pi \cdot 10^{-7}\,\text{H/m} \qquad \varepsilon_0 \approx \frac{10^{-9}}{36\pi}\,\text{F/m} \qquad c_0 \approx 3 \cdot 10^8\,\text{m/s}$$

$$\frac{1}{\mu_0 \varepsilon_0} = c_0^2 \qquad \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad \eta_0 \approx 120\pi\,\Omega \approx 377\,\Omega$$

Potentialer

$$\begin{cases} \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} \\ \boldsymbol{E} = -\boldsymbol{\nabla} V - \dfrac{\partial \boldsymbol{A}}{\partial t} \end{cases}$$

$$V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho\left(\boldsymbol{r}', t - \left|\boldsymbol{r}-\boldsymbol{r}'\right|/c_0\right)}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|} \,\mathrm{d}v'$$

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi} \iiint \frac{\boldsymbol{J}\left(\boldsymbol{r}', t - \left|\boldsymbol{r}-\boldsymbol{r}'\right|/c_0\right)}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|} \,\mathrm{d}v'$$

Poyntings vektor

$$\boldsymbol{S}(\boldsymbol{r},t) = \boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)$$

Tidsharmoniska fält

Plan, tidsharmonisk våg

$$\begin{cases} \boldsymbol{E}\cdot\hat{\boldsymbol{x}} = E_x = E_{0x}\cos(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t + \phi), \quad \text{ögonblicksvärde för komponent} \\ \boldsymbol{E} = \boldsymbol{E}_0\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}, \quad \text{komplexvärde} \\ \boldsymbol{E}_0\cdot\hat{\boldsymbol{x}} = E_{0x}\mathrm{e}^{\mathrm{i}\phi} \end{cases}$$

Utbredningshastighet

$$v = \frac{1}{\sqrt{\mu_r\mu_0\varepsilon_r\varepsilon_0}} = \frac{\omega}{k} \qquad k = |\boldsymbol{k}|$$

Vågimpedans, oledande rymd

$$\eta = \sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}}$$

Komplexa strålningsvektorn

$$\boldsymbol{S}(\boldsymbol{r}) = \frac{1}{2}\left[\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}^*(\boldsymbol{r})\right]$$

Några vektoridentiteter

1. $\boldsymbol{A}\cdot(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}\cdot(\boldsymbol{C}\times\boldsymbol{A}) = \boldsymbol{C}\cdot(\boldsymbol{A}\times\boldsymbol{B})$
2. $\boldsymbol{A}\times(\boldsymbol{B}\times\boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A}\cdot\boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A}\cdot\boldsymbol{B})$
3. $\boldsymbol{\nabla}(\psi V) = \psi\boldsymbol{\nabla}V + V\boldsymbol{\nabla}\psi$
4. $\boldsymbol{\nabla}\cdot(\psi\boldsymbol{A}) = \psi\boldsymbol{\nabla}\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\boldsymbol{\nabla}\psi$
5. $\boldsymbol{\nabla}\times(\psi\boldsymbol{A}) = \psi\boldsymbol{\nabla}\times\boldsymbol{A} + \boldsymbol{\nabla}\psi\times\boldsymbol{A}$
6. $\boldsymbol{\nabla}\cdot(\boldsymbol{A}\times\boldsymbol{B}) = \boldsymbol{B}\cdot(\boldsymbol{\nabla}\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\boldsymbol{\nabla}\times\boldsymbol{B})$
7. $\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}V = \nabla^2 V = \Delta V$

8. $\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{A} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \boldsymbol{A}) - \boldsymbol{\nabla}^2 \boldsymbol{A}$
9. $\boldsymbol{\nabla} \times \boldsymbol{\nabla} V = 0$
10. $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{A}) = 0$
11. $\iiint_V \boldsymbol{\nabla} \cdot \boldsymbol{A} \; \mathrm{d}v = \iint_S \boldsymbol{A} \cdot \hat{\boldsymbol{n}} \; \mathrm{d}S$ Gauss sats
12. $\iiint_V (\psi \nabla^2 \varphi - \varphi \nabla^2 \psi) \; \mathrm{d}v = \iint_S (\psi \nabla \varphi - \varphi \nabla \psi) \cdot \hat{\boldsymbol{n}} \; \mathrm{d}S$ Greens formel
13. $\iint_S \; (\boldsymbol{\nabla} \times \boldsymbol{A}) \cdot \hat{\boldsymbol{n}} \; \mathrm{d}S = \oint_C \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$ Stokes sats
14. $\boldsymbol{\nabla} \left(\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} \right) = -\boldsymbol{\nabla}' \left(\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|} \right) = -\frac{\boldsymbol{r}-\boldsymbol{r}'}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$

Koordinatsystem

Kartesiska koordinater (x, y, z)

Ortsvektor $\boldsymbol{r} \; = x\,\hat{\boldsymbol{x}} + y\,\hat{\boldsymbol{y}} + z\,\hat{\boldsymbol{z}}$

Linjeelement $\mathrm{d}\boldsymbol{l} = \mathrm{d}x\,\hat{\boldsymbol{x}} + \mathrm{d}y\,\hat{\boldsymbol{y}} + \mathrm{d}z\,\hat{\boldsymbol{z}}$

Volymelement $\mathrm{d}v = \mathrm{d}x \; \mathrm{d}y \; \mathrm{d}z$

Differentialoperatorer

$$\begin{aligned}
\boldsymbol{\nabla} V &= \hat{\boldsymbol{x}} \frac{\partial V}{\partial x} + \hat{\boldsymbol{y}} \frac{\partial V}{\partial y} + \hat{\boldsymbol{z}} \frac{\partial V}{\partial z} \\
\boldsymbol{\nabla} \cdot \boldsymbol{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\boldsymbol{\nabla} \times \boldsymbol{A} &= \hat{\boldsymbol{x}} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{\boldsymbol{y}} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{\boldsymbol{z}} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\
\boldsymbol{\nabla}^2 V &= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}
\end{aligned}$$

Cylinderkoordinater $(r_{\mathrm{c}}, \phi, z)$

Ortsvektor $\boldsymbol{r} \; = r_{\mathrm{c}}\,\hat{\boldsymbol{r}}_{\mathrm{c}} + z\,\hat{\boldsymbol{z}}$

Linjeelement $\mathrm{d}\boldsymbol{l} = \mathrm{d}r_{\mathrm{c}}\,\hat{\boldsymbol{r}}_{\mathrm{c}} + r_{\mathrm{c}} \; \mathrm{d}\phi\,\hat{\boldsymbol{\phi}} + \mathrm{d}z\,\hat{\boldsymbol{z}}$

Volymelement $\mathrm{d}v = r_{\mathrm{c}} \; \mathrm{d}r_{\mathrm{c}} \; \mathrm{d}\phi \, dz$

Differentialoperatorer

$$
\begin{aligned}
\nabla V &= \hat{\boldsymbol{r}}_\mathrm{c}\frac{\partial V}{\partial r_\mathrm{c}} + \hat{\boldsymbol{\phi}}\frac{1}{r_\mathrm{c}}\frac{\partial V}{\partial \phi} + \hat{\boldsymbol{z}}\frac{\partial V}{\partial z} \\
\nabla \cdot \boldsymbol{A} &= \frac{1}{r_\mathrm{c}}\frac{\partial}{\partial r_\mathrm{c}}(r_\mathrm{c}A_{r_\mathrm{c}}) + \frac{1}{r_\mathrm{c}}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\
\nabla \times \boldsymbol{A} &= \hat{\boldsymbol{r}}_\mathrm{c}\left(\frac{1}{r_\mathrm{c}}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_{r_\mathrm{c}}}{\partial z} - \frac{\partial A_z}{\partial r_\mathrm{c}}\right) \\
&\quad + \hat{\boldsymbol{z}}\frac{1}{r_\mathrm{c}}\left[\frac{\partial}{\partial r_\mathrm{c}}(r_\mathrm{c}A_\phi) - \frac{\partial A_{r_\mathrm{c}}}{\partial \phi}\right] \\
\nabla^2 V &= \frac{1}{r_\mathrm{c}}\frac{\partial}{\partial r_\mathrm{c}}\left(r_\mathrm{c}\frac{\partial V}{\partial r_\mathrm{c}}\right) + \frac{1}{r_\mathrm{c}^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}
\end{aligned}
$$

Sfäriska koordinater (r, θ, ϕ)

Ortsvektor $\boldsymbol{r} = r\,\hat{\boldsymbol{r}}$

Linjeelement $\mathrm{d}\boldsymbol{l} = \mathrm{d}r\,\hat{\boldsymbol{r}} + r\,\mathrm{d}\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,\mathrm{d}\phi\,\hat{\boldsymbol{\phi}}$

Volymelement $\mathrm{d}v = r^2\sin\theta\ \mathrm{d}r\ \mathrm{d}\theta\ \mathrm{d}\phi$

Differentialoperatorer

$$
\nabla V = \hat{\boldsymbol{r}}\frac{\partial V}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial V}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}
$$

$$
\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}
$$

$$
\begin{aligned}
\nabla \times \boldsymbol{A} &= \hat{\boldsymbol{r}}\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right] \\
&\quad + \hat{\boldsymbol{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]
\end{aligned}
$$

$$
\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}
$$

Samband mellan basvektorer

$(r, \theta, \phi) \longrightarrow (x, y, z)$

$$
\begin{cases}
\hat{\boldsymbol{r}} = \hat{\boldsymbol{x}}\sin\theta\cos\phi + \hat{\boldsymbol{y}}\sin\theta\sin\phi + \hat{\boldsymbol{z}}\cos\theta \\
\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{x}}\cos\theta\cos\phi + \hat{\boldsymbol{y}}\cos\theta\sin\phi - \hat{\boldsymbol{z}}\sin\theta \\
\hat{\boldsymbol{\phi}} = -\hat{\boldsymbol{x}}\sin\phi + \hat{\boldsymbol{y}}\cos\phi
\end{cases}
$$

$(x, y, z) \longrightarrow (r, \theta, \phi)$

$$\begin{cases} \hat{\boldsymbol{x}} = \hat{\boldsymbol{r}} \sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi \\ \hat{\boldsymbol{y}} = \hat{\boldsymbol{r}} \sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi \\ \hat{\boldsymbol{z}} = \hat{\boldsymbol{r}} \cos\theta \qquad\quad - \hat{\boldsymbol{\theta}}\sin\theta \end{cases}$$

$(r_\mathrm{c}, \phi, z) \longrightarrow (x, y, z)$

$$\begin{cases} \hat{\boldsymbol{r}}_\mathrm{c} = \hat{\boldsymbol{x}}\cos\phi + \hat{\boldsymbol{y}}\sin\phi = (\hat{\boldsymbol{x}}x + \hat{\boldsymbol{y}}y)/\sqrt{x^2+y^2} \\ \hat{\boldsymbol{\phi}} = -\hat{\boldsymbol{x}}\sin\phi + \hat{\boldsymbol{y}}\cos\phi = (-\hat{\boldsymbol{x}}y + \hat{\boldsymbol{y}}x)/\sqrt{x^2+y^2} \\ \hat{\boldsymbol{z}} = \hat{\boldsymbol{z}} \end{cases}$$

$(x, y, z) \longrightarrow (r_\mathrm{c}, \phi, z)$

$$\begin{cases} \hat{\boldsymbol{x}} = \hat{\boldsymbol{r}}_\mathrm{c}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi \\ \hat{\boldsymbol{y}} = \hat{\boldsymbol{r}}_\mathrm{c}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi \\ \hat{\boldsymbol{z}} = \hat{\boldsymbol{z}} \end{cases}$$

$(r, \theta, \phi) \longrightarrow (r_\mathrm{c}, \phi, z)$

$$\begin{cases} \hat{\boldsymbol{r}} = \hat{\boldsymbol{r}}_\mathrm{c}\sin\theta + \hat{\boldsymbol{z}}\cos\theta \\ \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{r}}_\mathrm{c}\cos\theta - \hat{\boldsymbol{z}}\sin\theta \\ \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \end{cases}$$

$(r_\mathrm{c}, \phi, z) \longrightarrow (r, \theta, \phi)$

$$\begin{cases} \hat{\boldsymbol{r}}_\mathrm{c} = \hat{\boldsymbol{r}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta \\ \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \\ \hat{\boldsymbol{z}} = \hat{\boldsymbol{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta \end{cases}$$

Några integraler

1. $\displaystyle\int x^n \,\mathrm{d}x = \frac{x^{n+1}}{n+1}, \qquad n \neq -1$

2. $\displaystyle\int \frac{1}{x}\,\mathrm{d}x = \ln x$

3. $\displaystyle\int \sqrt{x^2+a^2}\,\mathrm{d}x = \frac{1}{2}\left[x\sqrt{x^2+a^2} + a^2\ln\left(x+\sqrt{x^2+a^2}\right)\right]$

4. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$

5. $\displaystyle\int \frac{\mathrm{d}x}{(x^2+a^2)^{3/2}} = \frac{x}{a^2\sqrt{x^2+a^2}}$

6. $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a}$

7. $\displaystyle\int \frac{\mathrm{d}x}{x^2 + a^2} = \frac{1}{a}\arctan\frac{x}{a}$

8. $\displaystyle\int \frac{\mathrm{d}x}{\cos^2 x} = \tan x$

9. $\displaystyle\int \frac{\mathrm{d}x}{\sin x} = \ln|\tan\frac{x}{2}|$

10. $\displaystyle\int \ln x \; \mathrm{d}x = x\ln x - x$

Binomialutveckling

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \dots$$

Några trigonometriska formler

1. $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
2. $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
3. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
4. $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$
5. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
6. $\sin 2\alpha = 2\sin\alpha\cos\alpha$
7. $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$
8. $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$
9. $\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
10. $\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
11. $\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$
12. $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$
13. $\begin{cases} a\sin t + b\cos t = \sqrt{a^2+b^2}\,\sin(t+\phi) \\ \sin\phi = \dfrac{b}{\sqrt{a^2+b^2}}\ ,\ \cos\phi = \dfrac{a}{\sqrt{a^2+b^2}} \end{cases}$