# OFDM (part 2)

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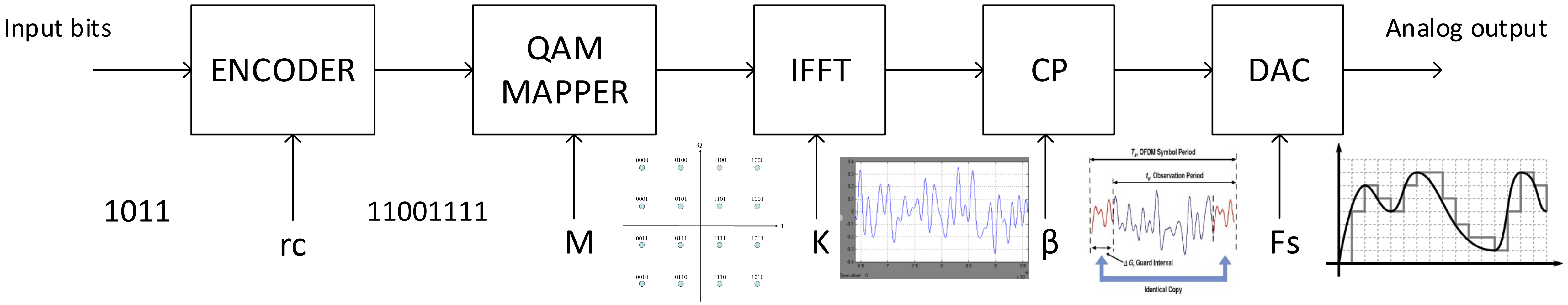
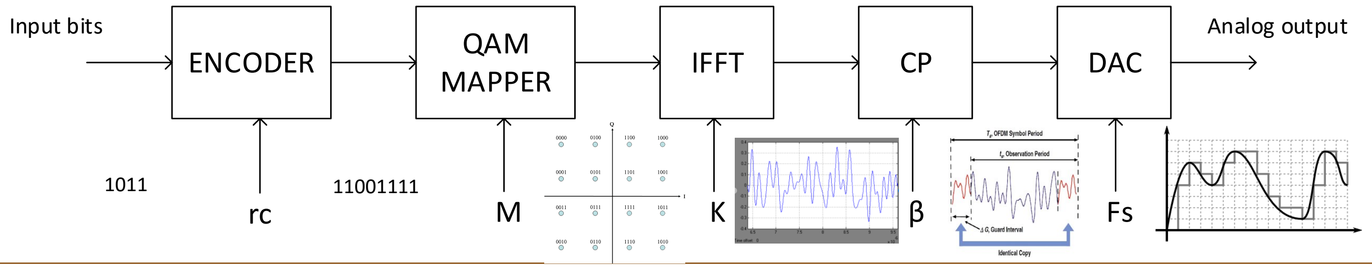
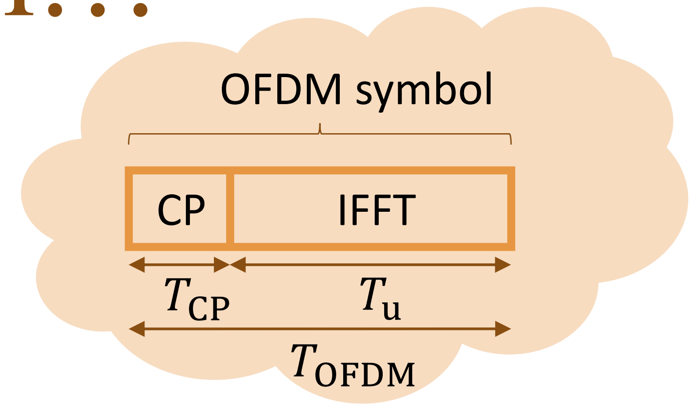
# Recalling from last session…



# Recalling from last session…




LTE example:

- $r_c$ = 1/3
- K = 1200
- M = 16
- $\Delta_f$ = 15KHz
- $T_{\mathrm{CP}}$ = 5us

$R$ and $BW_{tx}$?

$$R = \frac{\frac{1}{3}\cdot 1200 \cdot \log_2(16)\ bits}{\frac{1ms}{14}} = 22.4Mbps$$

$$BW_{tx} = 1200 \cdot 15kHz = 18MHz$$

$$BW_{channel} = \frac{BW_{tx}}{0.9} = \frac{18MHz}{0.9} = 20MHz$$

$$R = \frac{r_c K \log_2(M)}{T_{\mathrm{OFDM}}}$$

$$BW_{tx} = K\Delta_f$$

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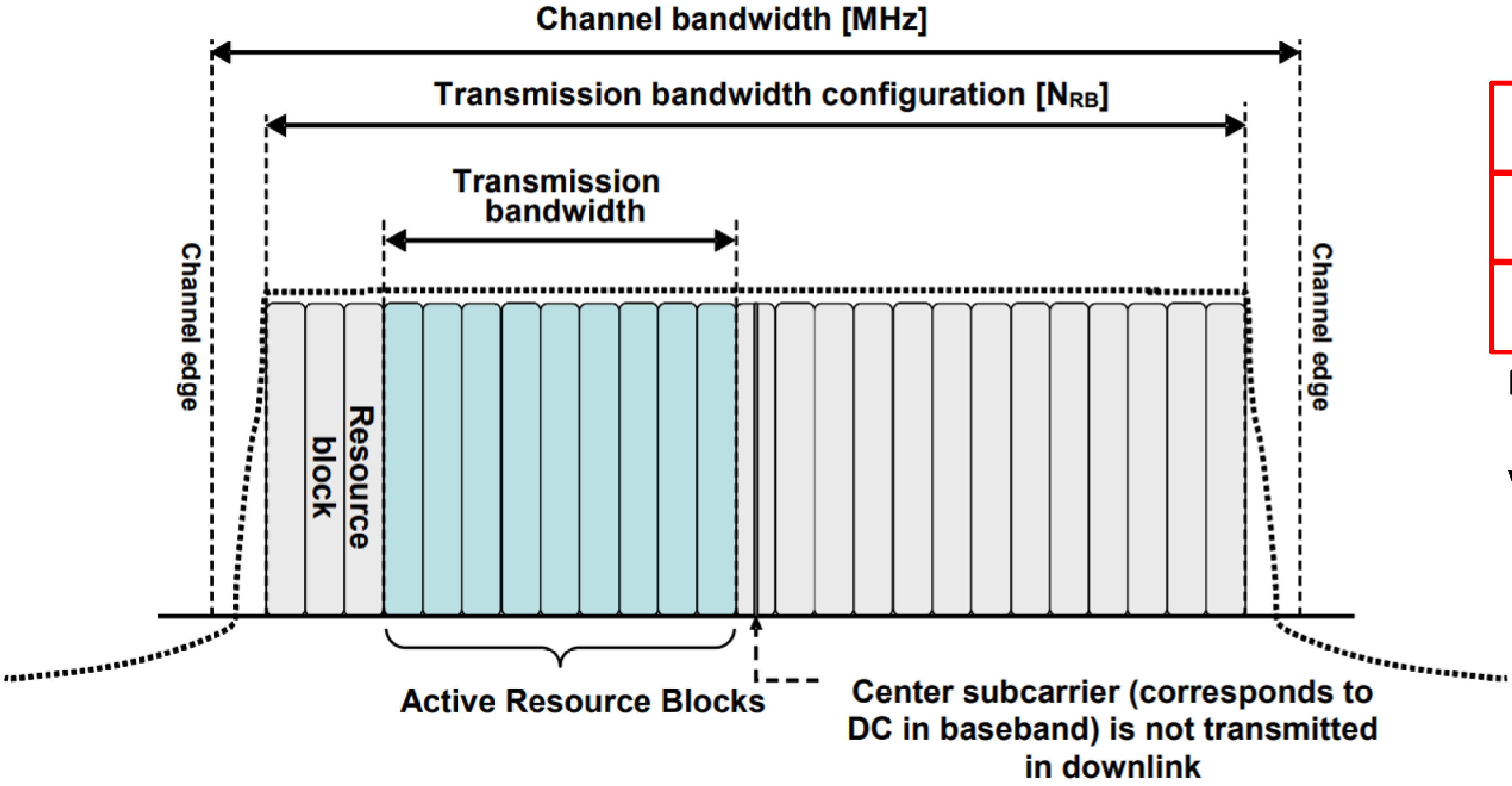
# $BW_{tx}$ and $BW_{channel}$

Transmission bandwidth configuration $N_{RB}$

| Channel bandwidth $BW_{Channel}$ [MHz] | 1.4 | 3 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|---|
| Transmission bandwidth configuration $N_{RB}$ | 6 | 15 | 25 | 50 | 75 | 100 |




$$N_{SC}^{RB} = 12$$

$$K = N_{SC}^{RB} N_{RB}$$

$$BW_{tx} = N_{SC}^{RB} N_{RB} \Delta_f$$

For

$BW_{channel} \geq 3MHz,$

we have

$BW_{tx} = 0.9BW_{channel}$

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# Review Exercise Set 01:

1. a) Estimate the number of sub-carriers in 3GPP LTE in case of a 10 MHz transmission bandwidth. Also determine the total number of coded bits carried by one OFDM signal if all sub-carriers are modulated with 64-QAM.

   b) In LTE, a transmitted OFDM signal duration typically is approximately equal to 71.7 $\mu s$, and the sub-carrier spacing is 15 $kHz$. Does this mean that the different sub-carriers are orthogonal over the entire OFDM symbol interval?

2. How many coded bits per second can be transmitted from an antenna within an LTE resource-block pair if we assume that 16-QAM is used in all sub-carriers? Is the obtained value reasonable for the uplink for a Category 3 terminal?

3. How many coded bits per second can be sent in LTE within a 20 MHz bandwidth if we assume that 64-QAM is used in all sub-carriers, and that eight transmitting and receiving antennas are used. Assume also normal cyclic prefix.

4. What is the required bandwidth to provide the peak rate of UE Category 9 devices?

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1. a) Estimate the number of sub-carriers in 3GPP LTE in case of a 10 MHz transmission bandwidth. Also determine the total number of coded bits carried by one OFDM signal if all sub-carriers are modulated with 64-QAM.

$BW_{tx} = 10\,MHz$ → $K = \frac{BW_{tx}}{15\,kHz} = \frac{10\,MHz}{15\,kHz} \approx 667$ SCs

$\Delta f = 15\,kHz$ for LTE

$M = 64$

$\#\text{coded bits} = K \log_2(M) = (667)(6) \approx 4002$ bits

$BW_{channel} = 10\,MHz$ $N_{RB} = 50$ $N_{sc}^{RB} = 12$ $K = (50)(12) = 600$ SC

$\#\text{coded bits} = (600)(6) = 3600$ bits

$R_{coded} = \frac{\#\text{coded bits}}{T_{OFDM}} = \frac{3600}{\left(\frac{T_{sf}}{14}\right)} = 50.4\,Mbps$

$T_{sf} = 1\,ms$

14 OFDM symbols of 1ms

$T_{cp}$ $T_u$

OFDM

$T_{OFDM} = \frac{1ms}{14}$

- https://www.etsi.org/deliver/etsi_ts/136100_136199/136101/15.14.00_60/ts_136101v151400p.pdf
- https://www.etsi.org/deliver/etsi_ts/136200_136299/136211/15.12.00_60/ts_136211v151200p.pdf

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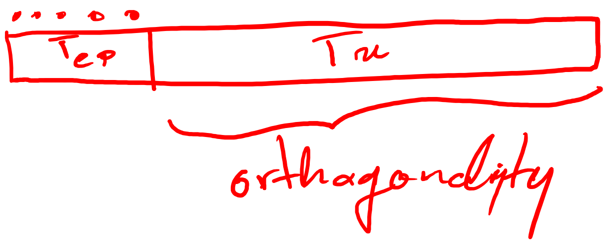
1. b) In LTE, a transmitted OFDM signal duration typically is approximately equal to 71.7 $\mu s$, 429 μs
and the sub-carrier spacing is 15 $kHz$. Does this mean that the different sub-carriers are orthogonal over the entire OFDM symbol interval?

No, orthogonality is ensured over the useful symbol time

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2. How many coded bits per second can be transmitted from an antenna within an LTE resource-block pair if we assume that 16-QAM is used in all sub-carriers?
Is the obtained value reasonable for the uplink for a Category 3 terminal?

$$R_{coded} = \frac{K \log_2(M)\, N_{sym}^{sf}}{T_{sf}}$$

$$T_{OFDM} = \frac{T_{sf}}{N_{sym}^{sf}}$$

$K = 12$ $T_{sf} = 1\,ms$

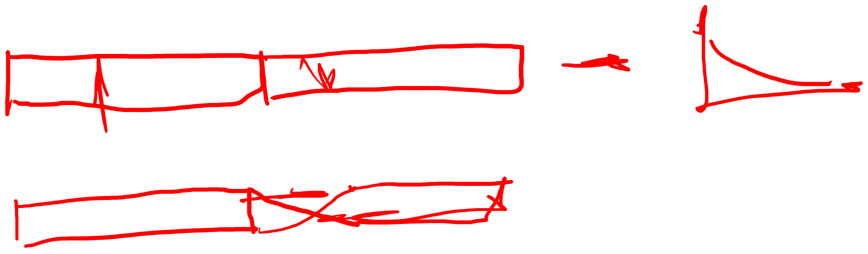
$M = 16$

if $N_{sym}^{sf} = 14$ (normal CP)

$R_{coded} = 672\,kbps$

if $N_{sym}^{sf} = 12$ (extended CP)

$R_{coded} = 576\,kbps$



- https://www.3gpp.org/keywords-acronyms/1612-ue-category

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2. According to 3GPP 36.306 table 4.1-2 we have

\# max UL-SCH transport block bits transmitted within a TTI (1ms) = 51024 bits

$T_{sf}$ = 1ms = 1 Transmission Time Interval

↳ $R_{data,cat3}$ (uncoded) $= \frac{51024 \text{ bits}}{1 \text{ms}} = 51.024$ Mbps

For 12 SCs, $R_{coded}$ = 672 kbps is reasonable for Cat3 UE (User Terminal)

For 1200 SCs, $R_{coded}$ = 67.2 Mbps

if $r = 1$, $R_{uncoded} = R_{coded}$

if $r = 1/2$ ↳ each info bit one more bit of redundancy

$R_{uncoded} = \frac{67.2 \text{ Mbps}}{2} = 33.6$ Mbps

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3. How many coded bits per second can be sent in LTE within a 20 MHz bandwidth if we assume that 64-QAM is used in all sub-carriers, and that eight transmitting and receiving antennas are used. Assume also normal cyclic prefix.

$M = 64 \rightarrow$ 64-QAM

$L = 8$ layers of information

$B_{channel} = 20\,MHz$

$B_{tx} = 18\,MHz \rightarrow K = 1200$ SCs

$N_{sym}^{sf} = 14$, $T_{sf} = 1\,ms$

$$R_{coded} = \frac{L K \log_2(M)}{T_{sym}} = \frac{L K \log_2(M) N_{sym}^{sf}}{T_{sf}} = 806.4\,Mbps$$

What is the required bandwidth to provide the peak rate of UE Category 9 devices?

# 4.

$R_{cat9} = 452.256$ Mbps → DL functioning

Under 64-QAM (max modulation order allowed for a Cat-9 UE based on 36.306)

we have in 36.213 tables 7.1.7.1-1, 7.1.7.2.1-1

that TBS_L1 = 75 376 bits per TTI → $R_{max}^{layer} = 75.376$ Mbps

Then, $R_{max}^{20MHz, 4\ layers} = 4 \cdot R_{max}^{layer} = 301.504$ Mbps is not enough, we gotta use Carrier Aggregation

20 MHz ———— 301.504 Mbps

$BW_{cat9}$ ———— 452.256 Mbps

$BW_{cat9} = 30$ MHz

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- https://www.etsi.org/deliver/etsi_ts/136200_136299/136213/15.11.00_60/ts_136213v151100p.pdf

5. What is the required coding rate $r_c$, number of subcarriers $K$ and the aggregated transmitted bandwidth $BW_{tx,total}$ to provide the peak rate for UE Category 9 devices in DL? Does everything agree with the calculations in exercise number 4? Hint: Consider the maximum rate per layer, number of layers, subcarriers, and constellation order for one LTE carrier on Category 9 devices.

* Assume normal CP → $N_{symb}^{sf} = 7$

First, recall from exercise 4 that, $R_{cat9} = 452.256$ Mbps, $L_{cat9} = 4$, $M_{cat9} = 64$

and that, from 36.213 tables 7.1.7.1-1 and 7.1.7.2.1-1,

we get TBS_L1 = 75376 → $R_{max}^{layer} = 75.376$ Mbps

and the maximum (uncoded) rate of all possible layers ($L_{cat9} = 4$) is

$$R_{max} = L_{cat9} \cdot R_{max}^{layer} = 301.504 \text{ Mbps}$$

→ for 1 component carrier (CC) in LTE (where 20MHz is the max $BW_{channel}^{CC}$) → $K_{max} = 1200$

We can derive the coding rate considering $R_{max}$ and our uncoded rate equation

$$R_{max} = \frac{r_{c,max} \cdot K_{max} \cdot L_{cat9} \cdot \log_2(M_{cat9})}{T_{symb}} \rightarrow r_{c,max} = \frac{R_{max} \cdot T_{sf}}{K_{max} \cdot L_{cat9} \cdot \log_2(M_{cat9}) \cdot N_{symb}^{sf}}$$

→ $r_{c,max} \approx 0.7478$

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5. Now from $R_{cat4} = \frac{r_{c,max} K_{cat4} L_{cat4} \log_2(M_{cat4}) N_{symb}^{sf}}{T_{sf}}$, we can find $K_{cat4}$

$\rightarrow K_{cat4} = \frac{R_{cat4} \cdot T_{sf}}{r_{c,max} \cdot L_{cat4} \cdot \log_2(M_{cat4}) \cdot N_{symb}^{sf}} = \boxed{1800}$. Since $K_{cat4} > K_{max} = 1200$, Carrier Aggregation (CA) is required (another carrier or carriers with max 20MHz BW)

$\rightarrow BW_{tx,total} = K_{cat4} \cdot \Delta f \approx \boxed{27\,MHz}$, with $\Delta f = 15\,kHz$

and $BW_{cat4} = \frac{BW_{tx,total}}{0.9}$, since for $BW_{channel} \geq 3\,MHz$, 10% allocated to guard bands

$\rightarrow \boxed{BW_{cat4} \approx 30\,MHz}$

we have two carriers here

↳ 2 channels → 1 of 20MHz, 1 of 10MHz } 30 MHz

↳ Calculation of the aggregated bandwidth for Cat4 devices, $BW_{cat4}$ agrees for both exercises 4 and 5 !!

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# Questions: Group discussion

$$R = \frac{r_c K \log_2(M)}{T_{\mathrm{OFDM}}}$$

- $T_{\mathrm{OFDM}} = \frac{1}{\Delta_{\mathrm{f}}}$?
- How to increase K while keeping $BW_{tx}$ constant?
- $R$ and $BW_{tx}$? $R$ and $BW_{channel}$?
- Maximum transmission rate?
- Maximum achievable rate (correctly received)?

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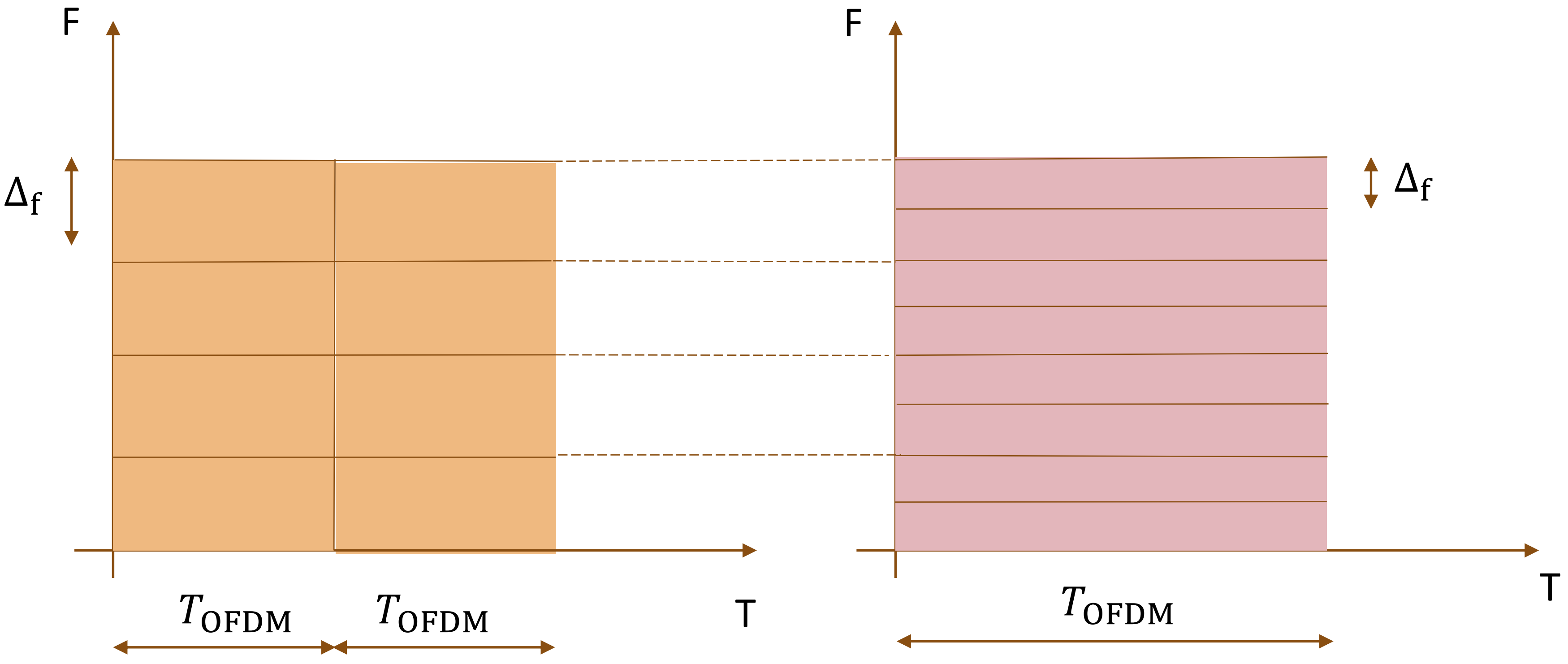
# Questions

$$R = \frac{r_c K \log_2(M)}{T_{\mathrm{OFDM}}}$$

Increase K while keeping $BW_{tx}$ constant?




$$2xK \rightarrow \frac{\Delta_f}{2} \rightarrow 2xT_u.\ If\ 2xT_{CP} \rightarrow 2xT_{\mathrm{OFDM}} \rightarrow = R$$

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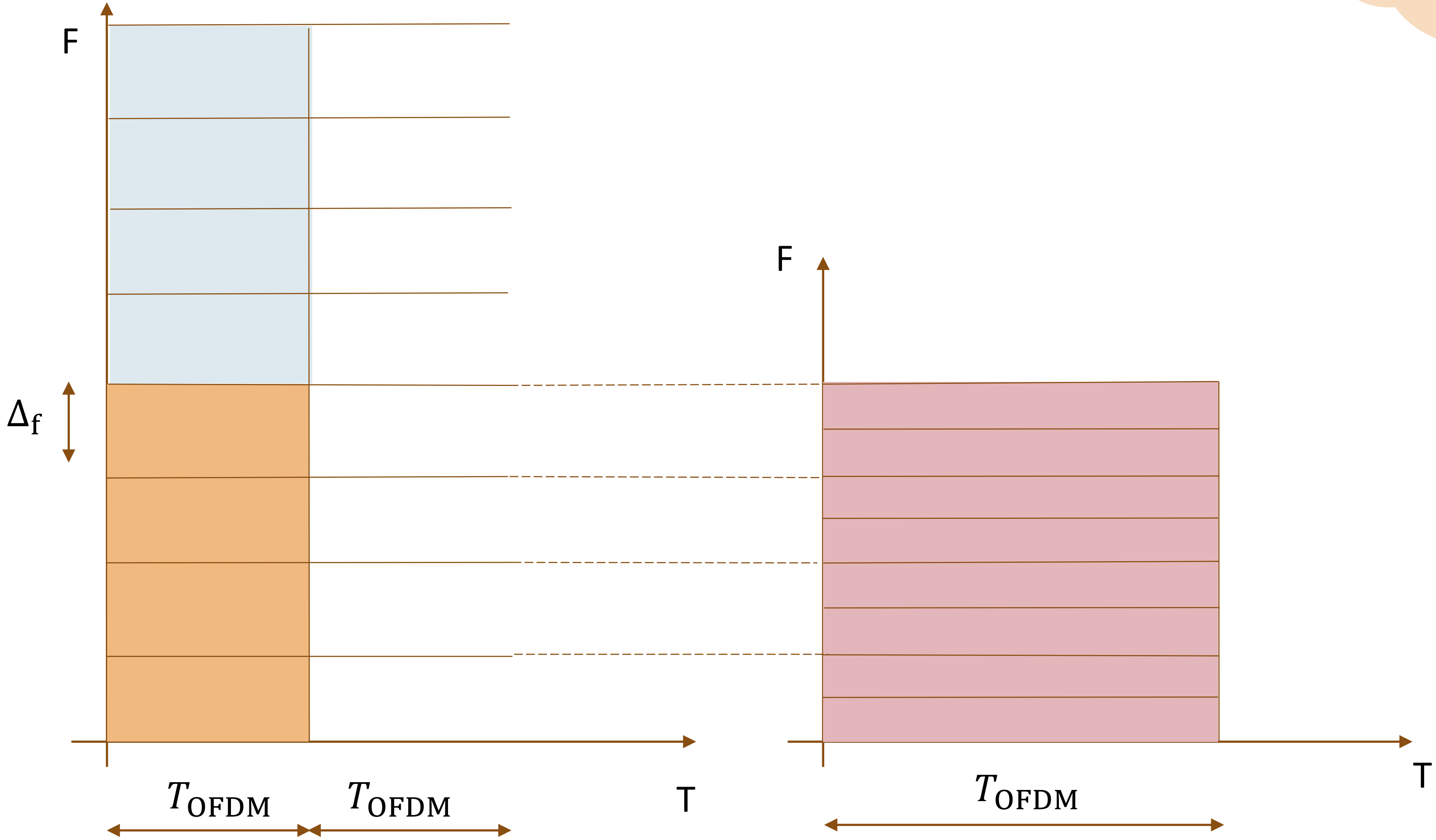
# Questions

$R$ and $BW_{tx}$? $R$ and $BW_{channel}$?

$$R = \frac{r_c K \log_2(M)}{T_{\text{OFDM}}}$$

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# Time structure: LTE, NR

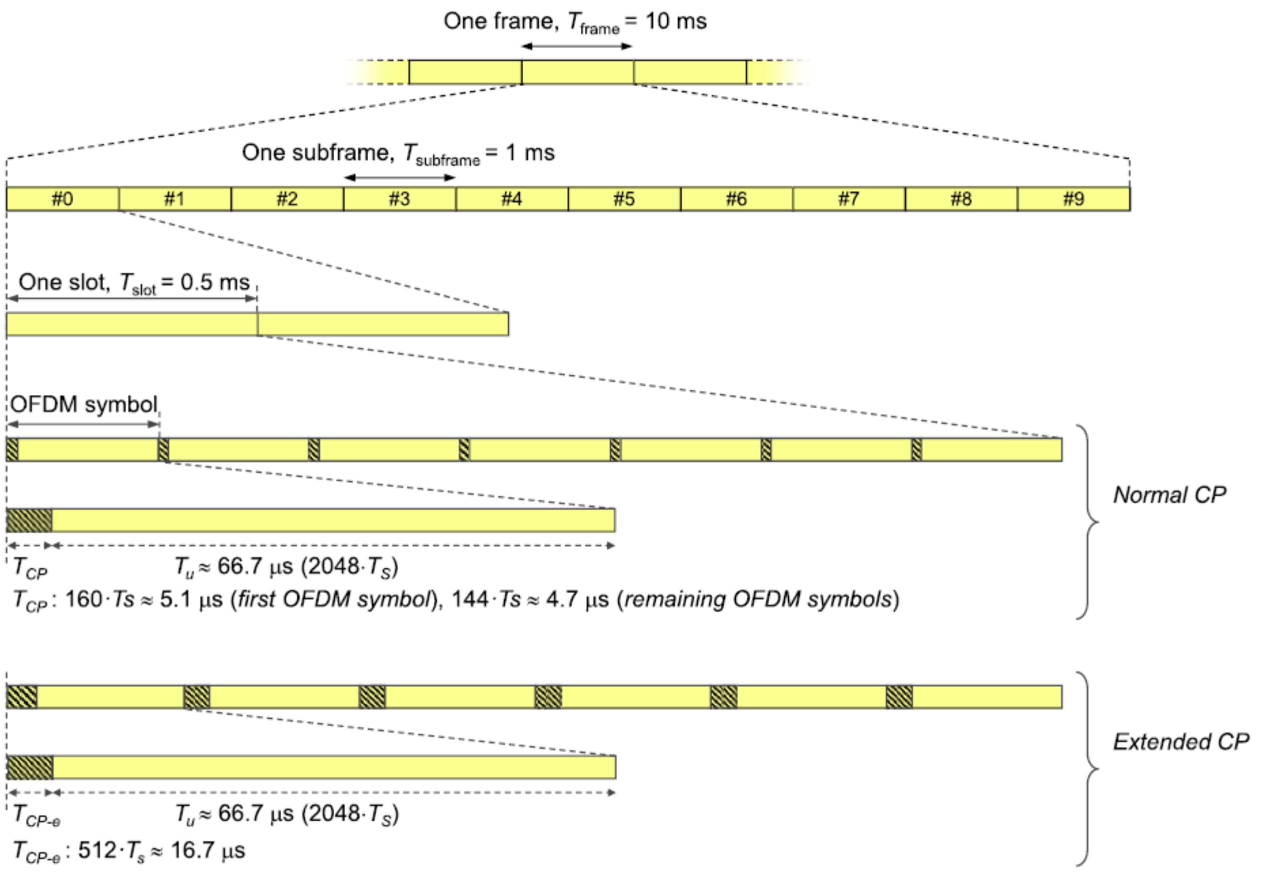


**FIGURE 5.1**

LTE time-domain structure.

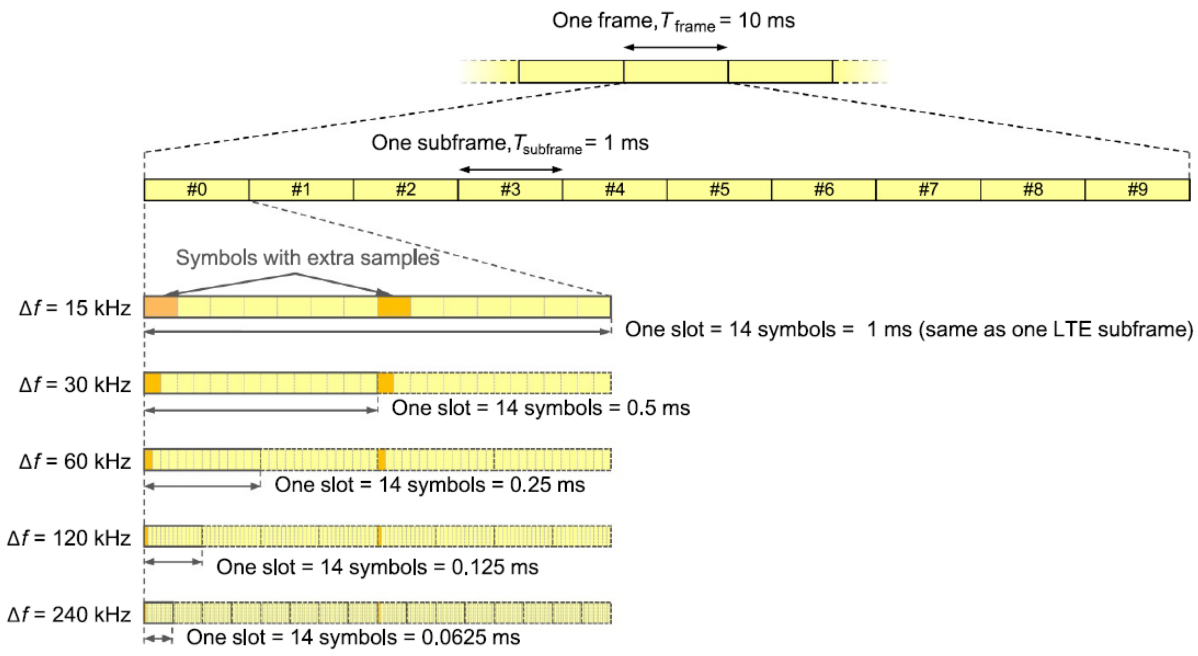


**Fig. 7.1** Frames, subframes, and slots in NR.

# Time-frequency grid: LTE, NR

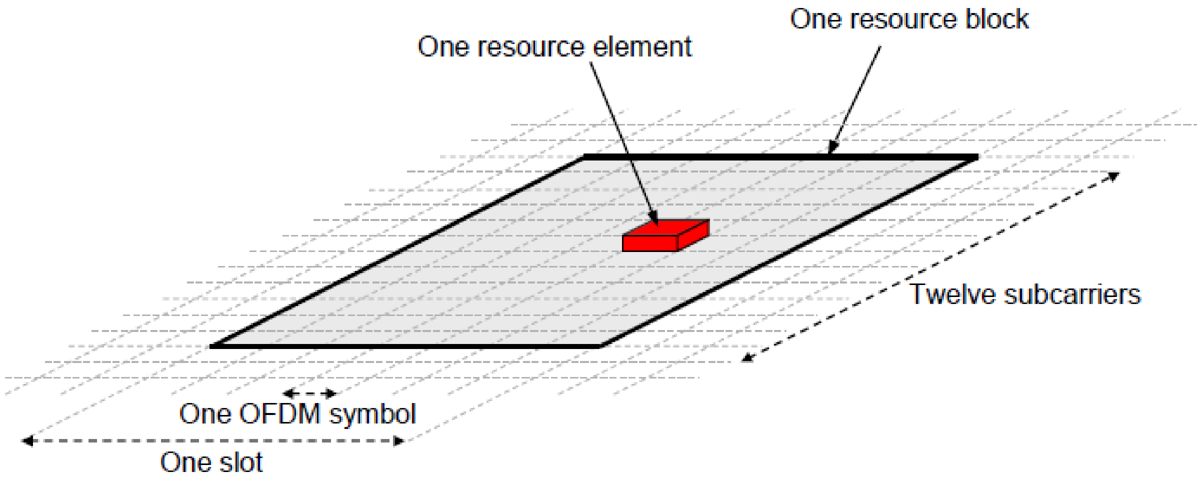


**FIGURE 5.2**

The LTE physical time–frequency resource.

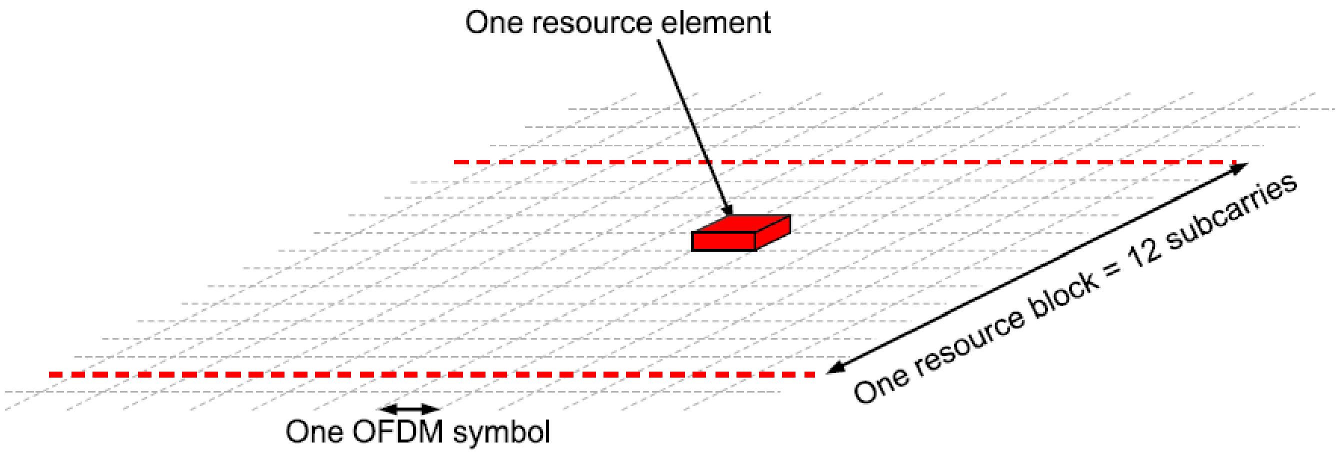


**Fig. 7.4** Resource element and resource block.

(NR)

# Maximum achievable rate (Capacity)

**Shannon Capacity Formula:**

$$C = BW_{channel} \log_2\left(1 + \frac{S}{N}\right)$$

**Received signal power**:
transmitted power, channel attenuation, receiver gain, …

**Received noise power:**

$$N = BW_{channel} \cdot N_o$$

Bandwidth (Hz)

noise density (W/Hz)

$$R \leq C$$

Error free communication

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# SNR

- Ratio between signal to noise power.
- No dimensions.
- Typically expressed in dB: $SNR_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$
- What if we use voltages instead? $SNR_{dB} = 20\log_{10}\left(\frac{V_{S,RMS}}{V_{N,RMS}}\right)$
- dB or linear?

$$C = BW_{channel}\log_2\left(1 + \frac{S}{N}\right)$$

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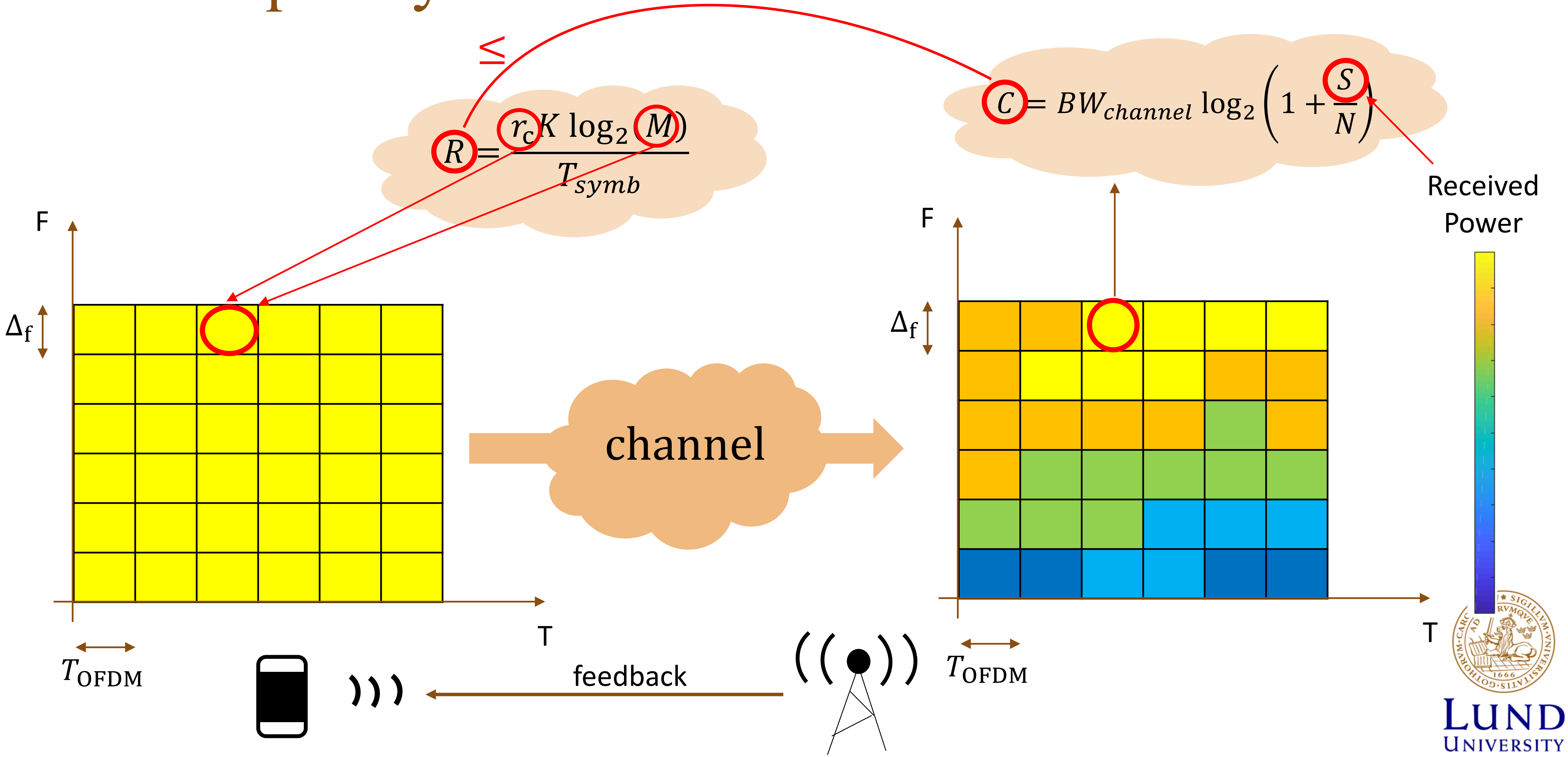
# Frequency-selective channel

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# Frequency-selective channel

Additional information on formulas:

Noisy channel coding theorem

$$R \leq C$$

https://en.wikipedia.org/wiki/Noisy-channel_coding_theorem

Shannon-Hartley theorem

$$C = BW_{channel} \log_2 \left(1 + \frac{S}{N}\right)$$

https://www.mdpi.com/1099-4300/16/9/4892/pdf

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# Frequency-selective channel

- The transmitter adjusts conveniently $r_c$ and $M$ based on the feedback. Modulation and Coding scheme (MCS)
- Feedback per individual subcarrier is not practical: group of subcarriers. We hope the channel does not change too much over those subcarriers!
- The feedback is independent per user.

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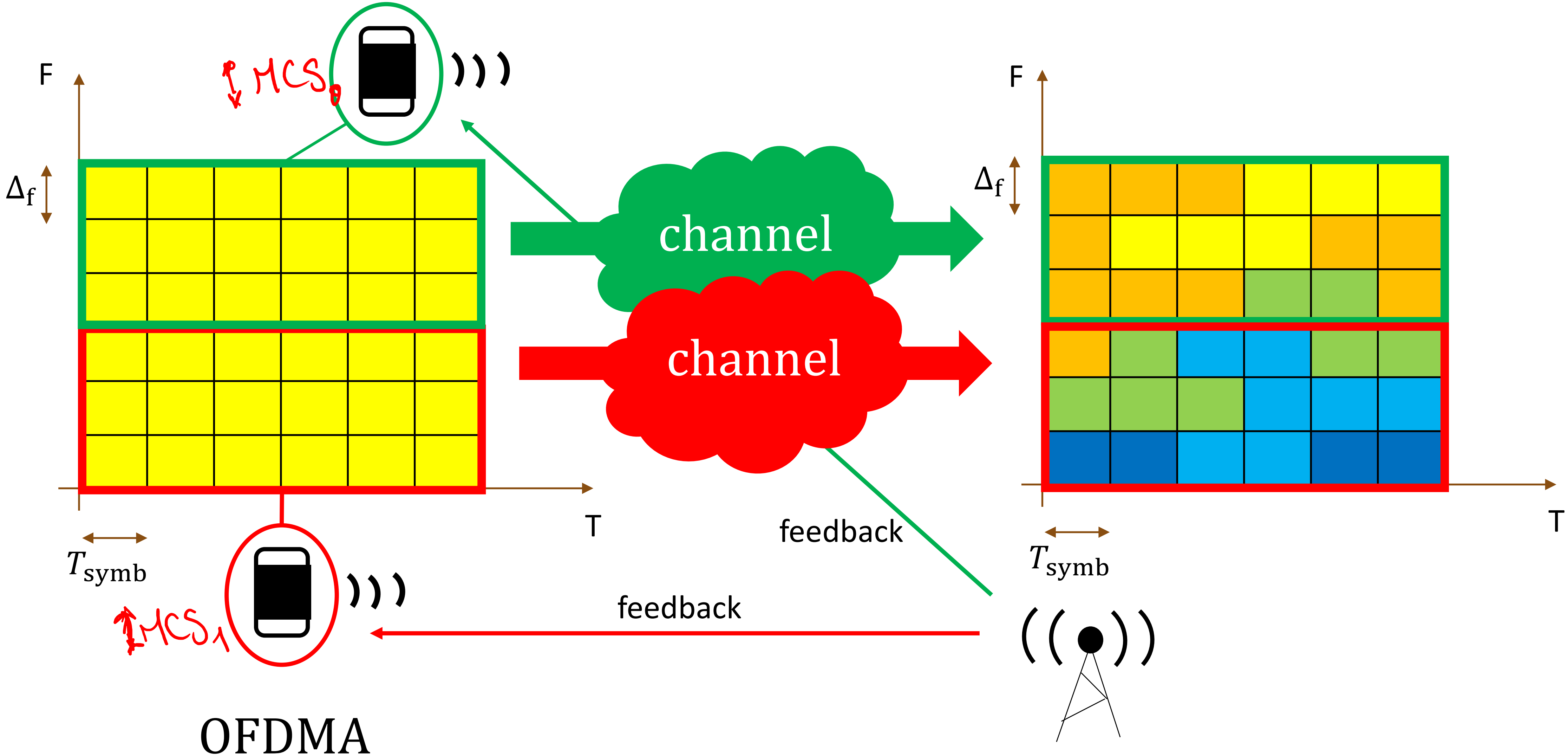
# Frequency-selective channel: multiuser

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