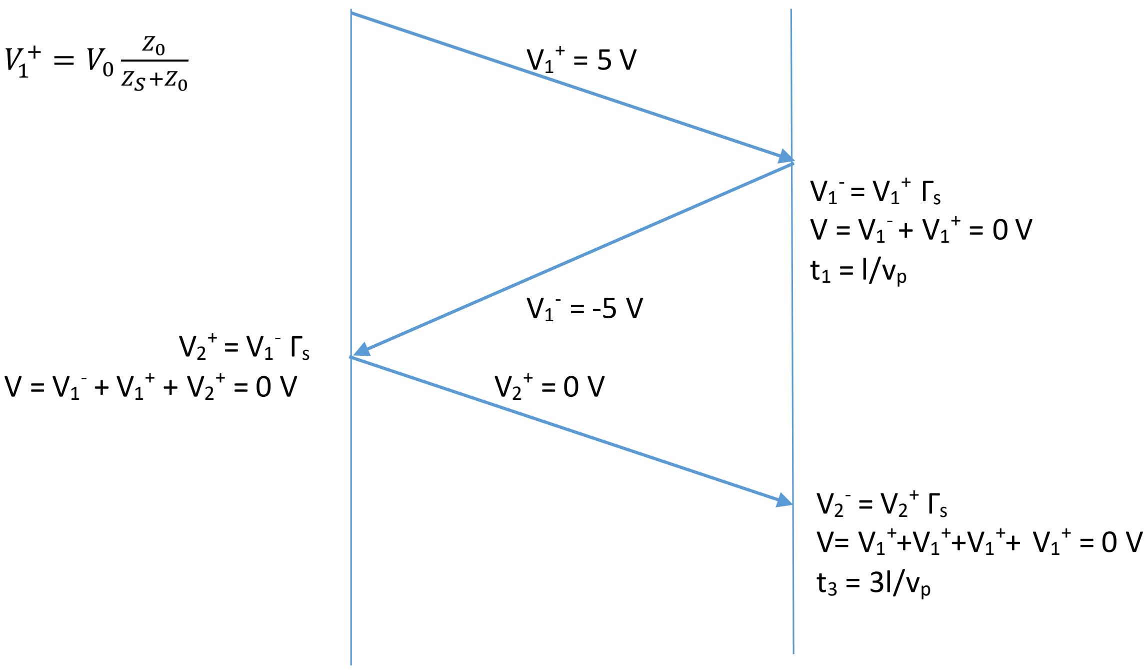
Abinaya Krishnaraja
abinaya.krishnaraja@eit.lth.se

High Speed Electronics 2019-Exercise 4 solutions

1. $V_0 = 10$ V
   $Z_s = 50\ \Omega$
   $Z_L = 0\ \Omega$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = 0$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$V_1^+ = V_0 \frac{Z_0}{Z_S + Z_0}$$



2. For an open line,

$$Z = -jZ_0 \frac{1}{\tan(\beta l)}$$

From the above equation, the length of the stub can be calculated as,

$$l = \frac{1}{\beta} \tan^{-1}\left(\frac{-jZ_0}{Z}\right)$$

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$$\beta = \frac{\omega}{v_p} = \frac{2\pi f\sqrt{\varepsilon_{eff}}}{c}$$

$$l = \frac{c}{2\pi f\sqrt{\varepsilon_{eff}}} tan^{-1}\left(\frac{-jZ_o}{Z}\right)$$

For a capacitance C= 1pF,

$Z = \frac{1}{j\omega C} = -j16.7$, Which gives, $l = 5.5mm$

For an Inductor L= 1nH,

$Z = j\omega L = j62.8$, Which gives, $l = 3.9mm$

3. $Z_o = 50\Omega, Z_1 = 75\Omega$

a) Injection/Reflection co-efficient

$$\Gamma_s = \frac{Z_1 - Z_0}{Z_1 + Z_0} = 0.2$$

Transmission co-efficient

$$T = \frac{2Z_0}{Z_1 + Z_0} = 1.2$$

b) $V_0^- = V_0^+\Gamma_s = 0.2V$ and $V_1^+ = V_0^+T = 1.2V$

c) $P_{in} = \frac{1}{2}\frac{V_0^{+2}}{Z_0} = 0.01W$

$$P_{avg} = \frac{1}{2}\frac{V_0^{+2}}{Z_0}(1 - \Gamma_s^2) = 0.009W$$

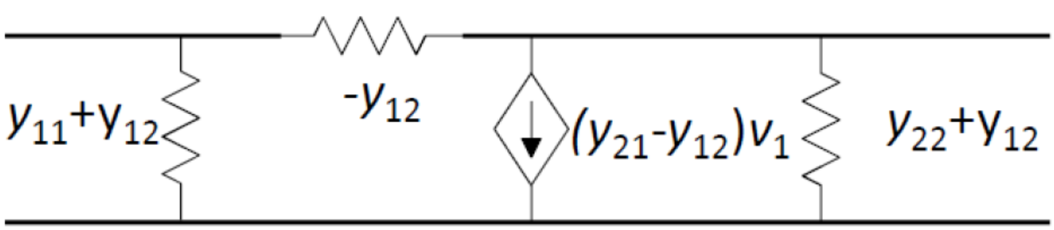
$$P_t = P_{in}\Gamma_s^2 = 0.0004W$$

$$P_{tr} = \frac{1}{2}\frac{V_1^{+2}}{Z_1} = 0.0096W$$

Energy is conserved because,

$$P_{in} - P_t = P_{tr} \;\&\; P_{tr} = P_{avg}$$

4. General hybrid pi model



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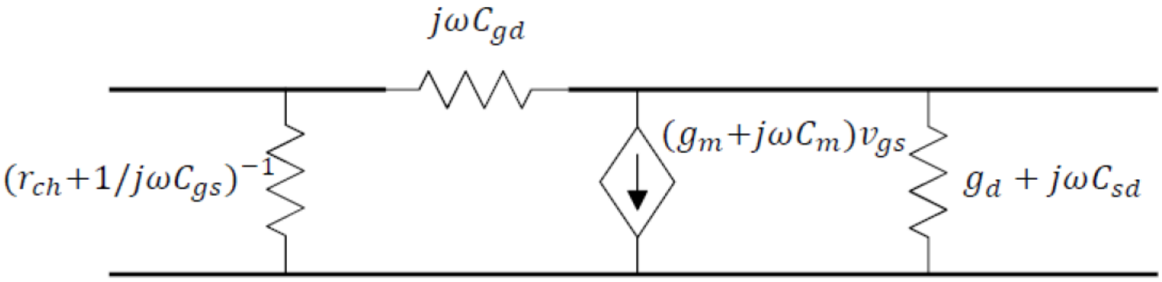
The corresponding simplified NQS model (R_S=R_D=R_G=0)



f_{max} is the frequency at which U=1,

$$U = \frac{|y_{21} - y_{12}|^2}{4[Re(y_{11})Re(y_{22}) - Re(y_{12})Re(y_{21})]}$$

Miller capacitance Cm is the feedback capacitance between gate and drain. So assuming Cm=0, y12=0

$$Re(y_{11}) = \frac{\omega^2 r_{ch}}{c_{gs}^2 + \omega^2 r_{ch}^2}$$

$$Re(y_{22}) = g_d$$

$$y_{21} - y_{12} = g_m(since\ C_m = 0)$$

$$Re(y_{12}) = 0$$

$$U = \frac{g_m^2(C_{gs}^2 + \omega^2 r_{ch}^2)}{4\omega^2 r_{ch} g_d} = 1$$

$$f_{max} = \frac{g_m C_{gs}}{2\pi\sqrt{4r_{ch}g_d - r_{cg}^2 g_m^2}}$$

5. $f_T = \frac{g_m}{2\pi C_{gg}}$,

For fT > 1THz, the minimum required gm is, $g_m = 2\pi C_{gg} f_T = 10^{12} * 2\pi * C_{gg}$

From 1a, the calculated Cgg = 0.23fF/µm.

Best case: When parasitic capacitances are of, $C'_{gs} = C'_{gd} = 0.1fF/\mu m$,

The total capacitance including parasitics is, $C'_{gg} = C_{gg} + C'_{gs} + C'_{gd} = 0.43fF/\mu m$

The minimum gm=2.7mS//µm

Worst case: When parasitics are of, $C'_{gs} = C'_{gd} = 0.5fF/\mu m$,

The total capacitance including parasitics is, $C'_{gg} = C_{gg} + C'_{gs} + C'_{gd} = 1.23fF/\mu m$

The minimum gm=7.72mS//µm

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Typical transconductances of quasi-ballistic FETs are 2-4mS/µm, thus it can be reached with the best-case condition considered. If the parasitic capacitances are too high, then it is not possible to achieve the desired transconductace for the given device.

6. De-normalizing the given parameters using the width W=10µm gives,

Gm=20mS

Gd=0.1mS

Ri=30Ω

Cgs=30fF

Cgd=3fF

To find the y-parameters at f=60GHz

$$y_{11} = \left(R_i + \frac{1}{j\omega C_{gs}}\right)^{-1} + j\omega C_{gd} = 0.0034 + j0.0113S$$

$\Omega y_{12} = -j\omega C_{gd} = -j0.0011S$

$$y_{21} = g_m = 0.02S$$

$$y_{22} = g_d = 0.1mS$$

Rollett's stability factor,

$$K = \frac{2[Re(y_{11})Re(y_{22}) - Re(y_{12})Re(y_{21})]}{|y_{12}y_{21}|} = 0.0304$$

Maximum stable gain

$$MSG = \left|\frac{y_{21}}{y_{12}}\right| = 12.47dB$$

Maximum available gain,

$$MAG = \left|\frac{y_{21}}{y_{12}}\right|\left(k - \sqrt{k^2 - 1}\right)$$

The device is unstable at 60GHs as k<1. Therefore, MAG gives complex number. To stabilize the device, add a passive feedback that cancels the y_{12}

The gain after unilateralization is

$$U = \frac{|y_{21} - y_{12}|^2}{4[Re(y_{11})Re(y_{22}) - Re(y_{12})Re(y_{21})]} = 24.64dB$$