Solutions to the Exam in EITP01 2020-01-09

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1. A) When the fermi level E_F lies within the energy band gap and far from the conduction and valence band edges. Then only the exponential tail of the Fermi-Dirac distribution will overlap with the finite density of states of the valence/conduction bands and thus the Fermi-Dirac distribution can be approximated by the exponential Boltzmann distribution. (**3p)**

   B) E_F-E_C = 0.1 eV means that the fermi level lies within the conduction band. Assuming 3D crystal, gives $n = N_{3D}\mathcal{F}_{\frac{1}{2}}\left(\frac{0.1q}{kT}\right)$. The Fermi-Dirac integral is the same for both materials, thus the difference lies in $N_{3D} = N_c = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} m_{de}^{3/2} \propto m_{de}^{\frac{3}{2}}$. The effective masses for Si and Ge are given by $m_{de}^{Si} = (6)^{\frac{2}{3}}(0.9163 * 0.1905^2)^{1/3} = 1.06$ and $m_{de}^{Ge} = \left(\frac{8}{2}\right)^{\frac{2}{3}}(1.588 * 0.0815^2)^{1/3} = 0.553$. Since $m_{de}^{Si} > m_{de}^{Ge}$ then the electron concentration is also larger in Si, actually its $(1.06/0.55)^{3/2}$ = 2.68x higher.

   Now what is n for the materials? The problem is that EF is inside the conduction band such that the Boltzmann approximation is not a good one. The Fermi-Dirac integral is difficult to calculate by hand but can be approximated in the degenerate case as $\mathcal{F}_{\frac{1}{2}}\left(\frac{0.1q}{kT} = 3.87\right) = \frac{3.87^{1+\frac{1}{2}}}{\Gamma\left(\frac{1}{2}+2\right)} = \frac{4}{3\sqrt{\pi}}7.61 = 5.73$, (Boltzmann would give 48 here! i.e. a gross overestimation)
   This gives for Ge: $N_c = 1.03 * 10^{19}\ cm^{-3}$ → n = $N_c * 5.73$ =5.9*10^{19} cm^{-3}, for Si $N_c = 2.75 * 10^{19}\ cm^{-3}$, n= 1.6*$10^{20}$ cm^{-3}. (**3p**)

   C) $N(k) = \frac{2V(k)}{V_k}$ where $V(k) = \pi k^2$ and $V_k = \left(\frac{2\pi}{L}\right)^2 \rightarrow N(k) = \frac{L^2k^2}{\pi}$

$$k = \sqrt{\frac{2m^*(E-E_c)}{\hbar^2}} \rightarrow \frac{\partial k}{\partial E} = \frac{1}{2}\sqrt{\frac{2m^*}{\hbar^2(E-E_c)}}$$

$$D_{2D} = \frac{1}{L^2}\frac{\partial N}{\partial E} = \frac{1}{L^2}\frac{\partial N}{\partial k}\frac{\partial k}{\partial E} = \frac{1}{L^2}\frac{L^2 2k}{\pi}\frac{1}{2}\sqrt{\frac{2m^*}{\hbar^2(E-E_c)}} = \frac{1}{\pi}\sqrt{\frac{2m^*(E-E_c)}{\hbar^2}}\sqrt{\frac{2m^*}{\hbar^2(E-E_c)}} = \frac{2m^*}{\pi\hbar^2}$$ (**4p**)

2. A) The semiconductor capacitance C_s or C_q originates in the finite density of states of the semiconductor. In other words, to change the channel charge one will need to shift the energy levels in relation to the fermi level. This requires some voltage to be applied. Thus $C_q = dQ_{ch}/dV_g$. The charge centroid capacitance C_c originates from the fact that having a nonzero charge in the quantum well will impact the band bending of the well, thus affecting the bound energy levels in the well, and in turn how much voltage needs to be applied to get a certain charge in the well. (**4p**)

   B) Current flows in both directions, from the source given by the following equation

$$J^{+} = \frac{q\sqrt{2m^*}}{2\pi^2\hbar^2}\frac{4}{3}\left(E_{fs} - E(0) = 0.2eV\right)^{\frac{3}{2}} = 1141\ A/m$$

And from the drain given by the following equation:

$$J^{-} = \frac{q\sqrt{2m^*}}{2\pi^2\hbar^2}\frac{4}{3}\left(E_{fs} - E(0) - qV_{DS} = 0.05\ eV\right)^{\frac{3}{2}} = 143\ A/m$$

The total current is the difference between these two currents giving:

$J_{tot} = J^{+} - J^{-} = 999\ A/m$, which with W = 10 µm gives $\underline{I = WJ_{tot} = 10\ mA.}$ **(4p)**

C) **4p**

	Origin	How affected by small L_G and why
R_{on}	Comes from limited channel conductance, i.e. limited number of carriers due to limited density of states. Can also have contributions from series resistances (access and contact resistance)	Shorter gate length should not affect the current in a ballistic device, as this is set by the number of injected carriers, not by resistance of the channel. However, in a diffusive FET R_{on} reduces with gate length for this reason. Series resistances are not affected by the gate length and thus if Ron is limited by these then there is no effect of reducing gate length.
g_d	Output conductance originates in that the drain bias is affecting the channel potential, thus lowering the barrier for carrier injection.	The shorter gate length the shorter distance between the top of the barrier E(0) and the drain, thus the gate will have a harder time to control the potential of the channel and the effect of the drain will be larger. Thus g_d should increase as L_G becomes very small.
BV_{ds}	The breakdown voltage is due to band-to-band tunnelling from the valence band in the channel to the drain conduction band, which happens at high electric fields.	As L_G decreases (given a certain V_{DS}) the electric field across the device increases. Thus breakdown will happen earlier and BV_{DS} decreases.

D) Current is independent of L_G in a ballistic device. If T < 1, then there is some amount of scattering, thus there will be a gate length dependence, in which shorter gate lengths have higher currents. (**3p**)

3. A) Assuming $L_G \gg \lambda$ means that one can assume that $C_{GG} = WL_G C_G'$. C_G' is given by the series capacitances C_{ox}, C_q and C_c → $C_G' = \left(\frac{1}{C_{ox}} + \frac{1}{2C_q} + \frac{1}{C_c}\right)^{-1}$ → $C_{GG} = 3.0\ fF, C_{GD} = \frac{C_{GG}}{10} = 0.30\ fF$. C_{dg}=C_{dd} = 0 because operated in saturation. This and the other parameters give the y-parameters as:

$$Y = \begin{bmatrix} j3 * 10^{-5} & -j3 * 10^{-6} \\ 0.02 & 0.002 \end{bmatrix}$$ (**3p**)

B) The elements of the hybrid-pi model are given by the following:

Left leg: y11+y12 = $0.02 + j0.018$

$$\left(\frac{1}{j\omega C_{gs}} + R_i\right)^{-1} = \frac{\omega^2 C_{gs}^2 R_i + j\omega C_{gs}}{1 + \omega^2 C_{gs}^2 R_i^2} = 0.02 + j0.018$$

➔ Equation system with two equations and two variables:

Real part: $\frac{\omega^2 C_{gs}^2 R_i}{1+\omega^2 C_{gs}^2 R_i^2} = 0.02,$ (1)

Imaginary part: $\frac{\omega C_{gs}}{1+\omega^2 C_{gs}^2 R_i^2} = 0.018$ (2)

Combine (1) and (2) to solve for C_{gs} and R_i →

$$R_i = 28\ \Omega$$
$$C_{gs} = 128\ fF$$

Right leg: y22+y12 = 0.006- j0.002

$$g_d = Re(y22 + y12) = 6\ mS$$
$$C_{sd} = -\frac{Im(y22 + y12)}{\omega} = 6.4\ fF$$
$$g_m = Re(y21) = 30\ mS$$
$$C_{gd} = -\frac{Im(y12)}{\omega} = 6.4\ fF$$

(**4p**)

C) $f_T = \frac{g_m}{2\pi C_{gg}}$ if excluding source and drain resistances which also reduces f_T. Thus one should reduce C_{gg} and increase g_m, as well as minimizing R_S and R_D. gm is increased by reducing the scattering in the channel as much as possible, or decreasing gate length if there is scattering. Importantly one must maintain good control of the channel potential while doing so, which means that going towards a finFET or gate-all-around nanowire structure will help. The current through the transistor should also be high, thus one should use a highly doped source and drain, and choose a material with as high DoS as possible. Parasitic $C_{gg} = C_{gs} + C_{gd}$ is reduced by adding spacers between the gate and the source and drain, and minimizing the overlap between the gate and the S/D. R_S and R_D is minimized by optimizing the current path from the contacts to the channel, i.e. minimizing distance, access resistance and contact resistance. (**4p**)

D) Old design:

$f_T = \frac{1}{2\pi}\left(\frac{C_{gg}}{g_m} + \frac{C_{gg}}{g_m}(R_S + R_D)g_d + (R_S + R_D)C_{gd}\right)^{-1}$ = 513 GHz

$$A_{old} = 70\ nm * 50\ nm = 3500\ nm^2, R_G = \frac{\frac{1}{3}W\rho_G}{A_{old}} = 143\ \Omega$$

$$f_{max} = \sqrt{\frac{f_T}{8\pi R_G C_{gd}\left(1+\frac{2\pi f_T}{C_{gd}}\Psi\right)}}, \Psi = \frac{(R_D+R_S)C_{gg}^2 g_d^2}{g_m^2} + (R_D+R_S)C_{gg}C_{gd}\frac{g_d}{g_m} + \frac{C_{gg}^2 g_d}{g_m^2}$$

$$f_{max} = 299\ GHz$$

New design: $A_{new} = 50\ nm * 70\ nm + 100\ nm * 100\ nm = 13500\ nm^2$

$$R_{G,new} = \frac{\frac{1}{3}W\rho_G}{A_{new}} = 37\ \Omega$$

But we lose some in parasitic capacitances Cgs = Cgs,i + Cgs,p and Cgd = Cgd + Cgd,p.

$$\rightarrow f_{T,new} = 476\ GHz \rightarrow f_{max,new} = 542\ GHz.$$

So we lose some in f_T but gain a lot in f_{max}. We get $\frac{f_{max,new}}{f_{max}} = 181\ \%$ (**4p**)

4. $A)\ Z_{opt} = R_{opt} + jX_{opt}$

$$R_{opt} = \sqrt{R^2 + \frac{R}{\gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2}, \omega_T = 2\pi f_T = \frac{g_m}{C_{gg}}, R = R_i + R_G = \frac{1}{1.4 g_m} + 50 = 121\ \Omega$$

$$\rightarrow R_{opt} = 597\ \Omega$$

$$X_{opt} = \frac{1}{\omega C_{gg}} = 531\ \Omega$$

$$F_{min} = 1 + \frac{2\sqrt{\gamma}\omega}{\omega_T} = 1.377 \rightarrow NF = 10\log(F_{min}) = 1.4$$

(**4p**)

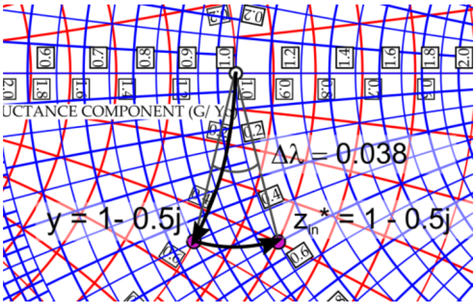
B) $K = \frac{1-|S_{11}|^2-|S_{22}|^2+|S_{11}S_{22}-S_{12}S_{21}|^2}{2|S_{12}S_{21}|} = 0.65 < 1$, i.e. transistor is not stable.

Since not stable use MSG = $\frac{|S_{21}|}{|S_{12}|} = 5$ (**3p**)

C) Target to change source impedance from Z_0=50 Ω to Z_{in}* = 50 – j25 Ω. $z_{opt} = \frac{Z_{in}^*}{50} = 1 - j0.5$ is at 1.85 cm from the center of Smith chart meaning that a TRL rotation from that point will intersect the conductance=1 circle at y = 1 - j0.5. Thus, we have to add a parallel stub to move from the center point at $Y_0 = 1/Z_0$ to this point, i.e. add $Y_{stub}Z_0 = y_{stub} = -j0.5$, i.e. a inductance. Let's use a shorted stub with $Y_{stub} = \frac{1}{Z_{stub}} = -j\frac{1}{Z_{TRL}\tan(\beta l_{stub})}$

For the TRL technology: $\varepsilon_{r,eff} = 2.2$ and $Z_{TRL} = 55.6\ \Omega$ ($W < H$). At f = 50 GHz → $\lambda = 4\ mm$ and $\beta = \frac{2\pi}{\lambda}$. Thus we can extract $l_{stub} = \frac{1}{\beta}\text{atan}\left(\frac{Z_0}{Im(y_{stub})Z_{TRL}}\right) = 680\ \mu m.$

From y=1 -0.5j, a series TRL should then rotate towards load by Δλ = 0.145 - 0.107 = 0.038. Thus $l_{TRL} = \frac{\Delta\lambda}{\beta} = 24$ µm. See graphical solution below. Note: The simplest solution is of course to add a series capacitance to move directly to z = 1 – 0.5j, but a series capacitor cannot be done with TRL technology only, but requires a real capacitor.



(**5p**)

D) In general building it with a $\frac{\lambda}{4}$ transformer would be preferable as it does not move to high Q values, thus giving very narrow band matching. However, in this particular example the resistance is already matched, meaning that one would only need a single capacitor to match it. (**3p**)