## Solution Written Examination EITP01 2019-03-21

1. Ballistic Transport in QWFET
a. (2p) $E_{n}=\frac{\hbar^{2} \pi^{2} n^{2}}{2 m^{*} t_{w}} \rightarrow \mathrm{E}_{1}=0.38 \mathrm{eV}, \mathrm{E}_{2}=1.51 \mathrm{eV}$
b. (3p) $\mathrm{C}_{\mathrm{ox}}=0.035 \mathrm{~F} / \mathrm{m}^{2}, \mathrm{C}_{\mathrm{c}}=0.069 \mathrm{~F} / \mathrm{m}^{2}, \mathrm{C}_{\mathrm{q}}=0.027 \mathrm{~F} / \mathrm{m}^{2}$

In on-state $C_{G}=\left(\frac{1}{C_{o x}}+\frac{1}{C_{C}}+\frac{1}{C_{q}}\right)^{-1}=\underline{0.013 \mathrm{~F} / \mathrm{m}^{2}}$.
However, in saturation $C_{G}=\left(\frac{1}{C_{o x}}+\frac{1}{C_{C}}+\frac{2}{C_{q}}\right)^{-1}=0.0085 \mathrm{~F} / \mathrm{m}^{2}$. The question was not clear on what was asked for, so both answers are correct. However, the result in saturation should be used in d-f.
c. (3p) As the drain bias increases, the drain fermi level is pulled down and less and less carriers will be injected from the drain into the channel. The reason is that the top of the band in the channel ends up further and further above the drain fermi level where in the limit a negligible amount of electrons reside. Then the only carriers in the channel will be the carriers injected from the source traveling from source to drain. Assuming no short channel effects, this amount of electrons is not affected by further change in drain bias, thus the current saturates.
d. (3p). $\mathrm{V}_{\mathrm{T}}=3.0-3.2+0.38 \mathrm{~V}=0.18 \mathrm{~V} . V_{d s, s a t}=\frac{V_{g s}-V_{T}}{1+\frac{C_{q}}{2 C_{o x}^{\prime}}}=0.21 \mathrm{~V}<\mathrm{V}_{\mathrm{DS}} \rightarrow$ Yes.
e. (2p) $I_{D S}=T * I_{D S, \text { ballistic }}=0.6 * 15.7 \mathrm{~mA}=10 \mathrm{~mA}$
f. (2p) $g_{m}=\frac{\partial I_{D S}}{\partial V_{G S}}=\frac{\partial}{\partial V_{G S}}\left(T * I_{D S, \text { ballistic }}\right)=44 \mathrm{mS}$
2. High frequency operation
a. (4p) Implicitly assumes operation in saturation $\rightarrow$ Use $\mathrm{C}_{\mathrm{q}} / 2 \rightarrow \mathrm{C}_{\mathrm{G}}=0.0082 \mathrm{~F} / \mathrm{m}^{2} \rightarrow \mathrm{C}_{\mathrm{gg}}$ $=\mathrm{C}_{\mathrm{G}}{ }^{*} \mathrm{~W}^{*} \mathrm{~L}_{\mathrm{G}}=7.4 \mathrm{fF} . f_{T}=\frac{C_{g g}}{2 \pi g_{m}}=1.2 \mathrm{THz}(1190 \mathrm{GHz})$
b. (6p) $R_{S}=R_{D}=62$ Ohm. ( $\left.R_{\text {contact }}=16 \Omega, R_{\text {access }}=3.1 \Omega, R_{\text {spacer }}=43 \Omega\right)$. The undoped spacer region is the main contributor to the resistance.

$$
\begin{aligned}
& \rho_{s, \text { access }}=\frac{1}{\sigma_{s, a c c e s s}}=\frac{1}{q n_{\text {access }} \mu_{\text {access }}}=1.25 * 10^{-5} \Omega \mathrm{~m} \\
& \rho_{s, \text { spacer }}=\frac{1}{\sigma_{s, \text { spacer }}}=\frac{1}{q n_{\text {spacer }} \mu_{\text {spacer }}}=5.2 * 10^{-4} \Omega \mathrm{~m}
\end{aligned}
$$

$$
R_{S H}=\frac{\rho_{s, a c c e s s}}{t_{w}}=2080 \Omega, L_{T}=\sqrt{\frac{\rho_{\sigma}}{R_{S H}}}=69 \mathrm{~nm}
$$

$$
\begin{aligned}
& R_{\text {contact }}=\frac{\sqrt{R_{S H} \rho_{\sigma}}}{W} \operatorname{coth}\left(\frac{L_{C}=100 \mathrm{~nm}}{L_{T}=69 \mathrm{~nm}}\right)=16 \Omega \\
& R_{\text {access }}=\frac{\rho_{s, a c c e s s} L_{\text {access }}}{t_{w} W}=\frac{1.25 E-5 * 15 \mathrm{~nm}}{6 n m * 10 \mu m}=3.1 \Omega \\
& R_{\text {spacer }}=\frac{\rho_{s, \text { spacer }} L_{\text {spacer }}}{t_{w} W}=\frac{5.2 E-4 * 5 \mathrm{~nm}}{6 n m * 10 \mu m}=43
\end{aligned}
$$

c. $\quad(5 p) C_{g g}=C_{g g, 0}($ from $2 a)+C_{g s, p}+C_{g d, p}=17.4 f F$. Add $R_{S}=R_{D}=62 O h m \rightarrow f_{T}=138 \mathrm{GHz}$.

## 3. Power gain

a. (2 p) The finite transconductance of the channel in a ballistic transistor induces a phase shift between input (gate) and output (drain) as well as limiting the power transfer. This is modelled by a channel resistance $R_{i}$.
b. (5 p) Parameters excluding $\mathrm{R}_{\mathrm{G}}, \mathrm{R}_{\mathrm{D}}$, and $\mathrm{R}_{\mathrm{s}}$ :

$$
\begin{gathered}
y_{11}=\left(\frac{1}{j \omega C_{g s}}+R_{i}\right)^{-1}+j \omega C_{g d} \\
y_{12}=-j \omega C_{g d} \\
y_{21}=\frac{g_{m}}{1+j \omega C_{g s} R_{i}}-j \omega C_{g d} \\
y_{22}=g_{d}+j \omega C_{g d} \\
\rightarrow Y=\left[\begin{array}{cc}
0.0010+0.011 j & -0.0031 j \\
0.040-0.0084 j & 0.0050+0.0031
\end{array}\right] \\
\rightarrow Z=\left[\begin{array}{cc}
17.26-28.1 j & 17.6+0.16 j \\
49.1+219 j & 58.8-6.1 j
\end{array}\right]
\end{gathered}
$$

Add $Z_{\text {res }}=\left[\begin{array}{cc}70 & 50 \\ 50 & 100\end{array}\right] \rightarrow Z^{\prime}=Z+Z_{\text {res }}=\left[\begin{array}{cc}87-28 j & 68+0.16 j \\ 99+219 j & 159-6.1 j\end{array}\right]$
c. (2 p) Using z-parameters including $R_{S}, R_{D}$ and $R_{G} \rightarrow k=1.29>1$. Yes, it is stable.
d. (2 p) For a stable device, use MAG. Use $z_{21}$ and $z_{12} \rightarrow$ MAG $=1.68=2.25 \mathrm{~dB}$.
e. $(4 \mathrm{p}) \mathrm{f}_{\mathrm{T}}=140 \mathrm{GHz} \rightarrow \mathrm{f}_{\max }=195 \mathrm{GHz}$ using model including $\mathrm{R}_{\mathrm{i}}(197 \mathrm{GHz}$ with standard model). Except for increasing $f_{T}$ more, one could work on reducing $R_{G}$ and $g_{d}$ as well as $\mathrm{C}_{\mathrm{gd}}$.
4. Designing a Low noise amplifier
a. (3 p) $\omega_{T}=\frac{g_{m}}{C_{g s}}=1.9 \mathrm{THz}, \omega=314 \mathrm{GHz} \ll \omega_{T}, Z_{\text {opt }}=351+j 303 \rightarrow z_{\text {opt }}=7+$ j6
$F_{-} \min =1.25 \rightarrow N F=0.96 d B$
b. (5 p) The distance from $Z_{0}$ (center) to $Z_{\text {opt }}$ is 6.5 cm in the Smith chart. Draw a circle with this radius and find the intercept going from $Z_{\text {opt }}$ "towards generator" (i.e. clockwise) with the "admittance $=1$ " at $z_{1}=0.09-0.28 j$. The angle between these corresponds to a phase shift (read scales) from $0.239 \lambda$ to $0.457 \lambda=0.218 \lambda$. The length of the microstrip TRL should thus be $l_{p h y s}=\frac{0.218}{\beta}=0.218 * \frac{c}{\sqrt{\varepsilon_{r, e f f}} \omega}$.
$\varepsilon_{r, e f f}=2.18 \rightarrow \lambda=4.06 \mathrm{~mm} \rightarrow \beta=\frac{2 \pi}{\lambda}=1.55 \mathrm{~mm}^{-1} \rightarrow$ length of TRL $=141 \mu \mathrm{~m}$.
A shorted stub is used to shift to from $z_{1}$ to the center $\left(z_{0}\right)$, since $z_{1} \rightarrow y_{1}=1+3.24 j$ this requires a normalized susceptance of $-\mathrm{j} 3.24 \rightarrow$ normalized impedance of j0.31. The characteristic impedance of the stub is $Z_{\text {stub }}=\approx \frac{60}{\epsilon_{r}} \ln \left(\frac{8 H}{W}+\frac{W}{4 H}\right)=60 \Omega \rightarrow$ a changing $l_{\text {stub }}=\frac{1}{\beta} \operatorname{atan}\left(\frac{Z_{0}}{Z_{\text {stub }}}\right)=160 \mu \mathrm{~m}$. (one can also use an open stub of length 1.0 mm ).
c. (5 p) The $\lambda / 4$ transformer should transform from $Z_{0}=50 \Omega$ to $351 \Omega . \rightarrow Z_{\text {trans }}=132 \Omega$. This means $W=54 \mathrm{~nm} \rightarrow \varepsilon_{r, e f f}=2.09 \rightarrow \lambda=4.1 \mathrm{~mm} \rightarrow \beta=1.52 \mathrm{~mm}^{-1} \rightarrow$
$l_{p h y s}=\frac{0.25}{\beta}=\underline{165 \mu \mathrm{~m}}$.

Then a positive normalized reactance of $6 j$ is needed to move from $z=351 / 50$ to $Z_{\text {opt }}$. An inductor with $\mathrm{L}=0.95 \mathrm{nH}$ in series can be used to shift to $\mathrm{Z}_{\text {opt }}$. Alternatively one can view the process in reverse, but have then to start at $Z_{o p t}^{*}$.
d. (2p) The network in c is more preferable from a robustness perspective since the path to $Z_{\text {opt }}$ stays close to the center line of the Smith chart $\rightarrow$ low $Q$ factor. This means that the design works for a larger frequency band around $f=100 \mathrm{GHz}$. (However, the narrow width of this line $\sim 50 \mathrm{~nm}$ means that it may be problematic to make in practice).

