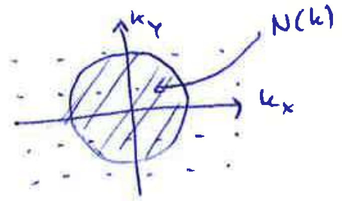
1 a) k-spacing $\frac{2\pi}{L}$ in 2D $\Rightarrow A_k = \left(\frac{2\pi}{L}\right)^2$: area of single state.

The number of states up to a specific k value is thus given by the area of a circle $\pi k^2 / A_k$



$$N(k) = \underbrace{2}_{\text{for spin}} \cdot \frac{\pi k^2}{A_k} = \frac{2 L^2 \pi k^2}{(2\pi)^2}$$

$$\frac{dN}{dE} = [\text{chain rule}] = \frac{dN}{dk} \cdot \frac{dk}{dE} = \frac{\cancel{4} L^2 \cancel{\pi} k}{\cancel{4} \pi^{\cancel{2}}} \cdot \frac{dk}{dE} = \frac{L^2 k}{\pi} \frac{dk}{dE}$$

$$E = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k = \frac{\sqrt{2m^* E}}{\hbar} \Rightarrow \frac{dk}{dE} = \frac{\sqrt{2}\, m^*}{2\sqrt{mE}\,\hbar}$$

$$\Rightarrow \frac{dN}{dE} = L^2 \frac{\cancel{\sqrt{2 m^* E}}}{\pi \hbar} \cdot \frac{\cancel{\sqrt{2}}\, m^*}{\cancel{2} \cancel{\sqrt{m^* E}}\, \hbar} = L^2 \frac{m^*}{\pi \hbar^2}$$

$$\boxed{D_{2D}(E) = \frac{1}{L^2}\frac{dN}{dE} = \frac{m^*}{\pi \hbar^2}}$$

b)

$$n_s = \underbrace{N_{2D}}_{\frac{m^* \cancel{kT}}{\pi \hbar^2}} \sum_n \underbrace{\left(1 + e^{(E_F - E_n)/kT}\right)}_{\approx \frac{E_F - E_n}{\cancel{kT}}} = \frac{m^*}{\pi \hbar^2}\overbrace{\left(E_F - E_1 + E_F - E_2\right)}^{\text{two bands only}}$$

$$E_n = \frac{\hbar^2 \pi^2}{2m^* t_w^2} \cdot n^2$$

$$= \frac{m^*}{\pi \hbar^2}\left(2E_F - \frac{\hbar^2 \pi^2}{2m^* t_w^2} - \frac{4\hbar^2\pi^2}{2m^* t_w^2}\right) = \frac{2m^*}{\pi \hbar^2} E_F - \frac{5\pi}{2 t_w^2}$$

$$\Rightarrow E_F = \frac{\pi \hbar^2}{2m^*}\left(n_s + \frac{5}{2}\frac{\pi}{t_w^2}\right)$$

$$\left.\begin{aligned} n_s &= 6 \cdot 10^{12}\ \text{cm}^{-2} \\ t_w &= 8 \cdot 10^{-9}\ \text{m} \end{aligned}\right\} \Rightarrow \boxed{E_F = 0{,}54\ \text{eV}}$$

But is this ok? $E_1 = 0{,}145\,\text{eV}$

$E_2 = 0{,}582$ eV ← higher than E_F... thus only one band.

$$n_s = \frac{m^*}{\pi \hbar^2} E_F - \frac{\pi}{2 t_w^2} \Rightarrow \boxed{E_F = 0{,}50\ \text{eV}}$$

2. a) The current in a MOSFET depends on the number of carriers, *n*, in the channel and their average velocity, v_{av}.

In a ballistic MOSFET, ***n* is given by the injected carriers from the source and drain, distributed in energy and k-vector according to the equilibrium Fermi-Dirac distribution of the source/drain. Electrons from the source all have positive k-vectors while electrons from the drain have negative.**

Since there is **no scattering in the channel**, these two injected carrier distributions will travel in opposite directions through the channel each with a certain average injection velocity, **creating two currents that counteract each other.**

The current from the drain will diminish as the drain bias increases as the number of carriers will be greatly reduced (exponentially) with increased drain bias. (The remaining carriers will have higher average velocity as they reside at larger k-values, however this increase is only by $\sqrt{E}$.) **In the limit the total current will be given only by the current from the source, thus leading to current saturation.**

b) As described above, the source and drain current density in saturation is given by the injected current from the source to the drain:

$$J^+ = qn_s^+ v_T$$

i.e. as the **function of both the carrier density and the velocity of the carriers.**

The injected carrier density is determined by the **density of states** of the source and channel material and the **source Fermi level** that can be adjusted by the doping level in the source. A high density of states leads to a high carrier concentration, and is achieved by a high effective mass. Furthermore, the injected charge density is determined by the **height of the channel barrier**, which is controlled via the **gate electrode through capacitive coupling**. A **high gate capacitance** is thus important to achieve efficient coupling.

The carrier velocity is $v_T = \sqrt{\frac{2k_BT}{\pi m^*}}$. Thus one should have **a low effective mass to achieve a high velocity**.

Overall in 2D, there is an optimum effective mass that balances density of states and carrier velocity.

3) a) $C_{gg} = W \cdot L \left(C_f \,//\, C_{ox} \,//\, C_c \right)$

C_f $\quad$ $C_{f/2}$ in saturation.

$C_{ox} = \frac{\varepsilon_{ox} \cdot \varepsilon_0}{t_{ox}} = 0{,}0266\ F/m^2$

$W = 20\,\mu m$

$L = 40\,nm$

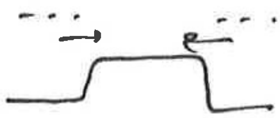
$C_f = \frac{m^* q^2}{\pi \hbar^2} = \begin{cases} 0{,}02697 \\ 0{,}01345 \end{cases} F/m^2$ $\Rightarrow$

$C_c = \frac{\varepsilon_r \cdot \varepsilon_0}{0{,}36 \cdot t_w} = 0{,}0399\ F/m^2$

$\Rightarrow$ $C_{gg} = 8\ fF \quad (V_{DS} = 0)$

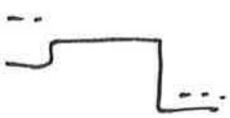
$5{,}8\ fF \quad (V_{DS} \geq V_{DS,sat})$

b) $V_{DS} = 0$:



$Q_S = Q_D \Rightarrow C_{gs} = C_{gd} = \frac{C_{gg}}{2}$

$V_{DS} \geq V_{DS,sat}$



No effect of V_D since $Q_D = 0$ $\Rightarrow$ $C_{gs} = C_{gg}$, $C_{gd} = 0$

c) $C_{ox}' = C_c // C_{ox} = 0{,}0159\ F/m^2$

$C_f = 0{,}01345$

$V_{GS} - V_T = 0{,}5$

$$i_{ds} = \frac{q^{5/2}\, W \cdot 2\sqrt{2 m^*}}{3 \pi^2 \hbar^2} \underbrace{\left[\frac{C_{ox}'}{C_{ox}' + C_f/2} \right]^{3/2}}_{0{,}39} \underbrace{(V_{GS} - V_T)^{3/2}}_{0.33} = 43\ mA$$

$m^* = 0{,}04 \cdot m_0$

d) $\lambda_0 = \frac{2kT}{q} \cdot \mu_n / v_T = 9{,}8 \cdot 10^{-7} \approx \underline{\underline{10nm}}$

μ_n: 500, v_T: $2.6 \cdot 10^7$

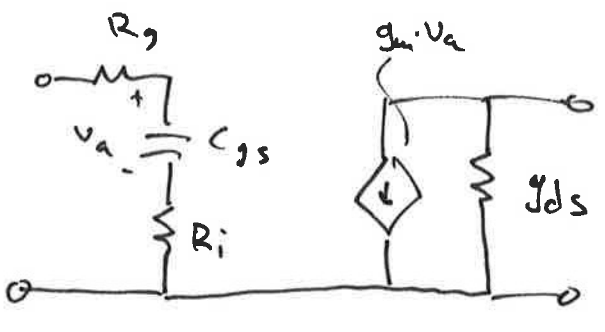
$$T = \frac{\lambda_0}{\lambda_0 + L_g} = \frac{10}{10+40} = \underline{\underline{0{,}2}}$$

$\Rightarrow$ $i_{ds,scattering} = 0{,}2 \cdot \underline{i_{ballistic}}$

e) Only thing in ballistic i_{ds} which depends on ~~the~~ t_w is $\underline{C_c}$!

C_c increases w/ decreasing t_w $\rightarrow$ higher current!

4)



$R_i = \frac{1}{1{,}4 \cdot g_m}$: Set $R_i + R_g = R$

a) $Y_{11} = \left.\frac{i_1}{V_1}\right|_{V_2=0} = \frac{1}{Z_{in}} = \left(R + \frac{1}{j\omega C_{gs}}\right)^{-1} = \left(\frac{1 + j\omega C_{gs} \cdot R}{j\omega C_{gs}}\right)^{-1} =$

$= \frac{j\omega C_{gs}}{1 + j\omega C_{gs} \cdot R} = \underline{\frac{j\omega C_{gs} + \omega^2 C_{gs}^2 \cdot R}{1 + (\omega C_{gs} \cdot R)^2}}$

$Y_{12} = \left.\frac{i_1}{V_2}\right|_{V_1=0} = \underline{0}$

$V_a = V_1 \frac{1/j\omega C}{R + 1/j\omega C}$

$Y_{21} = \left.\frac{i_2}{V_1}\right|_{V_2=0} = g_m V_a = \frac{g_m}{1 + j\omega C_{gs} \cdot R} = \underline{\frac{g_m(1 - j\omega C_{gs} \cdot R)}{1 + (\omega C_{gs} R)^2}}$

$Y_{22} = \left.\frac{i_2}{V_2}\right|_{V_1=0} = \underline{g_{ds}}$

b) $MSG = \left|\frac{Y_{21}}{Y_{12}}\right| \to \underline{\underline{\infty}}$ Since $Y_{12} = 0$, T is always stable!

→ Instead, use <u>MAG</u> or <u>U</u>!

Same!

4

c) $$|h_{21}| = \left|\frac{Y_{21}}{Y_{11}}\right| = \left|\frac{g_m}{\cancel{1+j\omega C_{gs}R}} \cdot \frac{\cancel{1+j\omega C_{gs}\cdot R}}{j\omega C_{gs}}\right| = \frac{g_m}{\omega\cdot C_{gs}}$$

$$f_T: |h_{21}| = 1 \Rightarrow f_T = \frac{g_m}{2\pi\cdot C_{gs}} = 157\,\text{GHz}$$

d) $$f_{max}:\ \overset{\text{(of unilateral)}}{U} = \frac{|Y_{21}|^2}{4\,\underbrace{Re(Y_{11})}_{\frac{\omega^2\cdot C_{gs}^2\cdot R}{1+(\omega C_{gs}R)^2}}\,\underbrace{Re(Y_{22})}_{g_{ds}}}$$

$Y_{12}=0$!

$$|Y_{21}|^2 = g_m^2\,\frac{(1-j\omega C_{gs}\cdot R)(1+j\omega C_{gs}\cdot R)}{(1+(\omega R C_{gs})^2)^2} = \frac{g_m^2}{1+(\omega R C_{gs})^2}$$

$$= \frac{g_m^2}{4\cdot\omega^2\cdot C_{gs}^2\cdot R\cdot g_{ds}}$$

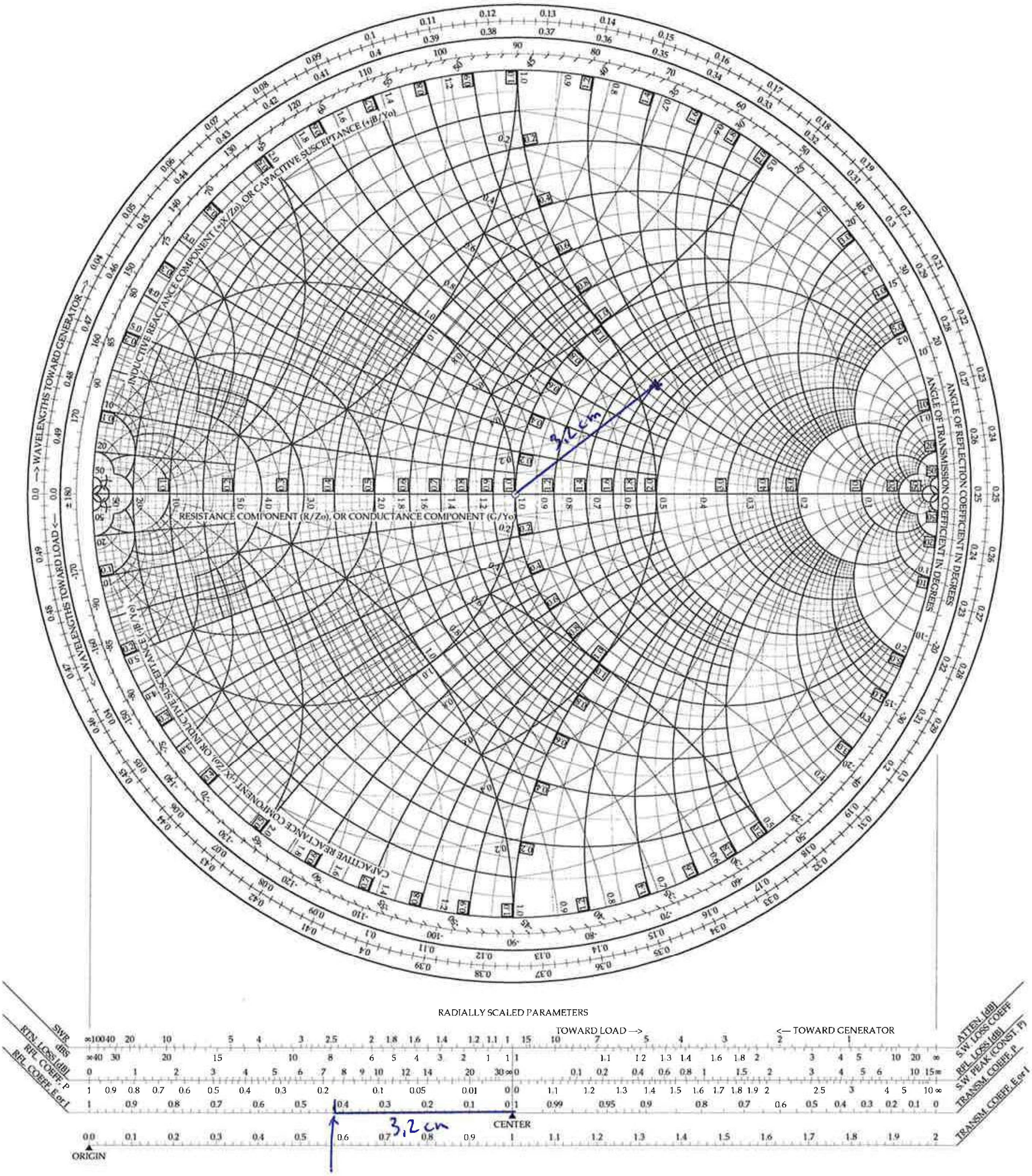
$$f_{max}: U=1 \Rightarrow f_{max} = \sqrt{\frac{g_m^2}{4\cdot R\cdot C_{gs}^2\cdot g_{ds}}} = 170\,\text{GHz}$$

e) $R \to R_i \Rightarrow f_{max} = 190\,\text{GHz}$

f) $$F_{min} = 1 + 2\sqrt{(R_g+R_i)\cdot r\cdot g_m}\cdot\frac{f}{f_T} = \begin{cases} 1{,}17 & R_g \neq 0 \\ 1{,}18 & R_g = 0 \end{cases}$$

5a) $Z(\omega = 10\,GHz) = 80 + j50 \rightarrow z = 1,6 + j1$

The Smith Chart



$\Gamma = 0,42$

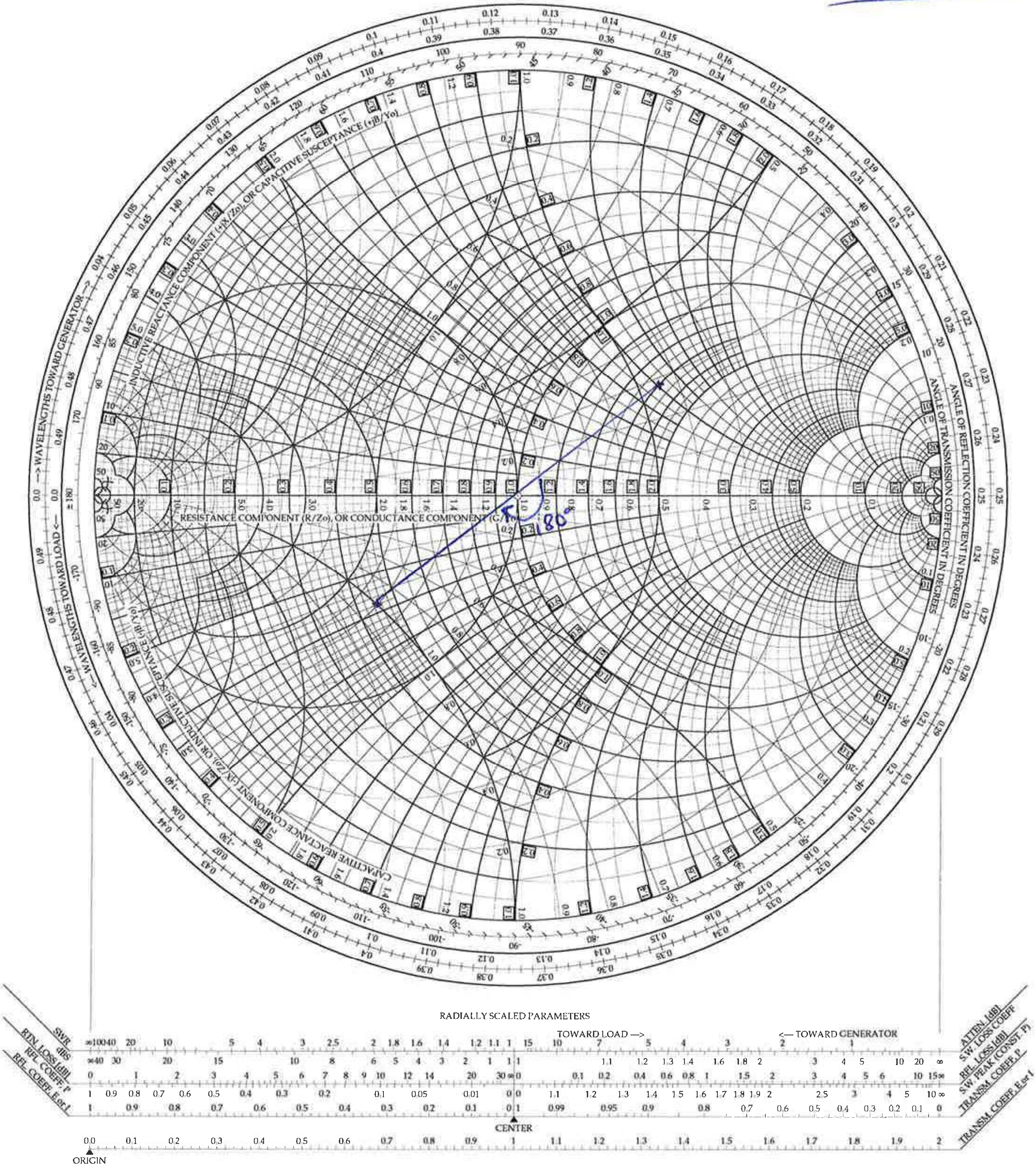
$$\Rightarrow \frac{P_r}{P_{in}} = |\Gamma|^2 = 0,42^2 = 0,18 = \boxed{18\%}$$

5b) $f = \frac{10 \cdot 10^9}{2\pi} = 1{,}59\ \text{GHz}$

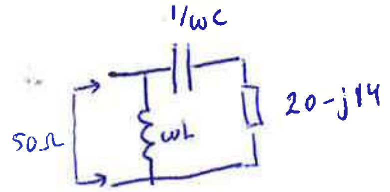
$\lambda = \frac{c}{\sqrt{\varepsilon_r}} \frac{1}{f} = 10{,}88\ \text{cm} \Rightarrow 57{,}1\ \text{cm} = 5{,}25\lambda \Rightarrow$ phase shift of $180°$

The Smith Chart

$\Rightarrow Z_{new} = 0{,}45 \cdot 50\Omega - j\,0{,}28 \cdot 50\Omega \Rightarrow \boxed{Z_{new} = 20 - j14\ \Omega}$



5c)



1. $\frac{1}{\omega C} = 0{,}22 \cdot 50\,\Omega = 11\,\Omega \rightarrow \boxed{C = 9\,pF}$

2. $\frac{1}{\omega L} = 1{,}075 \cdot \frac{1}{50\,\Omega} = 0{,}0215\,\frac{1}{\Omega} \Rightarrow L = 4{,}65\,nH \approx \boxed{4{,}6\,nH}$

The Smith Chart