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High Speed Electronics 2019 – Exercise 3 Solutions

1. a)

$$\omega_T = \frac{g_m}{C_{gs}} = 2 * 10^{12}\ rad/s$$

$$R = R_i + R_G = \frac{1}{1.4 g_m} + R_G = 40.7\Omega$$

For f = 10GHz,

$\omega = 2\pi f = 6.28*10^{10}$ rad/s ; $\omega << \omega_T$

$$F_{min} = 1 + 2\sqrt{\gamma}\frac{\omega}{\omega_T} = 1.06$$

NF = 10 log(F) = 0.25 dB

$$X_{opt} = \frac{1}{\omega C_{gs}} = 1592\ \Omega$$

$$R_{opt} = \sqrt{R^2 + \frac{R}{\gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2} = 1437\ \Omega$$

For f = 94GHz,

$\omega = 2\pi f = 0.6*10^{12}$ rad/s ; $\omega \approx \omega_T$

$$F_{min} = 1 + 2\sqrt{R\gamma g_m\left(\frac{\omega}{\omega_T}\right)^2 + g_m^2\gamma^2R^2\left(\frac{\omega}{\omega_T}\right)^4} + 2\gamma g_m R\left(\frac{\omega}{\omega_T}\right)^2 = 1.7$$

NF = 10 log(F) = 2.3 dB

$$X_{opt} = \frac{1}{\omega C_{gs}} = 167\ \Omega$$

$$R_{opt} = \sqrt{R^2 + \frac{R}{\gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2} = 156\ \Omega$$

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b) Assume $Z_s = R_s = 50\ \Omega$

$$F = 1 + \frac{v_n^2 + |Z_S|^2 i_n^2 + 2Re(v_n i_n^* Z_S^*)}{4kTR_S}$$

$$v_n^2 = 4kT\left(R + \frac{\gamma}{g_m}(1 + (\omega RC)^2)\right)$$

$$i_n^2 = \frac{4kT\gamma}{g_m}\left(\omega C_{gs}\right)^2$$

$$Re(v_n i_n^*) = \frac{4kT\gamma R}{g_m}\left(\omega C_{gs}\right)^2$$

For f = 10GHz,

$v_n^2 = 363$ kT
$i_n^2 = 8*10^{-5}$ kT
$Re(v_n i_n^*) = 3*10^{-3}$ kT
F = 2.82
NF = 4.5 dB

For f = 10GHz,

$v_n^2 = 375$ kT
$i_n^2 = 7*10^{-3}$ kT
$Re(v_n i_n^*) = 0.29$ kT
F = 3.11
NF = 4.9 dB

c)

$$v_n^2 = 4kT\left(R + \frac{\gamma}{g_m}(1 + (\omega RC)^2)\right)\Delta f$$

Assuming very low frequency, ωRC << 1

$$v_n^2 = 4kT\left(R + \frac{\gamma}{g_m}\right)\Delta f$$

At room temperature T = 293 K,

$$v_n^2 = 1.5*10^{-12}\ V^2$$

Smallest possible signal $v_n = 1.2\ \mu V$

d) Low frequency Δf = 1 Hz. C_{gs} looks open. So we need to calculate v_n^2 , i_n^2 and $v_n i_n^*$. We also need to move all the noise sources outside the circuit:

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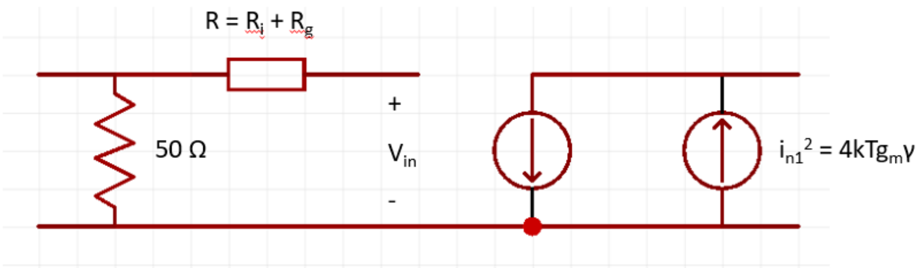
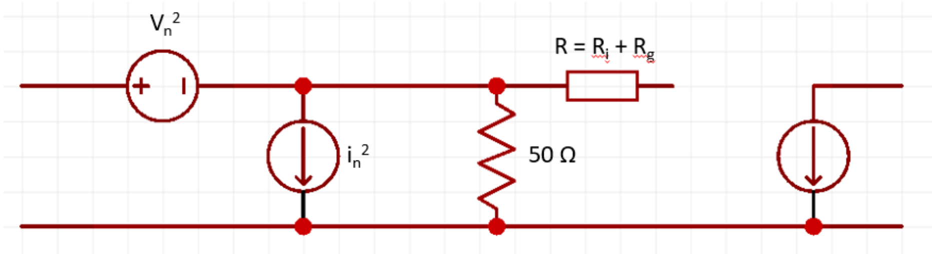
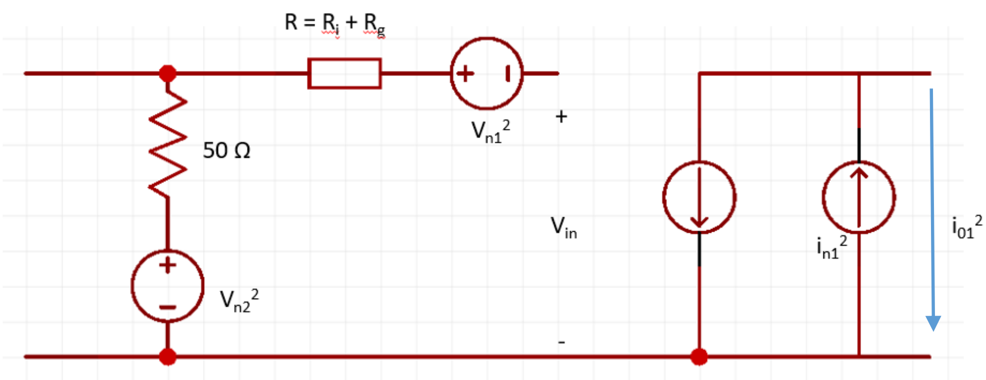


i_{n1}^2 represent the noise due to drift/diffusion in the FET
The aim is to find v_n and i_n associated with the noise due to 50Ω resistor and the channel.

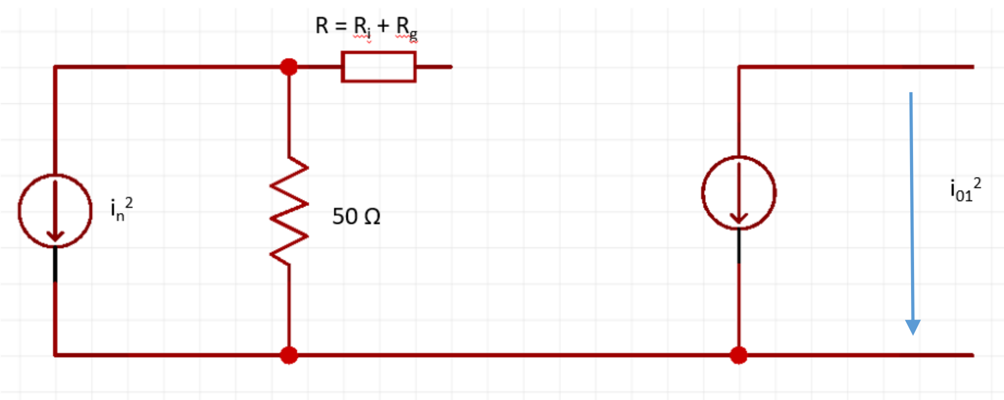


(i) To find i_n^2, consider input open:



$i_{01} = i_{n1} + (v_{n1} + v_{n2})*g_m$

Representing all the noise sources in the above circuit as i_n^2



$i_{01} = g_m i_n*50$

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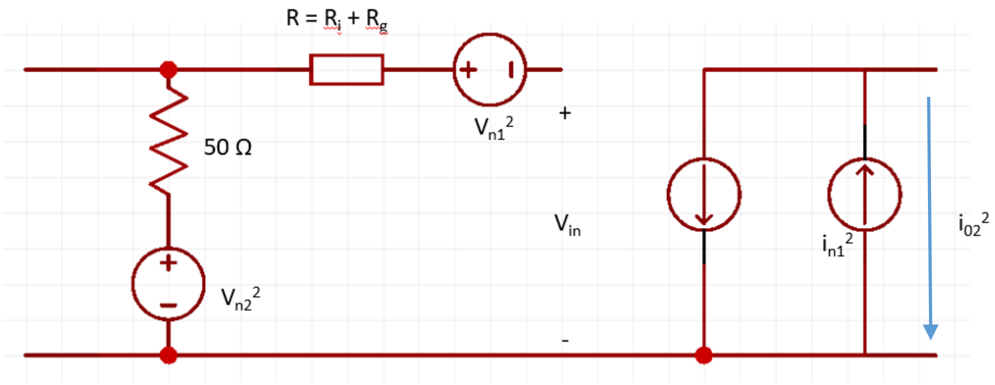
Equating i_{01} from the above circuits:

$$i_n^2 = \frac{i_{n1}^2}{g_m^2 50^2} + \frac{v_{n1}^2 + v_{n2}^2}{50^2} \quad (Assuming\ uncorrelated\ sources)$$

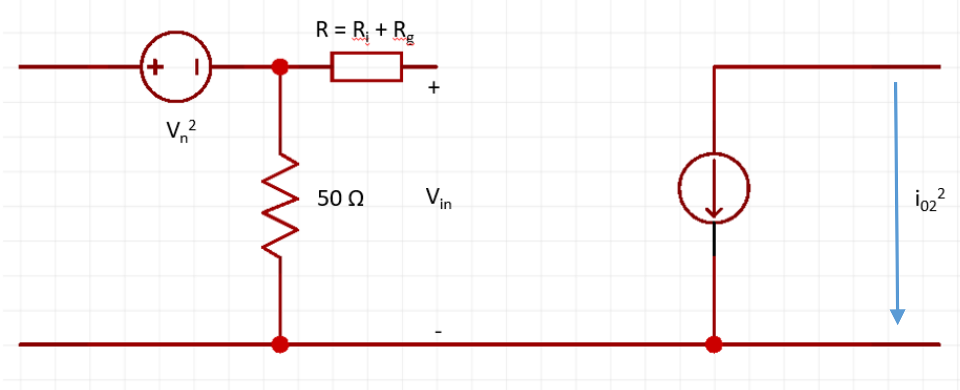
$$i_n^2 = 4kT\left(\frac{\gamma}{g_m * 50^2} + \frac{R+50}{50^2}\right) = 0.23\ kT$$

(ii) To find v_n^2, consider input shorted:



$i_{02} = i_{n1} + v_{n1}g_m$

Representing all the noise sources in the above circuit as v_n^2



$i_{02} = g_m v_n$

Equating i_{02} from the above circuits:

$$v_n^2 = v_{n1}^2 + \frac{i_{n1}^2}{g_m^2} = 4kT\left(R + \frac{\gamma}{g_m}\right) = 364\ kT$$

$$v_n i_n^* = \frac{v_{n1}^2}{50} + \frac{i_{n1}^2}{g_m^2 * 50} = 4kT\left(\frac{R}{50} + \frac{\gamma}{g_m * 50}\right) = 7.3\ kT$$

Thus the minimum noise figure for the device with a 50Ω resistance at the source is

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$$F = 1 + \frac{v_n^2 + |Z_s|^2 i_n^2 + 2Re(v_n i_n^* Z_S^*)}{4kTR_s} = 8.3$$

$$NF = 10\ log_{10}F = 9.2dB$$

2. a-c) Gate capacitance normalized to areas is given by

$$\frac{1}{C_g} = \frac{1}{C_{ox}} + \frac{1}{C_q} + \frac{1}{C_c}$$

Thus, the gate capacitance per unit width for a gate length of L_g =20nm

$$\frac{1}{C_g} = \left(\frac{1}{L_g C_{ox}} + \frac{1}{L_g C_q} + \frac{1}{L_g C_c}\right)$$

	m^*	ε_s	L_g*C_{ox} (fF/μ)	L_g*C_q (fF/μ)	L_g*C_c (fF/μ)	L_g (fF/μ)
InAs	0.023	15.15	0.888	0.307	0.952	0.184
$In_{0.53}Ga_{0.47}As$	0.041	13.9	0.888	0.548	0.856	0.242
GaN	0.2	8.9	0.888	2.672	0.547	0.301

d)

$$I_{ds,sat} = \frac{qW2\sqrt{2m^*}}{3\pi^2 h^2}\left(\frac{qC'_{ox}}{C'_{ox} + \frac{C_q}{2}}\right)^{\frac{3}{2}} (V_{GS} - V_T)^{\frac{3}{2}}$$

Considering all the common terms as a constant and substituting

C'_{ox}=$C_{ox}||C_c$=$\frac{C_{ox}C_c}{C_{ox}+C_c}$

$$I_{ds,sat} = K\sqrt{2m^*}\left(\frac{1}{1 + \frac{C_q(C_{ox} + C_c)}{2C_{ox}C_c}}\right)^{\frac{3}{2}}$$

$I_{ds,sat}$(GaN) = 0.06K

$I_{ds,sat}$($In_{0.53}Ga_{0.47}As$)=0.137K

$I_{ds,sat}$(InAs)=0.139K

Thus InAs has the highest saturation current given that there is no scattering.

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3. a) $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} j\omega C & -j\omega C \\ -j\omega C & jwC + R^{-1} \end{bmatrix}$

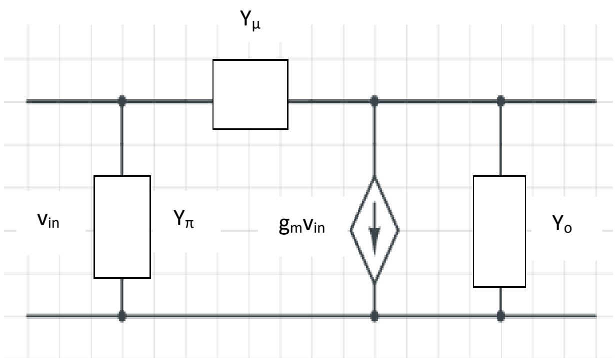
b) $Z = \frac{1}{Y_{11}Y_{22} - Y_{12}Y_{21}} \begin{bmatrix} Y_{22} & -Y_{21} \\ -Y_{12} & Y_{11} \end{bmatrix} = \begin{bmatrix} R + \frac{1}{j\omega C} & R \\ R & R \end{bmatrix}$

c) The added parasitic network is connected in series to the given RC network, so the Z parameters of the full system are sum of Z parameters of individual networks.

$$Z_{total} = \begin{bmatrix} R + j\omega C & R \\ R & R \end{bmatrix} + \begin{bmatrix} R_G + R_{SD} & R_{SD} \\ R_{SD} & 2R_{SD} \end{bmatrix} = \begin{bmatrix} 4R + \frac{1}{j\omega C} & 2R \\ 2R & 3R \end{bmatrix}$$

d) It is easier to convert from Y parameters to the hybrid-π model. Now the total Y parameters of network including the parasitics is given by,

$$Y = \frac{1}{Z_{11}Z_{22} - Z_{12}Z_{21}} \begin{bmatrix} Z_{22} & -Z_{21} \\ -Z_{12} & Z_{11} \end{bmatrix} = \frac{1}{R\left(8R + \frac{3}{j\omega C}\right)} \begin{bmatrix} 3R & -2R \\ -2R & 4R + \frac{1}{j\omega C} \end{bmatrix}$$



The hyrid parameters are, $Y_\pi = Y_{11} + Y_{12} = \frac{1}{\left(8R + \frac{3}{j\omega C}\right)}$

$$Y_\mu = -Y_{12} = \frac{2}{\left(8R + \frac{3}{j\omega C}\right)}$$

$$Y_o = Y_{22} + Y_{12} = \frac{2R + \frac{1}{j\omega C}}{R\left(8R + \frac{3}{j\omega C}\right)}$$

$$g_m = Y_{21} - Y_{12} = 0$$