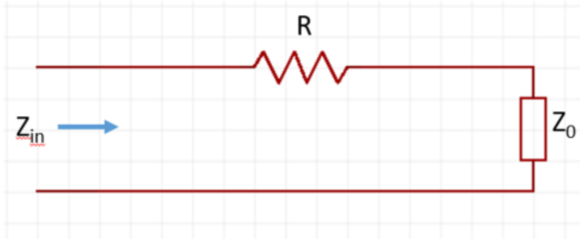
Abinaya Krishnaraja
abinaya.krishnaraja@eit.lth.se

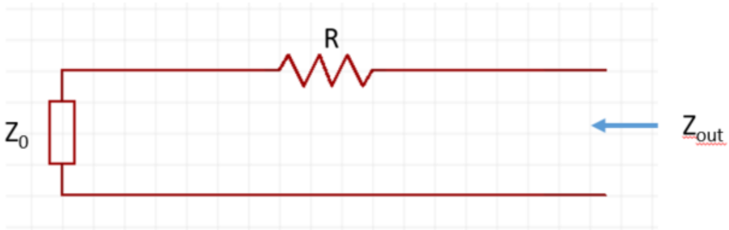
High Speed Electronics 2019 – Exercise 5 Solutions

1. a) To determine S_{11} (Input reflection co-efficient)



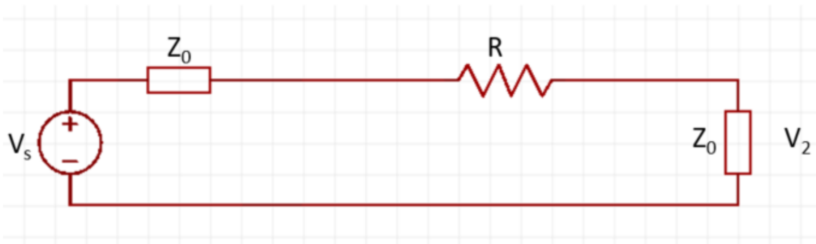
$$Z_{in} = R + Z_0$$
$$S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{R}{R + 2Z_0}$$

To determine S_{22} (Output reflection co-efficient)



$$Z_{out} = R + Z_0$$
$$S_{22} = \Gamma_{out} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} = \frac{R}{R + 2Z_0}$$

To determine S_{21} (Transverse gain)



$$V_2 = V_s \frac{Z_0}{R + 2Z_0}$$
$$S_{21} = \frac{2V_2}{V_s} = \frac{2Z_0}{R + 2Z_0}$$

To determine S_{12} (Reverse gain)

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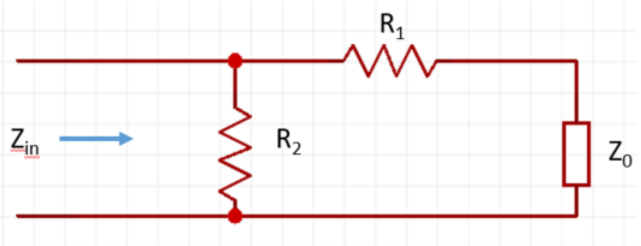
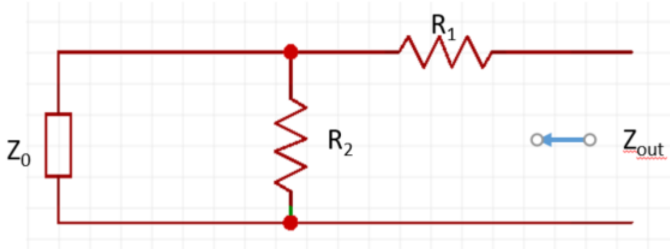
$$V_1 = V_s \frac{Z_0}{R + 2Z_0}$$
$$S_{12} = \frac{2V_1}{V_s} = \frac{2Z_0}{R + 2Z_0}$$

b) To determine S_{11} (Input reflection co-efficient)



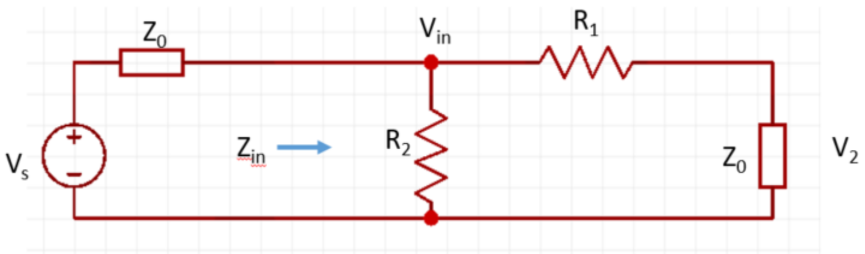
$$Z_{in} = R_2 || (R_1 + Z_0)$$
$$S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

To determine S_{22} (Output reflection co-efficient)



$$Z_{out} = R_1 + (R_2 || Z_0)$$
$$S_{22} = \Gamma_{out} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}$$

To determine S_{21} (Transverse gain)



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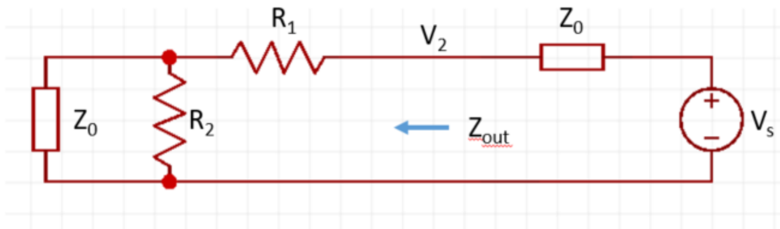
$$V_2 = V_{in}\frac{Z_0}{R_1 + Z_0}$$
$$V_{in} = V_s\frac{Z_{in}}{Z_{in} + Z_0}$$
$$S_{21} = \frac{2V_2}{V_s} = \frac{2Z_{in}Z_0}{(R_1 + Z_0)(Z_{in} + Z_0)}$$

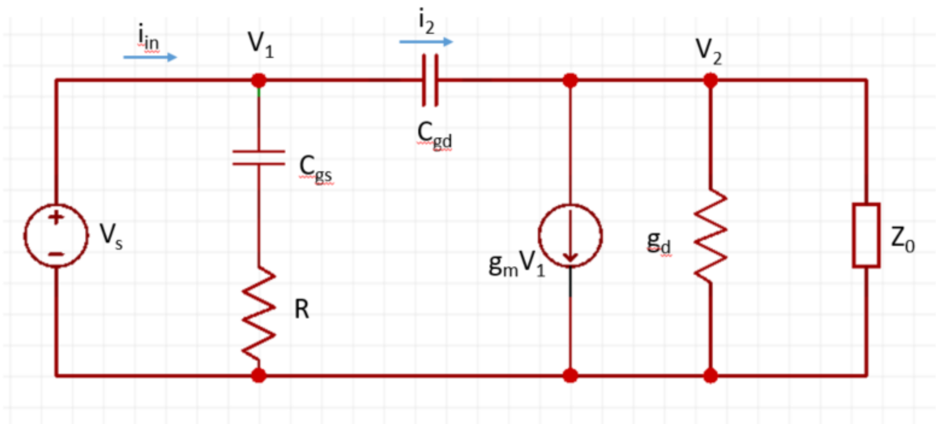
To determine S_{12} (Reverse gain)



$$V_1 = V_2\frac{Z_0||R_2}{R_1 + (R_2||Z_0)}$$
$$V_2 = V_s\frac{Z_{out}}{Z_{out} + Z_0}$$

$$S_{12} = \frac{2V_1}{V_s} = \frac{2Z_{out}(Z_0||R_2)}{(R_1 + (R_2||Z_0))(Z_{out} + Z_0)}$$

2. To determine S_{11}



$$S_{11} = \Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$Z_{in} = \frac{V_s}{i_{in}}$$

$$i_2 = (V_1 - V_2)j\omega C_{gd} = g_m V_1 + g_d V_2 + \frac{V_2}{Z_0}$$

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$$V_2 = \frac{V_1\left(j\omega C_{gd} - g_m\right)}{j\omega C_{gd} + g_d + \frac{1}{Z_o}}$$

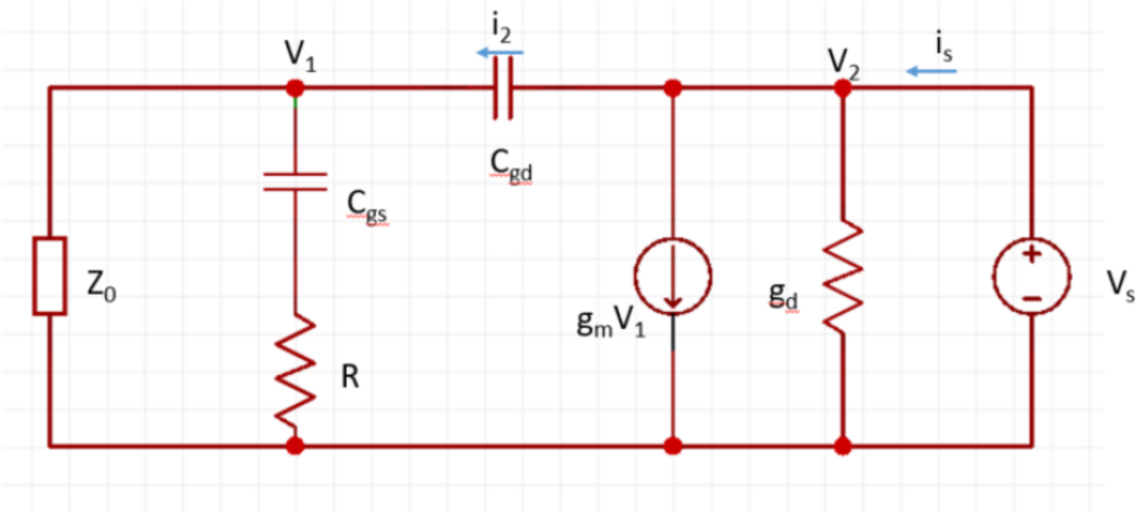
$$i_{in} = (V_1 - V_2)j\omega C_{gd} + \frac{V_1}{\left(R + \frac{1}{j\omega C_{gs}}\right)}$$

Substituting for V_2 in the i_{in} expression above gives

$$Z_{in} = \frac{V_1}{i_{in}}$$

From which S_{11} can be calculated.

To determine S_{22}



$S_{22} = \Gamma_{out} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}$, where $Z_{out} = \frac{V_s}{i_s}$

Writing KCL at node V1

$$i_2 = (V_2 - V_1)j\omega C_{gd} = \frac{V_1}{\left(R + \frac{1}{j\omega C_{gs}}\right)} + \frac{V_1}{Z_0}$$

$$V_1 = \frac{V_2 j\omega C_{gd}}{\left(R + \frac{1}{j\omega C_{gs}} + \frac{1}{Z_0} + j\omega C_{gd}\right)}$$

Similarly, KCL at node V2

$$i_s = (V_2 - V_1)j\omega C_{gd} + V_1 g_m + V_2 g_d$$

Substituting for V_1 in the i_s expression above gives $Z_{out} = \frac{V_s}{i_s}$ from which S_{22} can be computed.

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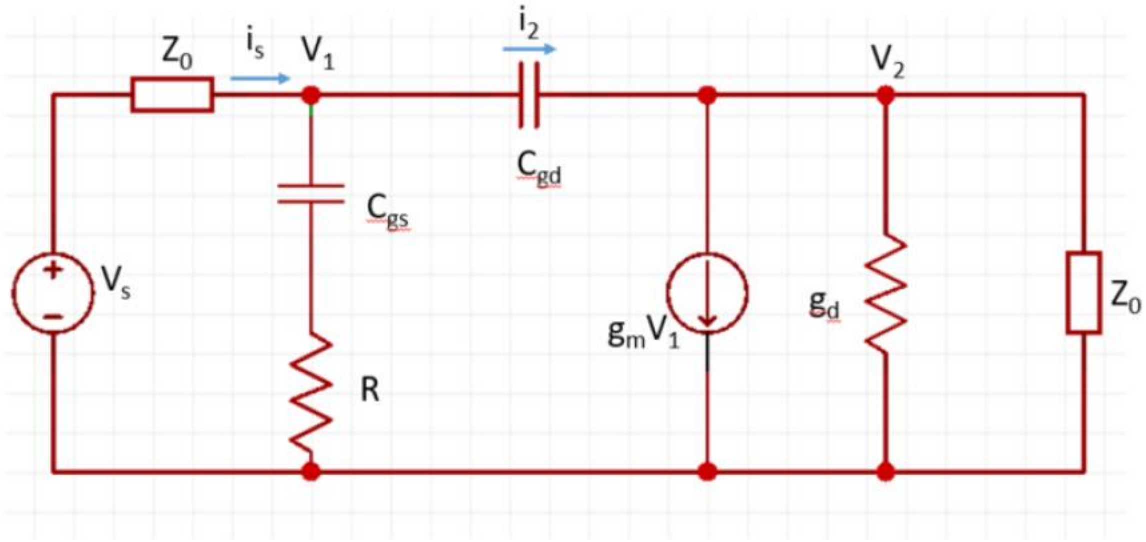
To determine S_{21}



$$S_{21} = \frac{2V_2}{V_s}$$

Now we try to write V2,Vs can be calculated from KCL at nodes V1 and V2.

$$i_2 = (V_1 - V_2)j\omega C_{gd} = \frac{V_2}{Z_0} + V_1 g_m + V_2 g_d$$

Which gives,

$$V_1 = \frac{V_2(j\omega C_{gd} + g_d + \frac{1}{Z_0})}{\left(j\omega C_{gd} - g_m\right)}$$

Also,

$$i_s = \frac{(V_s - V_1)}{Z_0} = \frac{V_1}{\left(R + \frac{1}{j\omega C_{gs}}\right)} + (V_1 - V_2)j\omega C_{gd}$$

Which results to,

$$\frac{V_s}{Z_0} = V_1\left(\frac{1}{Z_0} + \frac{1}{\left(R + \frac{1}{j\omega C_{gs}}\right)} + j\omega C_{gd}\right) - V_2(j\omega C_{gd})$$

Solving the above equations and rearrange them to get $S_{21} = \frac{2V_2}{V_s}$

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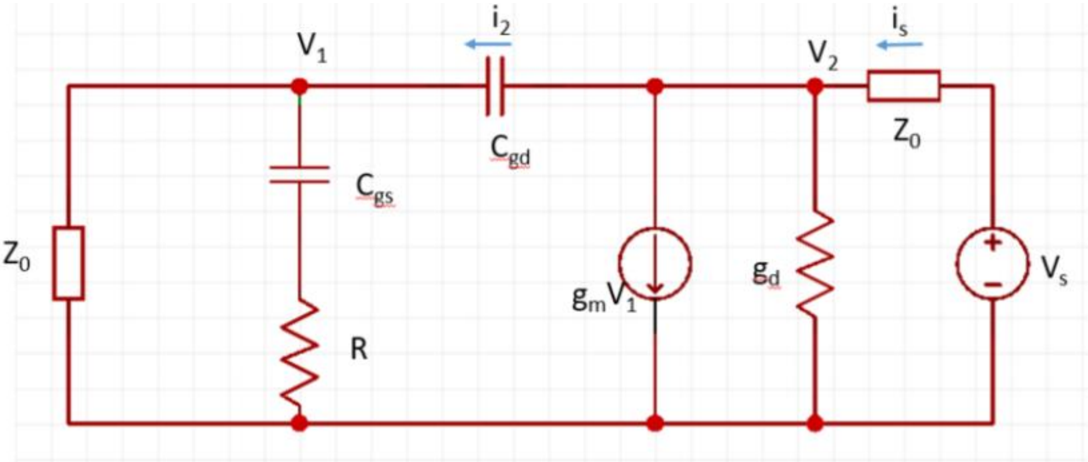
To determine S_{12}



$$S_{12} = \frac{2V_1}{V_s}$$

As usual, writing eh KCL equations at V1, V2 nodes

$$i_2 = \frac{V_1}{Z_0} + \frac{V_1}{\left(R + \frac{1}{j\omega C_{gs}}\right)} = (V_2 - V_1)j\omega C_{gd}$$

$$V_2 = \frac{V_1\left(j\omega C_{gd} + \frac{1}{Z_0} + \frac{1}{\left(R + \frac{1}{j\omega C_{gs}}\right)}\right)}{j\omega C_{gd}}$$

Also,

$$i_s = \frac{V_s - V_2}{Z_0} = (V_2 - V_1)j\omega C_{gd} + V_1 g_m + V_2 g_d$$

$$\frac{V_s}{Z_0} = V_1(g_m - j\omega C_{gd}) + V_2\left(g_d + j\omega C_{gd} + \frac{1}{Z_0}\right)$$

Substitute for V2 in the above expression and rearrange to get

$$S_{12} = \frac{2V_1}{V_s}$$

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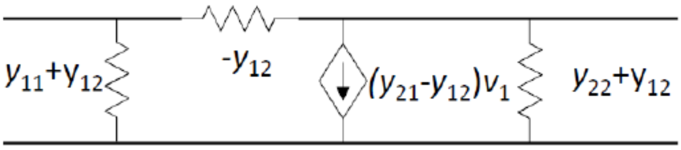
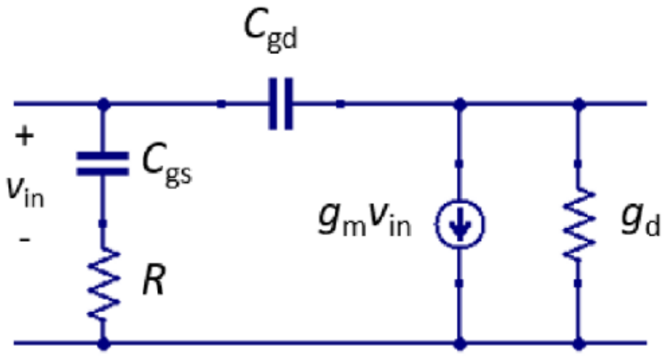
3. Since direct numerical calculations of S parameters is very tedious, let us compute the numerical y-parameters and convert them to S-parameters.

General hybrid π-model





Comparing the above two models, the y-parameters cab be extracted at

$$\omega = 2\pi f = 2\pi * 50 * 10^9 = 314 * 10^9$$

$$Y_{11} = \left(R + \frac{1}{j\omega C_{gs}}\right)^{-1} + j\omega C_{gd} = \frac{j\omega C_{gs}}{1 + j\omega R C_{gs}} + j\omega C_{gd}$$

$$= \frac{\omega^2 R C_{gs}^2}{1 + \omega^2 R^2 C_{gs}^2} + j\omega C_{gd} + \frac{j\omega C_{gs}}{1 + \omega^2 R^2 C_{gs}^2}$$

$$Y_{11} = 3.4 * 10^{-4} + j1.5 * 10^{-3}\ S$$

$$Y_{12} = -j\omega C_{gd} = -j1.57 * 10^{-3}\ S$$

$$Y_{21} = g_m - j\omega C_{gd} = 20 * 10^{-3} - j1.57 * 10^{-3}\ S$$

$$Y_{22} = g_d + j\omega C_{gd} = 5 * 10^{-3} + j1.57 * 10^{-3} S$$

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Using MATLAB y2s function, the parameters were converted and the absolute values of the S-parameters were obtained.

$$S_{11} = -0.8127 - j0.9502; |S_{11}| = 1.25$$

$$S_{12} = 0.1763 - j0.9397; |S_{12}| = 0.9561$$

$$S_{21} = 0.1674 - j0.9313; |S_{21}| = 0.9462$$

$$S_{22} = -0.8226 - j0.9063; |S_{22}| = 1.22$$

Since $|S_{11}| > 1$, the system is NOT unconditionally stable.

To draw the stability circle, the center and the radius of the circle should be computed using the following equations:

$$C = \frac{S_{22}^* - \Delta^* S_{11}}{|S_{22}|^2 - |\Delta|^2} = -0.6296 - \mathrm{j}0.0137$$

$$R = \frac{|S_{12}S_{21}|}{||S_{22}|^2 - |\Delta|^2|} = 0.3912$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.6530 + j1.8397$$

Converting the calculated C and R to z parameters and normalizing to $Z_0 = 50\ \Omega$ gives

C = 0.2272-j0.0103 and R = 2.29

The Smith chart shows the output stability circle. Since $|S_{11}| > 1$, the whole region outside the stability circle is unstable. A portion of the Smith chart being unstable confirms again that the system is NOT unconditionally stable. The Γ_L is also plotted in the Smith chart and it can be seen to lie in the unstable region. To make the system stable, the Γ_L is brought inside the stability circle by following the constant resistance circle. Thus a reactive element of -j0.57 can be added in series with the load to make it stable.

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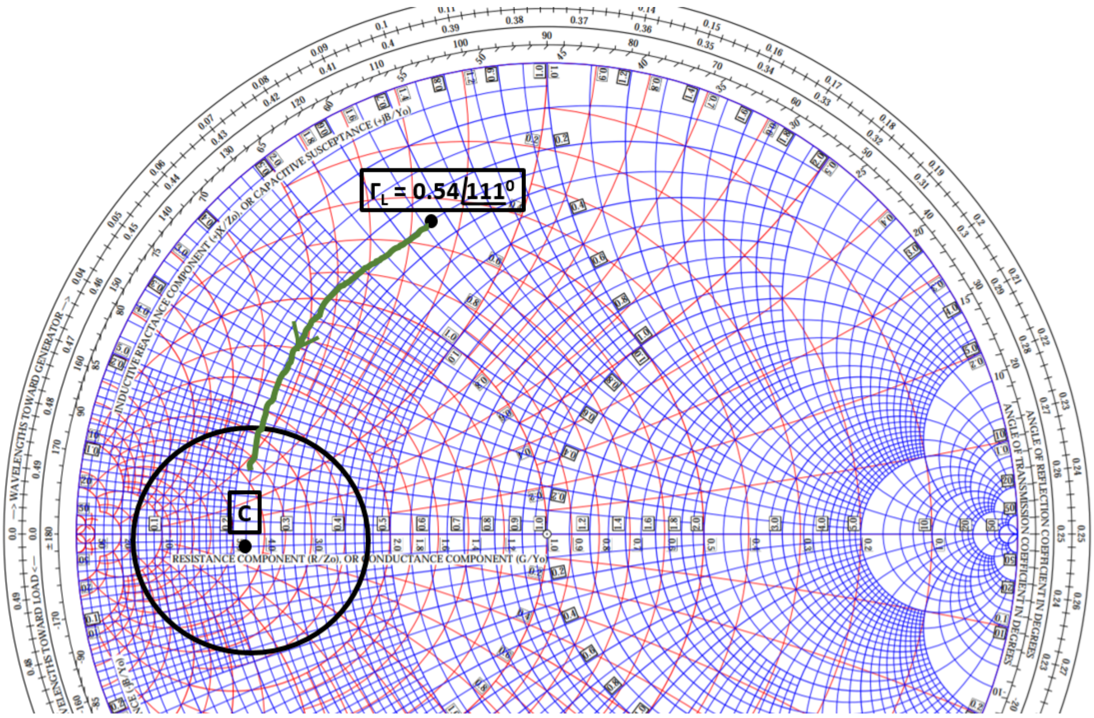

4. Z_{out} = 100+j200 Ω

Assuming characteristic impedance Z_0 = 50 Ω, the normalized Z_{out} = 2+j4 Ω

Z_{out}^* must be matched to Z_L = 50 Ω

Steps:

- Follow the constant resistance circle from Z_L which corresponds to a series inductor with reactance value j3.

$$Z = \frac{1}{Z_0} j\omega L \,;\, L = 24\, nH$$

- Follow the constant conductance circle downwards to reach Z_{out}^* which corresponds to a parallel capacitor with susceptance value j0.5.

$$Y = Z_0 j\omega C \,;\, C = 1.59\, pF$$

With the same L and C, at ω = 10 GHz, the impedance and admittance values will be Z = j 30.14 and Y = j5. Measuring from the center of Smith chart to the impedance point following the new L and C gives the magnitude of the reflection co-efficient (from the bottom of Smith chart). Γ = 1 complete reflection in this case.

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