EITP30 Modern Wireless Systems - 5G and Beyond

Carrier Frequency Offset

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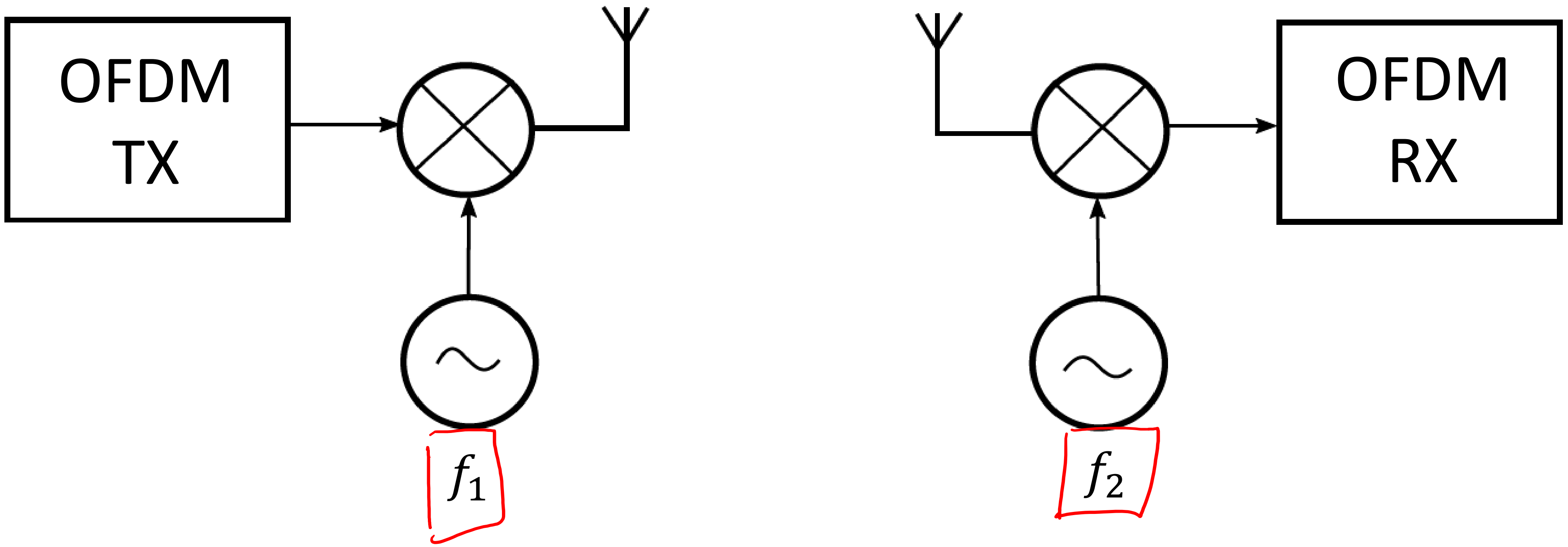
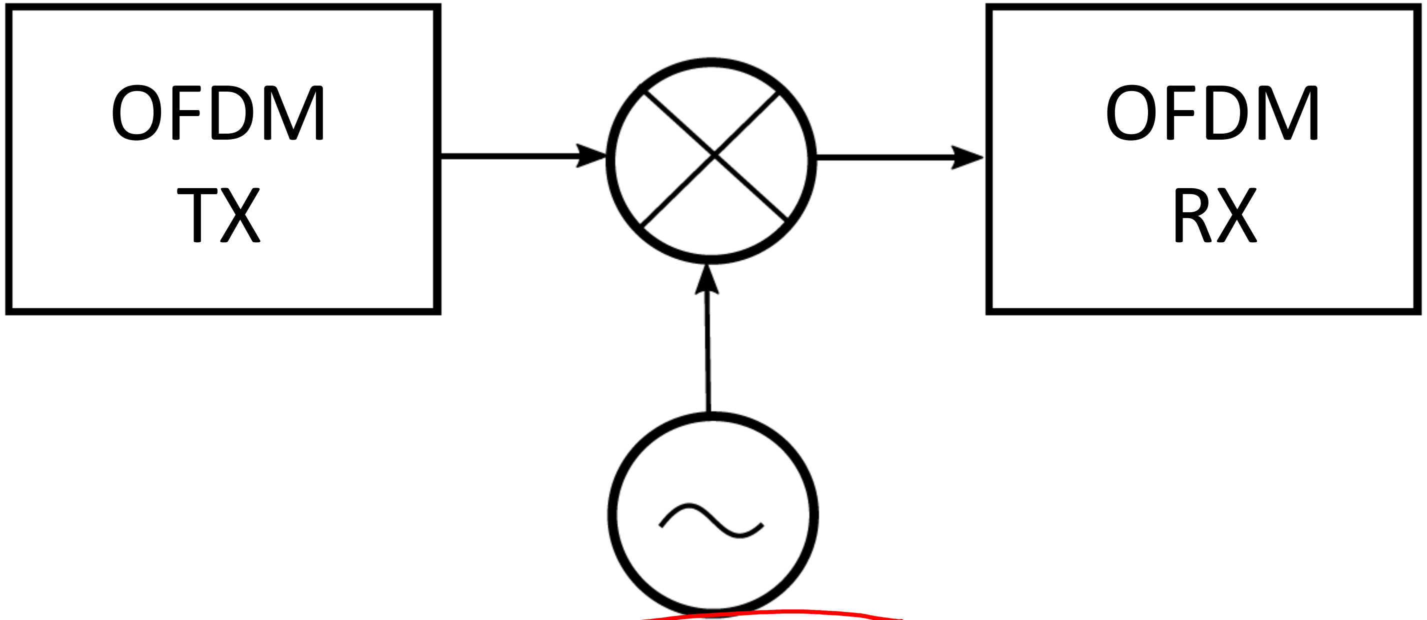
Carrier Frequency Offset (CFO)

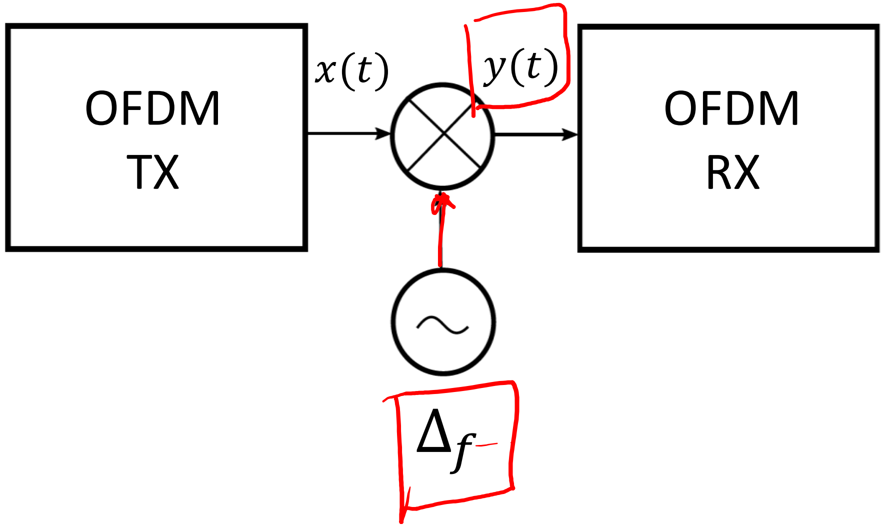


CFO: Equivalent scenario



$\Delta_f = f_2 - f_1$

CFO effect on transmitted symbol



IDFT: $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$

DFT: $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$

If one active subcarrier:
$X[m] = a$ (e.g. QPSK)
$X[i] = 0, i \neq m$

$i = 0, \dots, m, \dots, N-1$ (N: FFT size)

$x[n] = \frac{a}{N} e^{j\frac{2\pi}{N}mn}$

$y[n] = x[n]\, e^{j\frac{2\pi\Delta_f n}{f_s}}$

$x[n] = \frac{1}{N} X[m] \exp\left(j\frac{2\pi kn}{N}\right)$, $X[m] = a$

complex number

$Y[m] = a \cdot \frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi\Delta_f n}{f_s}}$

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CFO effect on transmitted symbol

$$y[n] = x[n]\exp\left(\frac{j2\pi\Delta f n}{f_s}\right)$$

$$\forall \alpha \in \mathbb{Z}$$

$$* \sum_{n=0}^{N-1} \exp\left(\frac{j2\pi\alpha n}{N}\right) = 0$$

$$Y[k] = \sum_{n=0}^{N-1} x[n]\exp\left(\frac{j2\pi\Delta f n}{f_s}\right)\exp\left(-j\frac{2\pi k n}{N}\right)$$

$$= \sum_{n=0}^{N-1} \frac{a}{N}\exp\left(\frac{j2\pi\Delta f n}{f_s}\right)\exp\left(\frac{j2\pi m n}{N}\right)\exp\left(-j\frac{2\pi k n}{N}\right)$$

$$Y[k] = \sum_{n=0}^{N-1} \frac{a}{N}\exp\left(\frac{j2\pi\Delta f n}{f_s}\right)\exp\left(\frac{j2\pi n(m-k)}{N}\right) \rightarrow \text{; when } m = k,\ \exp(0) = 1$$

$$\rightarrow Y[m] = \sum_{n=0}^{N-1} \frac{a}{N}\exp\left(\frac{j2\pi\Delta f n}{f_s}\right), \quad Y[i] = 0, \ i \neq m$$

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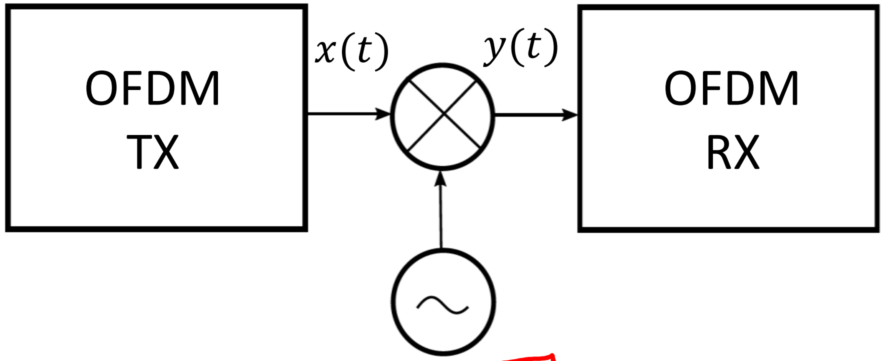
Small CFO approximation



$$\Delta_f \ll \frac{f_s}{N}$$

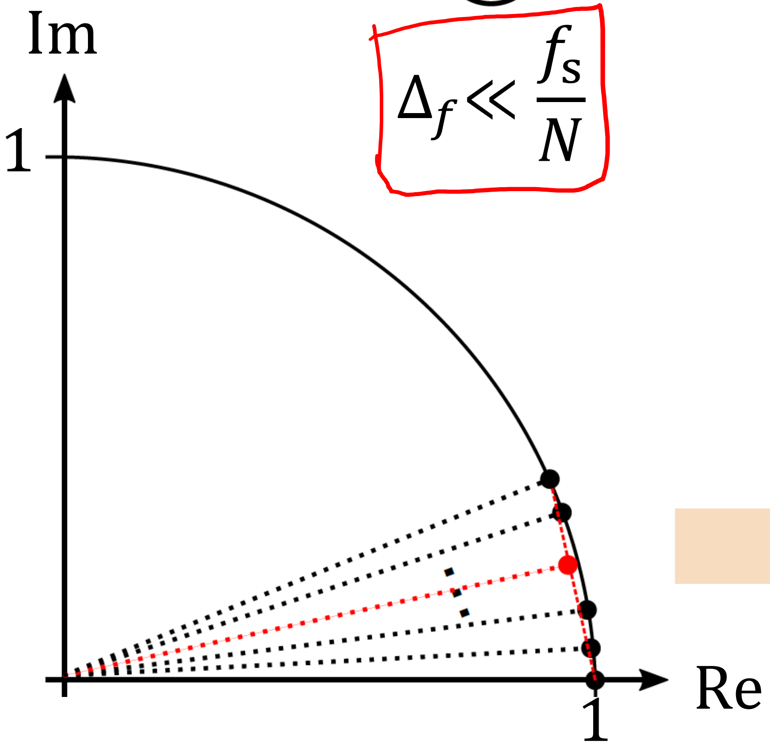
$$Y[m] = a \cdot \underbrace{\frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi\Delta_f n}{f_s}}}_{\text{average}}$$

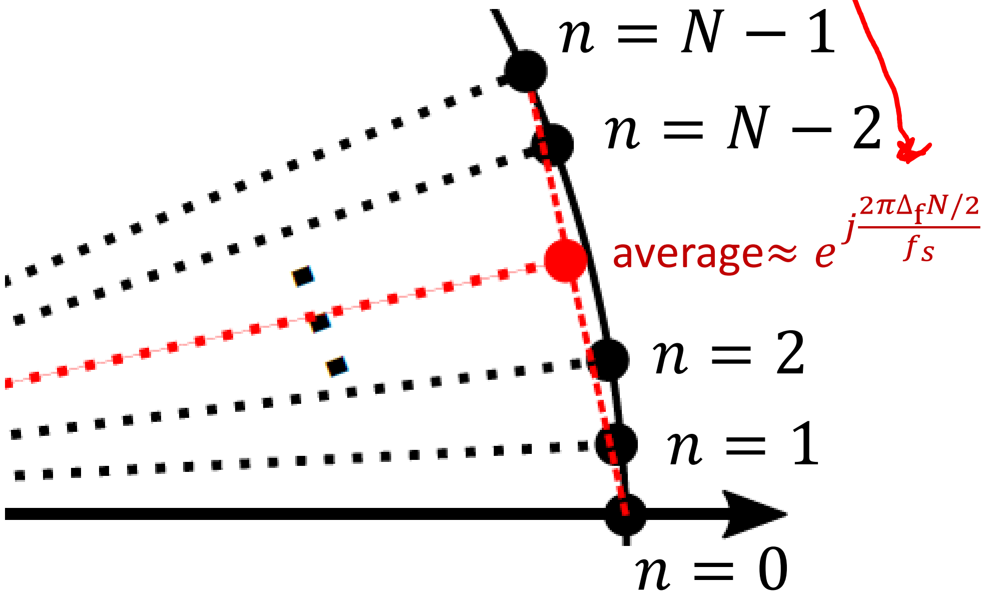
$n = 0, 1, \ldots, N-1$

$\Delta_f N \ll f_s$

$\Delta_f \ll \frac{f_s}{N}$

$Y[m] = a \cdot \exp\left(\frac{j\pi\Delta_f N}{f_s}\right)$





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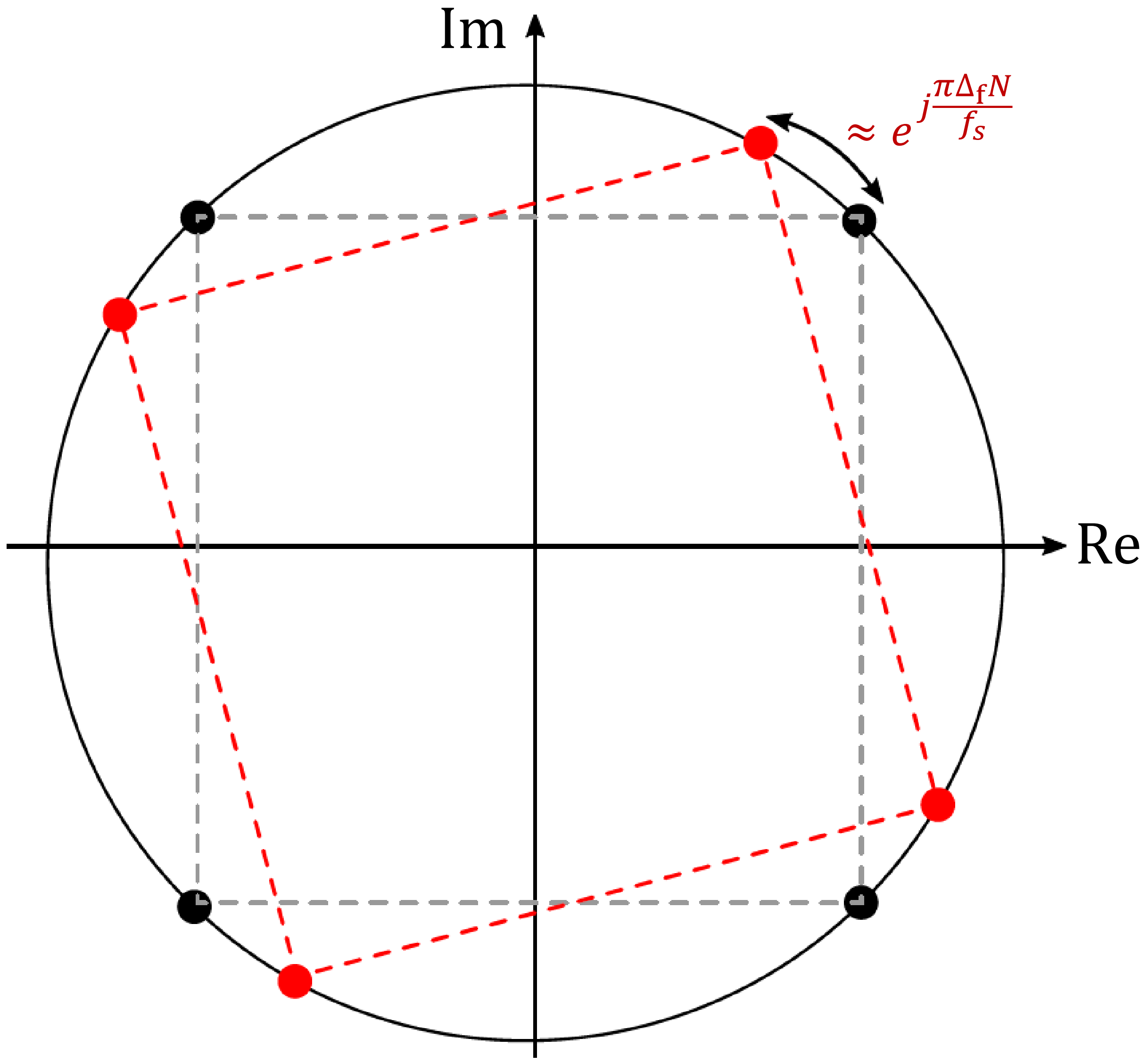
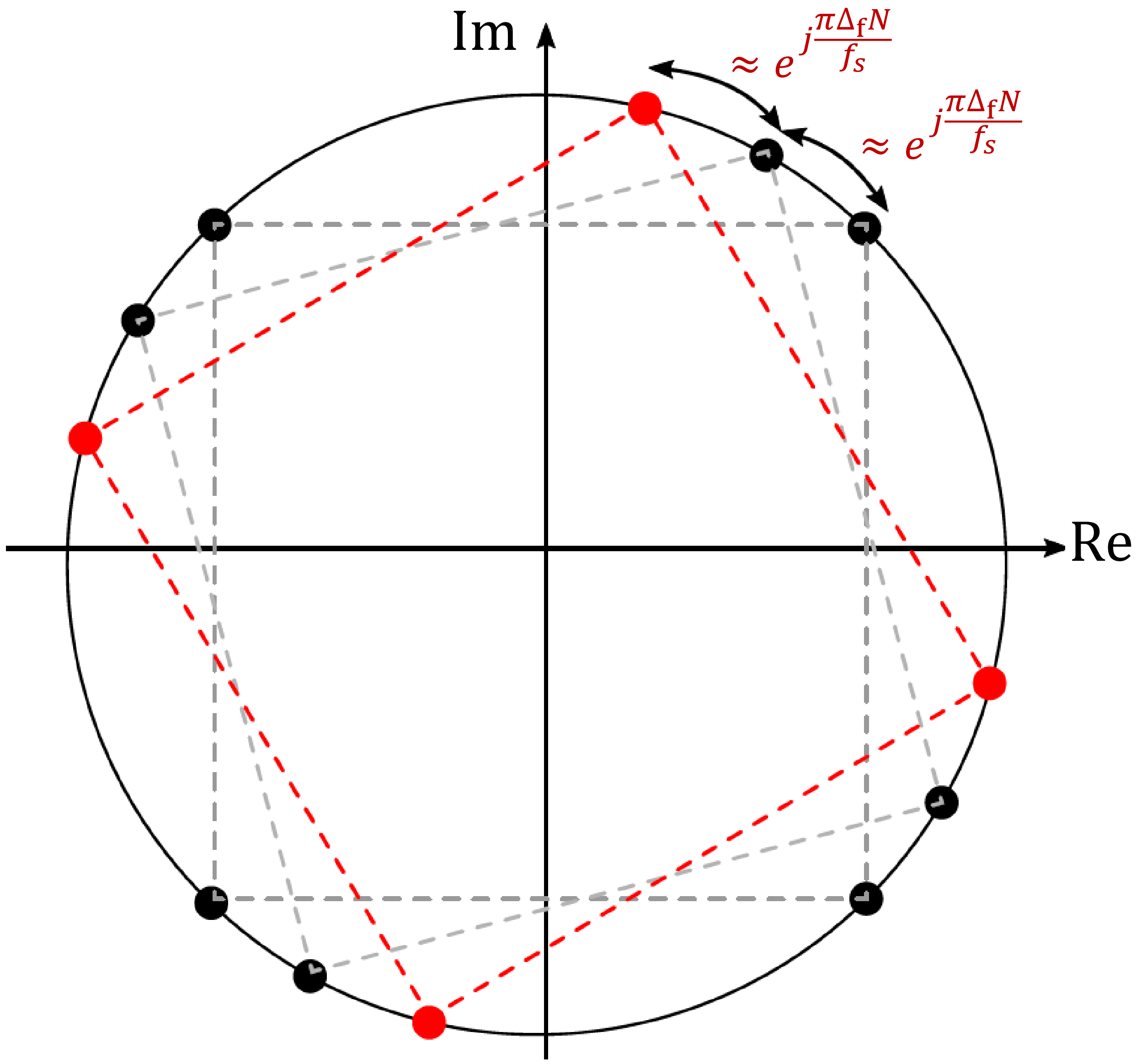
Rotation 1 OFDM symbol



Rotation 2 OFDM symbols



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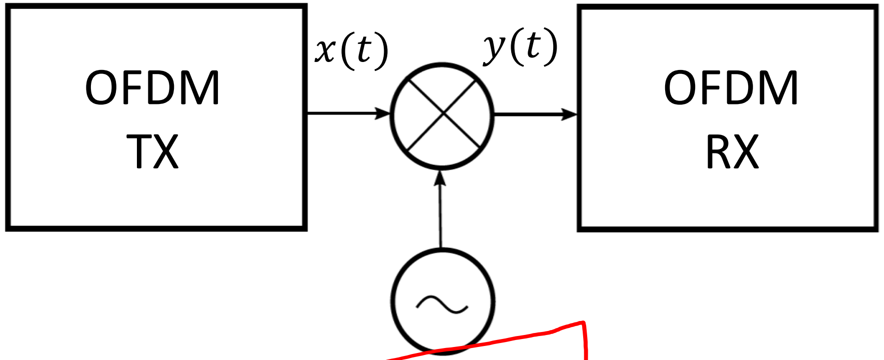
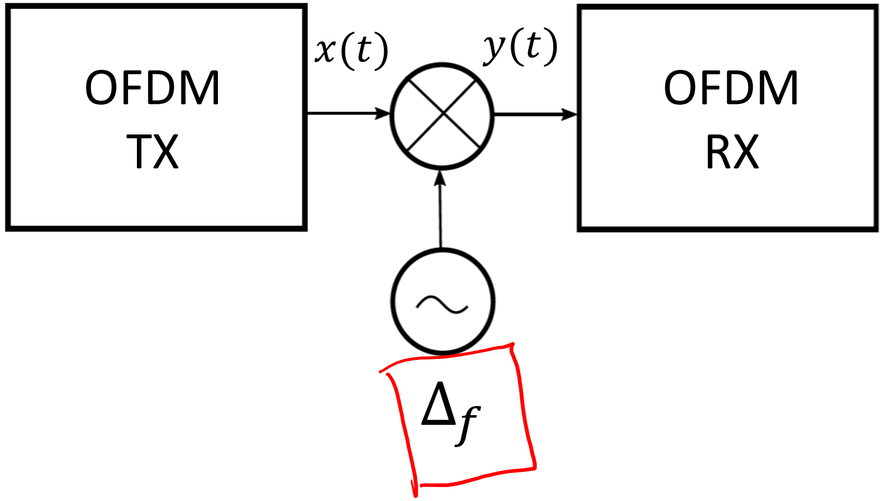
Rotation N_{symb} OFDM symbols



$$\Delta_f \ll \frac{f_s}{N}$$

$$\text{rotation} \approx \frac{\pi \Delta_\mathrm{f} N}{f_s} \cdot N_{symb}$$

Frequency stability



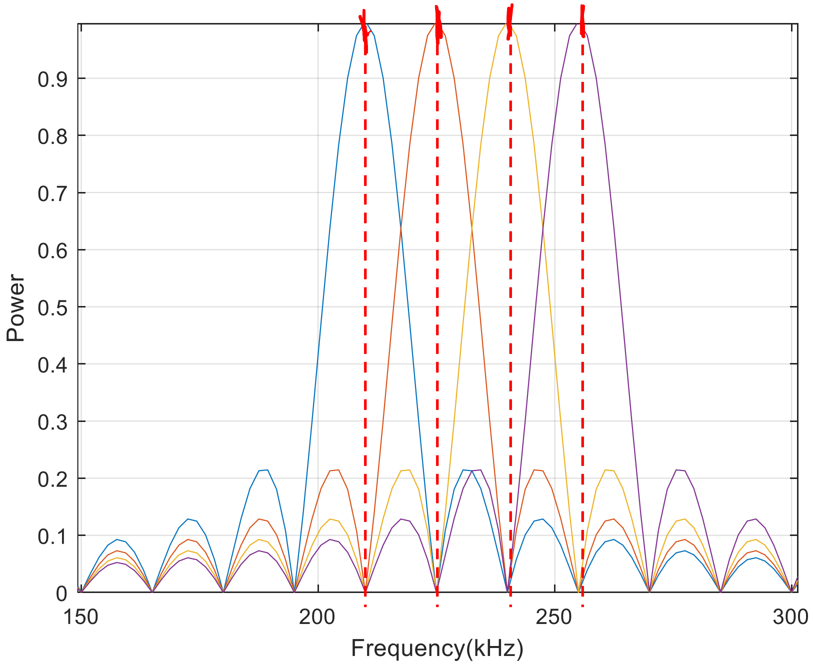
@28GHz

$$\Delta_f = \frac{f_c \cdot \text{ppm}}{10^6}$$

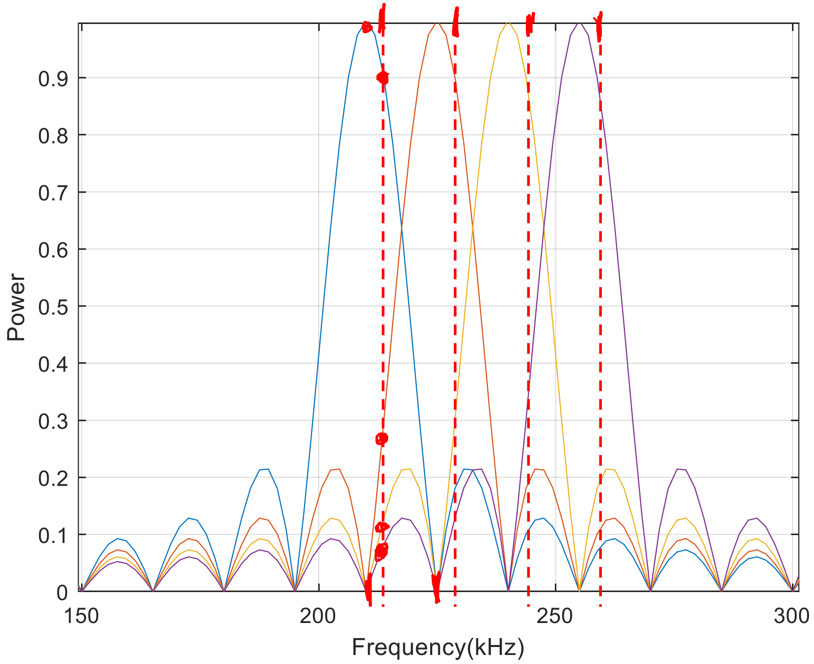
ppm: Parts per million

CFO: Frequency sampling

CFO = 0



CFO > 0



Potential issues?

Questions:

- Discuss about the possible cause of CFO.
- Potential issues of CFO.
- Who shall compensate it (eNB/UE)? How?
- For $f_c = 2\text{GHz}, \text{UE frequency stability} = 0.5\text{ppm}$, and LTE, calculate:
  - Maximum number of OFDM symbols, for BPSK, QPSK, and 16QAM, that the receiver can support without CFO compensation.
- Consequences of a CFO close in value to subcarrier spacing.