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High Speed Electronics 2019 – Exercise 1 Solutions

1. a) Effective conduction band density of states of $In_{0.53}Ga_{0.47}As$ $N_c = 2.1*10^{17}/cm^3$

   $n \approx N_d$ (Doner doping concentration)

   For $N_d = 10^{16}/cm^3$,

   $n < 0.05N_c$ and the Maxwell-Boltzmann statistics holds.

   Therefore,

$$E_f - E_c = kT \ln\left(\frac{n}{N_c}\right)$$

$$= -0.08 \text{ eV}$$

   For $N_d = 10^{18}/cm^3$,

   $n > 0.05N_c$ and so using Joyce-Dixon approximation,

$$E_f - E_c = kT[\ln\left(\frac{n}{N_c}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_c}\right) - 0.00495\left(\frac{n}{N_c}\right)^2]$$

$$= 0.08 \text{ eV}$$

   For $N_d = 10^{19}/cm^3$,

   $n > 0.05N_c$ and so using Joyce-Dixon approximation,

$$E_f - E_c = kT[\ln\left(\frac{n}{N_c}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_c}\right) - 0.00495\left(\frac{n}{N_c}\right)^2]$$

$$= 0.24 \text{ eV}$$

b)

Number of holes

$$p = N_v e^{\frac{E_v - E_f}{kT}}$$

$E_c - E_v + E_f - E_f = E_g$

$(E_v - E_f) = -E_g - (E_f - E_c)$

Effective conduction band density of states of $In_{0.53}Ga_{0.47}As$ N_v= $7.7*10^{18}/cm^3$
Bandgap of $In_{0.53}Ga_{0.47}As$ E_g = 0.74 eV

n ($/cm^3$)	**$E_f - E_c$ (eV)**	**$E_v - E_f$ (eV)**	**p ($/cm^3$)**
10^{16}	-0.08	-0.66	$5.9*10^{7}$
10^{18}	0.08	-0.82	$1.2*10^{5}$
10^{19}	0.24	-0.98	245

c)

By mass action law,

$$n_i^2 = \left(\sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}}\right)^2$$

$$= 5.65*10^{23}/cm^6$$

n ($/cm^3$)	**p ($/cm^3$)**	**np ($/cm^6$)**
10^{16}	$5.9*10^{7}$	$5.9*10^{23}$
10^{18}	$1.2*10^{5}$	$1.2*10^{23}$
10^{19}	245	$245*10^{19}$

Thus the mass action law is valid only when the Maxwell-Boltzmann approximation is valid.

2. The Fermi-Dirac integral can be calculated by substituting j = -0.5, 0, 0.5 for 1D, 2D and 3D systems. Taking GaAs (m^* = $0.067m_0$) as an example and assuming $E_f - E_c$ = 0.1 eV , the carrier concentrations can be calculated using the following equations:

$$F_j(\eta_F) = \eta_F^{j+1}/\Gamma(j+2)$$

$$n_{1D} = \frac{\sqrt{2m^* kT}}{h\sqrt{\pi}} F_{\frac{-1}{2}}(\eta_F)$$

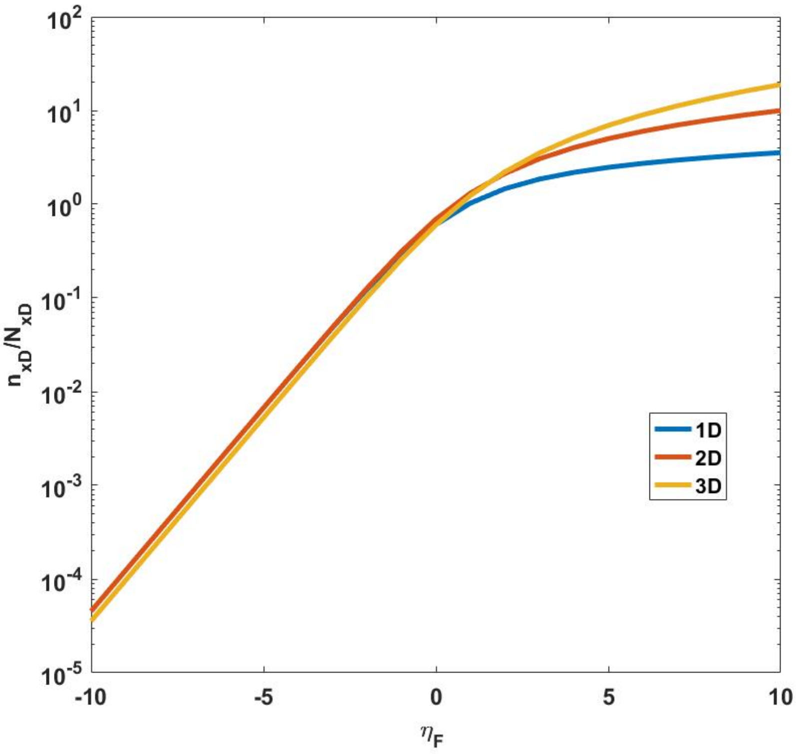
$= 28\ cm^{-3}$

$$n_{2D} = \left(\frac{m^* kT}{\pi h^2}\right) F_0(\eta_F)$$

$= 7*10^{8}\ cm^{-3}$

$$n_{3D} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{1.5} F_{\frac{1}{2}}(\eta_F)$$

$= 4*10^{18}\ cm^{-3}$



3. $$n_s = \int_{E_1}^{\infty} D_{2D} f_0(E, E_f) dE$$

For low temperature, the effective density of states reduces and hence Maxwell-Boltzmann condition (n < 0.05 N_c) is not valid. Hence considering degeneracy,

$$= \int_{E_1}^{\infty} D_{2D} (1 - \theta(E_f) dE$$

$$= \int_{E_1}^{E_f} D_{2D} dE$$

$$= \frac{m^*}{\pi h^2}(E_f - E_1)$$

$$E_f - E_1 = \frac{\pi h^2}{m^*} n_s$$

	$m^*(m_0)$	**$E_f - E_1$ (eV)**
InAs	0.026	0.28
GaAs	0.067	0.11
$In_{0.53}Ga_{0.47}As$	0.041	0.18

4. To avoid short channel effects,

   $L_g > 2\lambda$
   $L_g > 2*(t_{QW} + 2t_{ox})$ [since the dielectric constants are assumed to be equal]
   $t_{QW} < (L_g - 4t_{ox})/2$

   Thus the thickest possible quantum well to avoid short channel effects with the given $t_{ox} = 2\text{nm}$ and $L_g = 10\text{nm}$ is $t_{QW} = 1\text{nm}$.

5. a) Oxide capacitance $C_{ox} = \frac{\varepsilon_{ox}\varepsilon_0}{t_{ox}} = 0.089\ \text{F/m}^2$

   b) Quantum capacitance $C_q = \frac{q^2 m^*}{\pi h^2} = 0.027\ \text{F/m}^2$

   c) Charge centroid capacitance $C_c = \frac{\varepsilon_s\varepsilon_0}{0.36 * t_{well}} = 0.068\ \text{F/m}^2$

   Total gate capacitance $C_g = \frac{1}{C_{ox}^{-1} + C_q^{-1} + C_c^{-1}} = 0.016\ \text{F/m}^2$

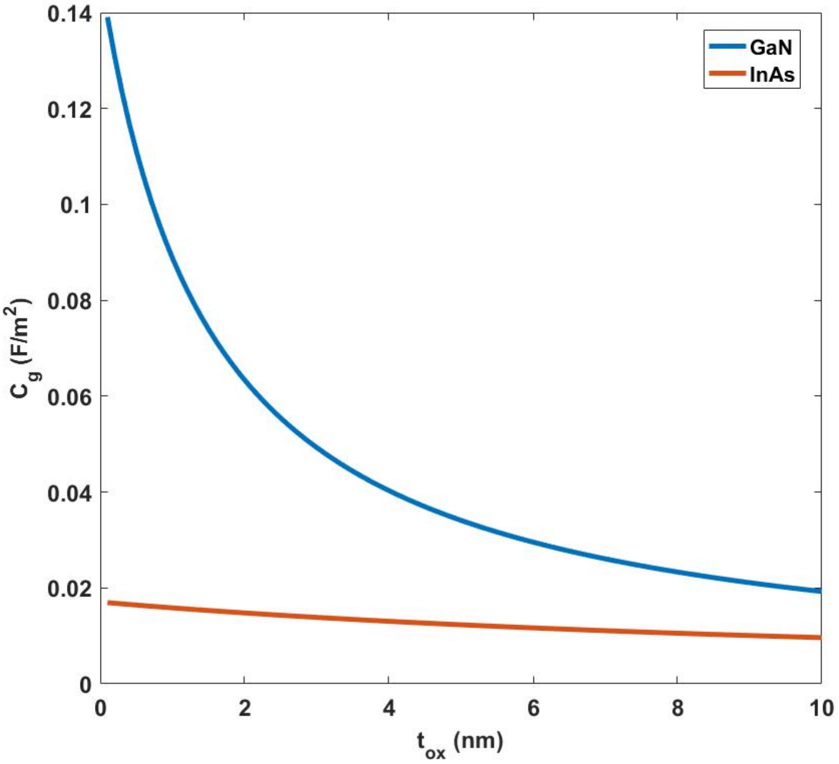
6. a) Ignore C_c

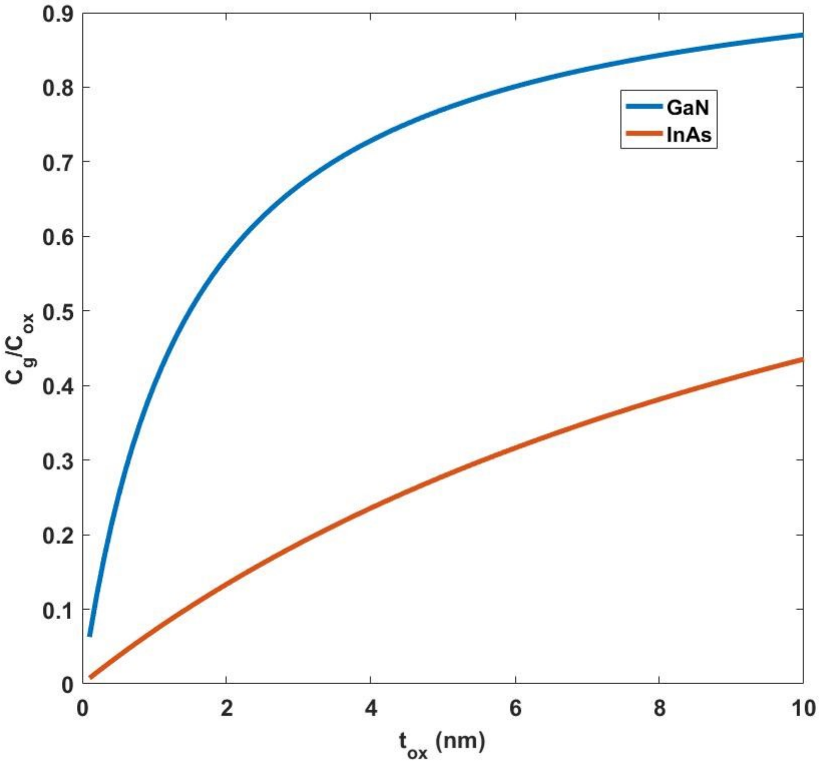
   $$C_g = \frac{1}{C_{ox}^{-1} + C_q^{-1}} = \frac{\varepsilon_{ox}\varepsilon_0}{t_{ox} + \Delta t_{ox}}$$

   $$\Delta t_{ox} = \frac{\varepsilon_{ox}\varepsilon_0 \pi h^2}{q^2 m^*}$$

   b-d) $C_g/C_{ox} = \frac{1}{1 + \left(\frac{\Delta t_{ox}}{t_{ox}}\right)}$

   Larger C_g/C_{ox} signifies $C_q > C_{ox}$. From the plots, GaN with larger m^* than InAs has higher C_g/C_{ox} even for thinner t_{ox} and so the gate stack is more scalable for GaN.


0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
C_g (F/m^2)
0
2
4
6
8
10
t_{ox} (nm)
GaN
InAs


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
C_g/C_{ox}
0
2
4
6
8
10
t_{ox} (nm)
GaN
InAs