

Quantum Computers

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Disclaimer





Quantum Computers



Classical Digital Computers

Why quantum computers?

Basics of QM

- Spin-Qubits
- Optical qubits
- Superconducting Qubits
- Topological Qubits

Classical Computing

- We build digital electronics using CMOS
- Boolean Logic
- Classical Bit 0/1 (Defined as a voltage level $0/+V_{dd}$)





Logic is built by connecting gates



Classical Computing

- We build digital electronics using CMOS
- Boolean Logic
- Classical Bit 0/1



- Very rubust!
- Modern CPU billions of transistors (logic and memory)
- We can keep a logical state for years
- We can easily copy a bit
- Mainly through irreversible computing

However – some problems are *hard* to solve on a classical computer



Computing Complexity





- P easy to solve and check on a classical computer ~O(N)
- NP easy to check hard to solve on a classical computer ~O(2^N)
- BQP easy to solve on a quantum computer. ~O(N)
- BQP is larger then P
- *Some* problems in P can be more efficiently solved by a quantum computer

It is very *hard* to build a quantum computer...

Basics of Quantum Mechanics



A state is a vector (ray) in a complex Hilbert Space

Dimension of space – possible eigenstates of the state

For quantum computation – two dimensional Hilbert space

Example - Spin $\frac{1}{2}$ particle - $|\uparrow\rangle$ or $\downarrow\rangle$

Spin can either be 'up' or 'down'

Basics of Quantum Mechanics

- A state of a system is a vector (ray) in a complex vector (Hilbert) Space.
- Denoted |x> using Dirac notation. (You can think of this as a vector in 3D space.)
- Orthonormal basis vectors various observables (Energy, position, momentum, spin direction...).
- A state can be in a superposition of the basis vectors







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Basics of Quantum Mechanics



Depending on the choise of basis – the state of an particle can be described using different basis vectors!

 $|\psi
angle = |a_1
angle$

 $|\psi\rangle = a|b_1\rangle + b|b_2\rangle + c|b_3\rangle + \cdots$

Ex: Difference between position and momentum eigenstates

Uncertainty relation $[x, p] = -i\hbar$

Coefficients α , β , γ are complex numbers

State is normalized: $|\alpha|^2 + |\beta|^2 + |\gamma|^2 + \cdots = 1$

Can be expressed in various bases (observables)

- Energy
- Position
- Momentum
- ...



For quantum computation – effective 2D dimensional Hilbert space This is called a quantum bit (qubit, qbit)

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

• Example - Spin $\frac{1}{2}$ particle - $|\uparrow\rangle\downarrow\rangle$: 2 dimensional Hilbert space

 $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

Matrix representation of a state

 α anb β – two complex numbers State is normalized: $\alpha^2 + \beta^2 = 1$

Basics of Quantum Mechanics



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Quantum Bit – two level system

Complex





Bloch Sphere representation

Computational basis (with spins)

- Spin up in z-direction
- Spin down in z-direction

 $|S_x\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ Spin in x-direction?

Quantum system with only two distinct (energy) states

• Spin ½ particle (spin)
• Double quantum dot (position)
• Atom (orbitals)

$$---- E_2$$

$$---- E_1$$
Normalized
Complex coeff.
 $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$
 $|\psi\rangle = \sin\left(\frac{\theta}{2}\right) |0\rangle + \cos\left(\frac{\theta}{2}\right) e^{i\phi} |1\rangle$
 $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

For one spin ¹/₂ particle – the Bloch sphere represent the spatial orientation of the spin!

У

X

|↓↑⟩





Qubit A	Tensor product $\bigwedge^{A} \qquad \downarrow^{B} \qquad H_{A}$ $ \psi\rangle = 0\rangle_{A} \otimes 1\rangle_{B} = 01\rangle \qquad H_{B}$				
Qudit B	Example: Two non-interacting qubits – <i>product</i> state				
$ \psi\rangle_A = \alpha_A 0\rangle_A + \beta_A 1\rangle_A$	$ \psi\rangle = \psi\rangle_A \otimes \psi\rangle_B = \alpha_A \alpha_B 00\rangle + \beta_A \alpha_B 10\rangle + \alpha_A \beta_B 01\rangle + \beta_A \beta_B 11\rangle$				
$ \psi angle_B=lpha_B 0 angle_B+eta_B 1 angle_B$	Example: Two correlated qubits – <i>entangled</i> state				
	$ \psi\rangle = \frac{1}{\sqrt{2}}(10\rangle + 01\rangle)$ This can NOT be written as a product state!				
N Coupled qbits requires 2^N complex numbers to describe	Example: General 2 qubit state $\uparrow \downarrow \downarrow \rangle$				
	$ \psi\rangle = \alpha 00\rangle + \beta 10\rangle + \gamma 01\rangle + \delta 11\rangle$				
N=100 : $2^{100} \approx 10^{30}$ complex numbers					

Superposition of 2-bit states



 $|\psi\rangle = \frac{1}{\sqrt{4}}[|00\rangle + |10\rangle + |01\rangle + |11\rangle]$ Two spins – 4 states

 $|\psi\rangle = \frac{1}{\sqrt{8}} [|000\rangle + |001\rangle + |010\rangle + |100\rangle + |011\rangle + |110\rangle + |101\rangle + |111\rangle]$ Three spins – 8 states

 $|\psi\rangle = \cdots$. 100 spin $\frac{1}{2}$: $2^{100} = 10^{30}$ dimensional space

 $|\psi\rangle = \cdots$. 300 spin $\frac{1}{2}$: $2^{300} = 10^{90}$ dimensional space

- There are 10⁸⁰ atoms in the observable universe
- Hilbert spaces are big!

A quantum state can represent a huge number of 0's and 1' simultaneously!



Quantum System Dynamics



 $|\psi(t)\rangle = \alpha(t)|a_1\rangle + \beta(t)|a_2\rangle + \gamma(t)|a_3\rangle + \cdots$



A quantum system evolves in time set by the Schrödinger Equation, where *H* is the Hamilton operator.

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = \hat{H}(t) |\psi(t)\rangle$$

$$\widehat{H} = rac{\widehat{p}^{-2}}{2m} + V(\widehat{x})$$
 Electron in potential
 $\widehat{H} = -\gamma B_z \sigma_z$ Spin in magnetic field

- This give a set of coupled differential equations for the coefficients α β γ...
- By applying an Hamiltonian to a quantum state move the state in the Hilbert space over time. We can in this way control a quantum state.
- This is *deterministic and time reversible!*

Measurement in Quantum Mechanics

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 $|\psi\rangle = \alpha |a_1\rangle + \beta |a_2\rangle + \gamma |a_3\rangle + \cdots$

Measurement – apparatus which determines an observable (Energy, position, Spin direction...)



 a_2

During a measurement of an observable \boldsymbol{a} State collapses to an eigenstate with probability

- $P_n = |\langle a_n | \psi \rangle|^2$
- Not deterministic and NOT time reversible!



Measurement of states

1



 $N = 100 \leftrightarrow 10^{30} states$

$$|\psi\rangle = \frac{1}{\sqrt{2^N}} \left[|000 \dots\rangle + |001 \dots\rangle + |010 \dots\rangle + |100 \dots\rangle + |011 \dots\rangle + |110 \dots\rangle + \cdots \right]$$



Will randomly select **ONE** of the states with equal probability!

Gives 100 bits of information or out 10³⁰

Direct application of quantum parallelism is not an easy task!

Entangeled state 1) Separate two entangled qubits (by light years (!)) 2) Measure qubit A (will show 0 or 1 with 50%) 3) If A shows $|0\rangle_A$ - we now know that qubit B is $|1\rangle_B$ $|\psi\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$ 4) Spooky action at a distance!

Single quantum Gate – Bloch Sphere rotation





$$|\psi\rangle = \sin\left(\frac{\theta}{2}\right)|0\rangle + \cos\left(\frac{\theta}{2}\right)e^{i\phi}|1\rangle$$
$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad {\alpha \choose \beta}$$

Matrix representation of a state



These can in general be build by applying time varying B/E field to a spin/position qbit in x, y and z direction for a fixed time.

NOT gate π rotation around x axis $|0\rangle \rightarrow |1\rangle \begin{bmatrix} 0 & 1\\ 1 & 0 \end{bmatrix}$

Hadamard gate:
$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$
 $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

Matrix representation of a operator

Matrix representation of a operator

These also work on superpositions: $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$

Quantum Gate – Controlled Gate







If x=0 do nothing to qubit y If x=1 apply rotation U to qubit y



CNOT Gate

If $x = |0\rangle$ then y=y If $x = |1\rangle$ then apply NOT to y

- All controlled gates will be unitary
- Controlled Gates also work on super positions quantum parallelism!

Universial set of quantum gates





These set of gates are 'universal' Can implement **all logic functions** NAND, NOR etc.

1 qbit gate – controlled rotation of Bloch Sphere

Controlled Gates – almost anyone will do, controlled NOT (CNOT) or controlled phase CPHASE most common.

$$\begin{aligned} H|0\rangle &: \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \\ |\psi\rangle &= \alpha |00\rangle + \beta |10\rangle + \gamma |01\rangle + \delta |11\rangle \\ & \langle \alpha \rangle \end{aligned}$$

Fig. 5: The Clifford+T gate quantum gate set.

Example - Deutsch's Algorithm





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f(x) takes a single bit {0,1} into a single output bit {0,1}.

If we don't know if f(x) is "balanced" or "constant" – but would like to know

Classical Computer – calculate f(0) and f(1) and compare – need to evaluate f twice!



General Quantum Computing

 $|x\rangle|0\rangle \rightarrow |x\rangle|f(x)\rangle$

Input register
$$\left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]^N = \frac{1}{2^{N/2}} \sum_{0}^{2^{N-1}} |x\rangle$$

$$|x\rangle|0\rangle \to \frac{1}{2^{N/2}} \sum_{0}^{2^{N}-1} |x\rangle|f(x)\rangle$$

 $\frac{1}{2^{N/2}} \sum_{0} |x\rangle |f(x)\rangle \rightarrow |x_{0}\rangle |f(x_{0})\rangle$ \bigwedge Measurement of $|x\rangle$

This can be done with *N* Hadamard gates

By just computing the f(x) once, f(x) is "calculated" for all 2^N values! N=100 - 10^{30} speedup!

However, a direct measurement of x gives only ONE random value of $f(x_0)$!

Need clever algorithms similar to Deutsch Problem to try and extract more information from the correlation between x and f(x)





Decoherence – random Bloch sphere rotation Group



A single qbit will couple to the environment (example for a single $\frac{1}{2}$ spin)

- Nuclear spins
- Stray B-fields
- Electrical fluctuations
- Interactions with photons
- Electrical Noise

Couples through spinorbit interactions

This will induce random H(t) which randomly shifts around the quantum state.

Quantum states can be very fragile. **nS – mS before the qbit is completely random.** Decoherance time (T1 / T2).

Decoherence – random Bloch sphere rotation **Electronics** Group

- T1 Spin relaxation Time
- T2 Decohrerance Time

Given two distinct energy states for

If particle initially in E_1 – how long time until it is found in E_0 ? T1 – Spin relaxation time

If particle in a superposition - how long until the phase relation between is lost? T2 – decoherence time

These are measures of the 'quality' of a qbit. We need to do our gate operations shorter then T1 and T2.



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E₁

 E_0

T1

Quantum Computers - implementations



- Spin qubits (compatible with Si processing!)
- Optical traps (Most stable!)
- Superconducting Qubits (Most qubits!)
- Majorna Zero Modes Topological Qubits (Most difficult to understand. Best?)

Ion Trap – Optical Qubits







Atomic lons trapped in RF/DC field Laser cooling Strongly isolated from environment +Very long decoherance T1 times (syears(!!) -Scalability? -Geometry?

↔ 10µm

Qubit Operation

B-field + 729 nm laser – excite between $|0\rangle$ and $|1\rangle$

Multi-qubit operation motivated through phonons

Readout – 397nm (and 866nm) laser Scattered light only if atom in state $|0\rangle$

 $|1\rangle$





Single ¹/₂ spin in B-field

Two energy eigenstates – Spin up and spin down

Energy split is small

B=1T (InAs, g^{*}=-10) gives $\Delta E_z = 0.5 meV$



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 E_2

 $\Delta E_z = g^* \mu_b B$



Thermal energy (~kT/q) should be smaller then this value

T=300K - 25 meV T=1K - 0.08 meV

We need to operate our qubits at cryogenic temperature!

Spin Qubits Manipulation

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By applying a varying RF magnetic field in x-y direction

Also works for field in x direction *only* – have to use pulses.

Spin position can be controllable shifted

Resonance – only effective is RF frequencies close to energy difference are effective in rotating the spin!



Can access different qubits by global B-field. Single qbit control!

$$\hbar\omega = \Delta E_z = g^* \mu_b B$$

Spin Qubit Implementation

0.6

0.4

0.2· 0.0·

200





Single Electron Transistor – can localize single electron in each gate

Applying RF B-field – rotates spin. Qbit control!



 $|\psi_{in}\rangle = |\downarrow\downarrow\rangle$

800

600

400

 $\tau_{\rm P}(\rm ns)$



Rabi oscillation – rotate the spin between up and down. Spin relaxation time T1 - μ S range

Two nearby spins will influence each other through exchange mechanism – can build CNOT gates!







FIG. 24. One-electron spin interacts with (a) a single nuclear spin in an atom and vs (b) many nuclear spins in a semiconductor quantum dot.

Single Electron will interact with a background of many nuclear spins

These are randomly distributed in the material *nS-µS decoherence time*

Electron moving in a electric field will experience a magnetic field. This leads to spin-orbitinteraction. Electrical noise (and phonons!) can thus couple to a spin. **Spin-orbit-interaction is stronger for heavy elements (III-Vs)** Spin qubits Materials with no nuclear spin

- Si²⁹
- C¹²

Light elements (C, Si – low SOI)

Spin Qubit Si²⁹, Si²⁹-P





Electron quantum dot Spin qubit T2=268 µS

Electron – Donor Qubit T2=567 µS

Nuclear Spin Qubit T2= 0.6S

This can be increased to $T_2 = 30S$ (!!) using pulse focusing techniques!

Spin Qubit Implementation

Spin qubit summary

+Can be made with standard Silicon Technology (!!)

+Small - 10 - 100 nm size

+Long decoherance times can be demonstrated

+CNOT gate

+ GHz resonance frequencies - fast

- Sensitive to single atomic defects difficult to control
- How to couple many qubits?

Currently - at a single qubit / single gate stage.



Superconducting Qbits



Superconducting Qbits

- Some metals becoms superconducting at low T
- Al *T*_C=1.12K Nb – *T*_C= 9K

Again – we need to operate qubits at mK temperatures!

- Zero resistivity
 - Induced current decay over very long times (millions of years..)
- BCS Theory
 - Electrons form Cooper pairs ($\uparrow+\downarrow$)
 - Energy-gap around the Fermi energy
 - Can move through the metal without any resistance

$$\psi(x) = \sqrt{n_s(x)} e^{j(\phi(x))}$$

- Macroscopic wavefunction
 - $n_{\rm s}$ Cooper pair concentration
 - ϕ global phase





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Quantum LC-circuit





when spring i unstretched

Position when spring is

(c)

(a)

www-

-////

 $\mathbf{v} = \mathbf{0}$

PE



Energy oscillates between capacitor and inductor Stored energy in *C*

gy in C Stored energy in L $H = \frac{1}{2C}Q^2 + \frac{1}{2L}\phi^2 \qquad [Q,\phi] = -i\hbar$

$$H = \frac{1}{2m^*}p^2 + \frac{1}{2}m^*\omega^2 x^2 \quad [p, x] = -i\hbar$$

The quantum version of the LC-circuit behaves as an Harmonic Oscillator!



Equi-distant energy separation between energy levels

$$\Delta E = \hbar \omega$$

Not a two-level system!

Josephson Junction



 $v_L = L \frac{di}{dt}$ Ordinary inductor

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- A JJ-junction will act as a non-linear inductor
- L increases when the stored flux increases
- This will lead to anharmonicity LC oscillator with different energies

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Tunable Josephson Junction - SQUID





 I_0 depends on how well the wave function overlaps in the insulating region. Can be difficult to control.



Solution – connect two JJ and apply an magnetic field. This can be shown to change $I_o \rightarrow I_0 \cos(kB)$. A local B-field can be created by running a DC-current through a conductor

We get a JJ-junction which can be tuned to adjust the non-linear L!

Superconducting Qubit





This forms an isolated island for charges 'Coper-pair box'

Sensitive to charge fluctuations & electrical noise

Solution – make C large! Variation in charge on the box – small change in energy Insensitive to charge noise – longer coherence time!

Transmon Qubit





Transmission Line Shunted Plasma Oscillation Qubit

Superconducting Qubit Control



 $v_{app}(t) = Acos(\omega_d - \phi)$

Capacitviely couple a RF signal to the transmon





- Frequency difference $\omega_q \omega_d$ rotation around z-axis
- Phase difference rotation around x and y axis
- Full control over Qbit states!
- ω_q is in the GHz range nS time / operation

Superconducting Qubit Resonator







- We integrate the qubit in a section of a microwave resonator
- The qbit will slightly shift the resonance frequency of the resonator depending on the qbit state $(|0\rangle |1\rangle)$.
- By measuring the resonance frequency of the resonator ω_r we can perform a measurement on the qbit!
- By applying RF pulses at the qbit resonance frequency ω_q we can manipulate the qbit state







- Qbit interaction is possible by coupling different qubits together
- This can be tunable through coupling via a DC SQUID
- CPHASE/CNOT qubit control can be implemented

Superconducting Qubit Control













Table 2. Summary of the achieved success probabilities for the implemented circuits, in percentages

Connectivity	Star shaped Superconducting			Fully connected Ion trap		
Hardware						
Success probability/%	Obs	Rand	Sys	Obs	Rand	Sys
Margolus	74.1(7)	82	75	90.1(2)	91	81
Toffoli	52.6(8)	78	59	85.0(2)	89	78
Bernstein–Vazirani	72.8(5)	80	74	85.1(1)	90	77
Hidden shift	35.1(6)	75	52	77.1(2)	86	57

Current state-of-the-art

Google Bristlecone – 72 qubits

Superconducting qubits – reasonable advanced circuits 50-75% success rate

Superconducting Qubit Control





Sources of Decoherance

- Emission of photon
- Critical current flucutations (I_0 in JJ)
- Dielectric losses
- Quasi-particle tunneling
- Charge noise

Decoherence Times in the 10-100 μ S time range.

Topological Qbits

- All qbits are sensitive to decoherence (nS-mS decoherence time)
- Local flucutations causes decoherance
- Can we build a qubit which is insensitive towards local pertubations?
 - (Microsoft thinks so!)
- These are so called Majorana Fermions 'Topological protected states'
- Should appear in a "p-type" superconductor
 - Cooper pairs with *same* spin
 - Can form at the edge of a p-type superconducting wire

 p-type superconductors do not really exist.... Nano

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Majorana Zero Modes





- The 1D nanowire will be spin polarized
- Superconductivity can be induced from the standard superconductor
- The nanowire system approximates a p-type superconductor!

Superconductor with epitaxial contact to nanowire

Nanowire (1D)with large spin-orbit interaction and large g-factor (InSb, InAs).

- Under very precise conditions we can then form Majorana Zeros Modes (Majorana Fermions) at the ends to the nanowire. (Very low T, exact position of Fermi energy, high quality SC-N interface, thin SC...)
- These are part of the same (quasi-)particle
- Highly non-local!
- To perturb the particle state one needs to perturb both ends of the wire – unlikely!
- 'Topological protected states'
- Potentially very long coherence times!



MZM qubit gates (?)





- The MZM adhers show 'non-abelian' statistics
- Interchange of two particles change the particle state in a non-trivial way
 - i.e. not only a change of phase
- Apply network of wires and gates to move Majorana states around each other.
- Could implement some qubit gates.
- This has never been demonstrated.

It is not clear which is the best way to implement Qubit or Gates...

MZM qubit gates



- Measurement could be done by 'fusing' two states
- Its own anti-particle will annihilate
- Nothing or creation of electron
- This has also never been demonstrated.

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Quantum Computing Conclusions

- Quantum Computers CAN do SOME things much better then a classical computer will be able to.
- Some IMPORTANT problems can get an EXPONENTIAL speed up
- Qubits are DIFFICULT to build
- Quantum Gates are even HARDER to build
- Plenty of very interesting and deep physics!
- Plenty of very interesting engineering challanges!
- There are alot of large companies (Google, Microsoft, Intel, IBM) working hard on this!





