Formulas for **High Speed Devices**
2018

1 Semiconductor Physics

1.1 3D Energy Bands

Kinetic Energy:

$$E - E_c = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2 + k_z^2)$$

Effective mass:

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

Group Velocity:

$$v_x = \frac{1}{\hbar}\frac{dE_k}{dk_x} = \frac{\hbar}{m^*}k_x$$

Density of states:

$$D_{3D}\frac{(2m^*)^{1.5}}{2\pi^2\hbar^3}\sqrt{E - E_c}$$

1.2 2D Energy Bands

Infinite quantum well energies:

$$E_n \approx \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$$

Sub-band energy dispersion:

$$E_k = \frac{\hbar}{2m^*}(k_x^2 + k_y^2) + E_n + E_c$$

Density of states:

$$D_{2D} = \sum \frac{m^*}{\pi\hbar^2}\Theta(E - E_n)$$

Effective density of states:

$$N_{2D} = \frac{m^* kT}{\pi \hbar^2}\ [m^2]$$

$$n_s(E_F) = \int_{E_c}^{\infty} D_{2D} f_0(E - E_n) dE = N_{2D} \sum_n ln\left[1 + e^{\frac{E_F - E_n}{kT}}\right]$$

1.3 1D Statistics

Carrier concentration:

$$n_L(E_F - E_C) = N_{1D} F_{-1/2}(\eta_F)$$

Effective density of states:

$$N_{1D} = \frac{\sqrt{2m^* kT}}{\hbar\sqrt{\pi}} m^{-1}$$

1.4 Crystal Growth

Lattice mismatch:

$$f = \frac{\Delta a}{a} - 0.014$$

Effective hole mass:

$$m_h^* = \frac{(m_{hh}^*)^{3/2} + (m_{lh}^*)^{3/2}}{(m_{hh}^*)^{1/2} + (m_{lh}^*)^{1/2}}$$

1.5 Fermi Level and F/D Statistics

Fermi-Dirac distribution:

$$f_0(E, E_F) = \frac{1}{1 + e^{(\frac{E - E_F}{kT})}}$$

Total concentration of free electrons:

$$n = \int_{E_c}^{\infty} D_{3D}(E) f_0(E - E_F) dE$$

Effective density of states:

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{1.5}$$

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{1.5}$$

Electron concentration:

$$\frac{n}{N_C} = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{\eta}}{1+e^{\eta-\eta_F}} d\eta = F_{1/2}(\eta_F)$$

$\eta_F = \frac{E_F - E_n}{kT}$

Maxwell-Boltzmann statistics, valid if $n < 0.05N_c$ or $E_c - E_F >> 3kT$:

$$n \approx N_c e^{(E_F - E_C)/kT}$$
$$p \approx N_v e^{(E_v - E_F)/kT}$$

If n is known:

$$\frac{E_f - E_C}{kT} \approx ln\left(\frac{n}{N_C}\right) + \frac{n}{N_C} \cdot \frac{1}{(64 + \frac{3.6n}{N_C})^{1/4}}$$

Joyce Dixon approximation:

$$\frac{E_f - E_C}{kT} \approx ln(\frac{n}{N_C}) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_C}\right) - 0.00495\left(\frac{n}{N_C}\right)^2 + ...$$

1.6 Intrinsic Semiconductors

Carrier concentration, only valid if $n < N_c$ and $p < N_V$:

$$n = p = n_i = \sqrt{N_V N_C} \cdot e^{-\frac{E_G}{2kT}}$$

1.7 Doped Semiconductors

n-doping ($N_d >> n_i$):

$$n = \frac{\sqrt{N_d^2 + 4n_i^2} + N_d}{2} \approx N_d$$

p-doping ($N_a >> n_i$):

$$p = \frac{\sqrt{N_a^2 + 4n_i^2} + N_a}{2} \approx N_a$$

Law of mass action:

$$np = n_i^2$$

For n-type material:

$$n_n \approx N_d >> n_i$$

$$p_n \approx \frac{n_i^2}{N_d} << n_i$$

1.8 Electrostatics

Poisson's equation:

$$\frac{d^2}{dx^2}v(x) = -\frac{q}{\epsilon_r \epsilon_0}(p(x) - n(x) + N_d(x) - N_a(x)) \quad \textbf{(1D)}$$

$$\Delta V(x, y, z) = \frac{\rho(x, y, z)}{\epsilon_r \epsilon_0} \quad \textbf{(3D)}$$

Relations between $\mathcal{E}$, V and ρ:

$$\mathcal{E}(x) = -\bigtriangledown V$$
$$\frac{q}{\epsilon_s}\rho(x) = \bigtriangledown \mathcal{E}$$

Potential energy of an electron:

$$E_p = -qV(x) + E_C$$

Potential energy of a hole:

$$E_p = (+)qV(x) + E_V$$

1.9 Drift and Diffusion

Long Devices - *accurate for low $\mathcal{E}$-fields, low carrier concentrations and long devices.*
Electron current density:

$$J = q\mu_n n\epsilon + qD_n \frac{dn}{dx}$$

Hole Current Density:

$$J = q\mu_p p\epsilon - qD_p \frac{dp}{dx}$$

Einsteins relation:

$$D_n = \frac{kT}{q} \cdot \mu_n$$

Drift velocity:

$$v_n = \mu_n \cdot \mathcal{E}$$

Short (hot) Devices - *accurate for short (hot) devices and for high $\mathcal{E}$-fields.*

Velocity saturation:

$$v_{el} \approx \frac{\mu_n \cdot \mathcal{E}}{1 + \mathcal{E}/\mathcal{E}_{crit}}$$

Critical $\mathcal{E}$-field around 3-4 kV/cm

Ballistic Devices - *accurate for nanoscale devices.* $\lambda > L$.

$$v_x = \frac{\hbar k_x}{m^*}$$

2 Transistor Fundamentals

2.1 n-type FET

On-resistance:

$$R_{on} = \left(\frac{dI}{dV_{ds}}\right)^{-1}\Bigg|_{V_{DS}\to 0V} [\Omega \mu m]$$

Output conductance:

$$g_d = \frac{dI_D}{dV_{DS}}\Bigg|_{V_{DS},V_{GS}}$$

Transconductance:

$$g_m = \frac{dI_D}{dV_{GS}}\Bigg|_{V_{DS}V_{GS}}$$

Geometric length scale:

$$\lambda = t_s + 2t_i$$
$$L > 2\lambda$$

where t_i is the thickness of the insulator, and t_s in the thickness of the channel.

3 2D FET Electrostatics

3.1 Charge Density FET

On-state:

$$qn_s = C_G(V_{GS} - V_T)$$

Off-state:

$$qn_s \approx N_{2D} e^{\frac{q}{kT}(V_{GS}-V_T)} \approx 0$$

3.2 Gate Capacitance

Quantum capacitance:

$$C_q = \frac{q^2 m^*}{\pi \hbar^2}$$

Quantum well charge:

$$n_s = N_{2D} F_0(\eta_F) \approx \frac{m^*}{\pi \hbar^2}(E_F - E_1)$$
$$q n_s \approx \frac{q^2 m^*}{\pi \hbar^2} \Psi_s = C_q \Psi_s$$

Oxide capacitance:

$$C_{ox} = \frac{\epsilon_{ox} \epsilon_0}{t_{ox}}$$

Centroid Capacitance:

$$C_c = \frac{\epsilon_r \epsilon_0}{0.36 t_w}$$

Gate Capacitance:

$$\frac{1}{C_G} = \frac{1}{C_q} + \frac{1}{C_{ox}} + \frac{1}{C_c}$$

Gate-Source Voltage:

$$V_{GS} = \frac{q n_s}{C_{ox}} + \frac{q n_s}{C_q} + \frac{q n_s}{C_c}$$

3.3 Threshold Voltage

Ideal MOS:

$$V_T = \phi_m - \chi + \frac{E_q}{q}$$

Ideal HEMT:

$$V_T = \phi_b - \Delta E_c + \frac{E_1}{q} - \phi_{00}$$

where

$$\phi_{00} = \frac{q N_D}{2 \epsilon_r \epsilon_0}(t_b - \delta)^2$$

4 Ideal Ballistic FETs

Drain current in saturation:

$$I^+ \approx \frac{qW2\sqrt{2m^*}}{2\pi^2\hbar^2}\left(\frac{q \cdot C'_{ox}}{C'_{ox} + \frac{C_q}{2}}\right)^{3/2}(V_{GS} - V_T)^{3/2}$$

Drain voltage in saturation:

$$V_{ds} \approx \frac{V_{GS} - V_T}{1 + \frac{C_q}{2C_{ox}}}$$

where $C'_{ox} = C_{ox}||C_c$

5 Real Ballistic devices

Mean free path:

$$\lambda_0 \approx v_T \cdot \tau$$

Transmission probability:

$$T = \frac{\lambda_0}{\lambda_0 + L_{eff}}$$

Scattering current:

$$I_{scattering} = T \cdot I_{ballistic}$$

6 Resistance

6.1 3D

$$R = \rho \cdot \frac{l}{A}$$

$$\rho_n = \frac{1}{qn\mu_n}$$

$$\rho_p = \frac{1}{qp\mu_p}$$

6.2 2D

$$R = \frac{\rho L}{tW} = R_{SH}\frac{L}{W}$$

$$n_s = \int_0^t n(x)dx$$

$$R_{SH} = \frac{1}{qn_s\mu_n}$$

6.3 Semiconductor Contacts

Contact Resistance:

$$R_C = \frac{\sqrt{R_{SH}\rho_\sigma}}{W} \coth\left(L_c\sqrt{\frac{R_{SH}}{\rho_\sigma}}\right)$$

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)}$$

Access resistance:

$$R_{lead} = \frac{R_{SH}L_{gs,gd}}{W}$$

Transfer length:

$$L_T = \sqrt{\frac{\rho_\sigma}{R_{SH}}}$$

Drain and Source resistance:

$$R_D = R_C + R_{lead,d}$$
$$R_S = R_C + R_{lead,s}$$

7 FET AC properties

7.1 Quasi-Static Operation

= *The charging time of the inversion layer is ignored.*

Gate current:

$$i_g = \frac{d}{dt}Q_G(t)$$

Drain Current:

$$i_D(t) = I_D(t) + \frac{d}{dt}Q_D(t) = I_D(t) + i_d(t)$$

Source current:

$$i_S = -I_D(t) + \frac{d}{dt}Q_S(t) = I_S(t) + i_s(t)$$

Charge neutrality:

$$Q_G(t) + Q_{CH}(t) = 0$$
$$Q_{CH}(t) = Q_D(t) + Q_S(t)$$
$$Q_G + Q_S + Q_D = 0$$
$$Q_S + Q_D = Q_G$$

Gate charge in saturation:

$$C'_G = \left(\frac{1}{C_{ox}} + \frac{2}{C_q} + \frac{1}{C_c}\right)^{-1}$$
$$Q_G = WLC'_G(v_{GS} - V_T)$$

7.2 Non Quasi-Static Operation

= It takes a finite time to charge the inversion layer due to finite velocity of the carriers. Introduces lag (resistors to C_{gg} and C_{gd}).

Channel resistance (resistive C_{gs}):

$$R_i = \frac{1}{2 \cdot 0.7WC_{gs}v^+} = \frac{1}{1.4g_m}$$

$$t_{1/2} \approx \frac{L_G}{2v^+}$$

8 Hybrid Pi Model

8.1 Large/Small Signal

$$i_G = I_{GS} + i_{gs} = I_{GS} + y_{11}v_{gs} + y_{12}v_{ds}$$
$$i_D = I_{DS} + i_{ds} = I_{DS} + y_{21}v_{gs} + y_{22}v_{ds}$$

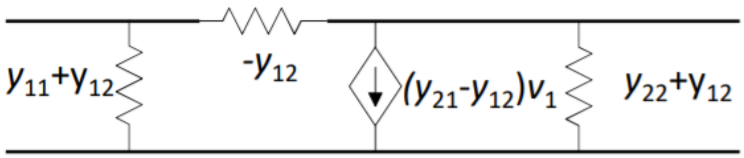
y-parameters:

$$y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0}$$
$$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$$
$$y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0}$$
$$y_{22} = \left.\frac{i_2}{v_2}\right|_{v_2=0}$$

Common source FET y-parameters:

$$y_{11} = j\omega C_{gg,t}$$
$$y_{12} = -j\omega C_{gd,t}$$
$$y_{21} = g_m - j\omega C_{dg,t}$$
$$y_{22} = g_d + j\omega C_{dd,t}$$

General hybrid-Pi:



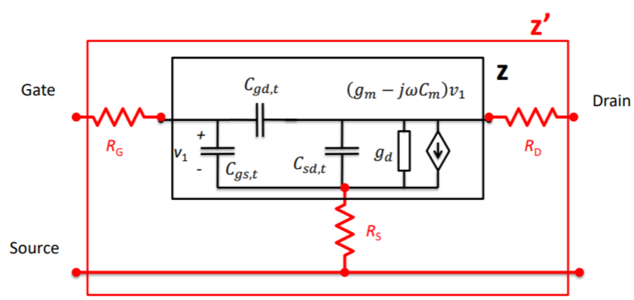
Capacitances:

$$C_{gs} = C_{gg} - C_{gd}$$
$$C_{sd} = C_{dd} - C_{dg}$$
$$C_m = C_{dg} - C_{gd}$$
$$In \quad saturation:$$
$$C_{gg} = C_{gs}$$
$$C_{dd} \approx 0$$
$$C_{dg} \approx 0$$

Parasitic drain and source resistance:

$$R_S = R_C + R_{lead,s}$$
$$R_D = R_C + R_{lead,d}$$

Complete small signal MOSFET model:



$$[z'] = [z] + \begin{bmatrix} R_G + R_S & R_S \\ R_S & R_S + R_D \end{bmatrix} = \begin{bmatrix} R_G + R_S + \frac{y_{22}}{\Delta y} & R_S - \frac{y_{12}}{\Delta y} \\ R_S - \frac{y_{21}}{\Delta y} & R_S + R_D + \frac{y_{11}}{\Delta y} \end{bmatrix}$$

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}$$

9 Gain and Stability

9.1 Current Gain

No parasitic resistances ($R_S = R_D = 0$):

$$h_{21} = -\frac{z_{21}}{z_{22}} = \frac{g_m - j\omega C_{dg,t}}{j\omega C_{gg,t}} \approx \frac{g_m}{j\omega C_{gg,t}}$$

$$f_T = \frac{g_m}{2\pi C_{gg,t}}$$

With parasitic resistances (R_S & $R_D > 0$):

$$h'_{21} = \frac{g_m}{j\omega[C_{gg,t} + (R_S + R_D)C_{gg,t}g_d + (R_D + R_D)C_{gd,t}g_m]}$$

$$f_T = \frac{1}{2\pi}\left(\frac{C_{gg,t}}{g_m} + \frac{C_{gg,t}}{g_m}(R_S + R_D)g_d + (R_S + R_D)C_{gd,t}\right)^{-1} \approx \frac{g_m}{2\pi C_{gg,t}}$$

where $C_{gg,t} = C_{gg}(L)\ (intrinsic)\ + C_{gg,p}\ (parasitic)$

9.2 Gain Expressions

Available gain:

$$G_A = \frac{P_{out}}{P_{AVS}}$$

Operating gain:

$$G_P = \frac{P_L}{P_{in}}$$

Transducer gain:

$$G_T = \frac{P_L}{P_{AVS}}$$

9.3 Stability

Sterns Stability factor, stable if k>1:

$$k = \frac{2Re(y_{11}) \cdot Re(y_{22}) - Re(y_{12}) \cdot Re(y_{21})}{|y_{12} \cdot y_{21}|}$$

Stability factor, S-parameters:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2}{2|S_{21}S_{S_12}|}$$
$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

9.4 Power Gain Definitions

Maximum Available Gain:

$$MAG = \left|\frac{y_{21}}{y_{12}}\right| \cdot (k - \sqrt{k^2 - 1})$$

Maximum Stable Gain:

$$MSG = \left|\frac{y_{21}}{y_{12}}\right|$$

Unilateral Gain:

$$U = \frac{|\theta_{21} - \theta_{12}|^2}{4[Re(\theta_{11}) \cdot Re(\theta_{22}) - Re(\theta_{12}) \cdot Re(\theta_{21})]}$$

where θ_{ij} is y, z, h, ...

$$f_{max} = \sqrt{\frac{f_T}{8\pi R_G C_{gd,t}\left[1 + \frac{2\pi f_T}{C_{gd,t}}\Psi\right]}}$$

$$\Psi = (R_D + R_S)\frac{C_{gg,t}^2 \cdot g_d^2}{g_m^2} + (R_D + R_S)\frac{C_{gg,t} \cdot C_{gd,t} \cdot g_d}{g_m} + \frac{C_{gg,t} \cdot g_d}{g_m^2}$$

10 Noise

10.1 Thermal Noise

In all resistors.

$$\overline{v^2} = 4kTR\Delta f$$
$$\overline{i^2} = \frac{4kT}{R}\Delta f$$

Total noise power:

$$P = \frac{\overline{v}^2}{R} = 4kT\Delta f$$

10.2 Shot Noise

Discrete nature of electron charge.

$$\overline{i^2} = 2qI_D\Delta f$$

10.3 1/f Noise

Semiconductor defects cause trapping of electrons.

$$\overline{i^2} = K_1\frac{I^a}{f}\Delta f$$

$a \approx 0.5 - 2$

10.4 Noise In a Diffusive FET

$$\overline{i^2} = 4kT\gamma g_0 \approx 4kT\frac{2}{3}g_m\Delta f$$

where $\frac{2}{3} < \gamma < 5$.

$$\overline{v^2} = 4kTR_{gs}\Delta f$$

10.5 Signal to Noise Ratio

$$P_{signal} = \frac{v_{signal}^2}{4R_S}$$

$$P_{g,noise} = kT$$

$$P_{amp,noise} = \frac{\overline{V_{total}^2}}{4R_S}$$

$$SNR_1 = \frac{P_{signal}}{P_{g,noise}}$$

$$SNR_2 = \frac{P_{signal}}{P_{g,noise} + P_{amp,noise}}$$

Noise factor:

$$F = \frac{SNR_1}{SNR_2} = \frac{P_{g,noise} + P_{amp,noise}}{P_{g,noise}} = 1 + \frac{P_{amp,noise}}{kT}$$

Noise figure:

$$NF = 10log(F)$$

10.6 Minimum NF and Optimal Z_S

$$F_{min} = 1 + \frac{1}{4kT}\left[2\sqrt{\overline{v_n^2 i_n^2} - (Im(\overline{v_n i_n^*}))^2} + 2Re(\overline{v_n i_n^*})\right]$$

$$Z_{opt} = R_{opt} + jX_{opt} = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}} - \left(\frac{Im(\overline{v_n i_n^*})}{\overline{i_n^2}}\right)^2} - j\frac{Im(\overline{v_n i_n^*})}{\overline{i_n^2}}$$

10.7 Simple Transistor Noise Model

$$\overline{v_n^2} = 4kT\left(R + \frac{\gamma}{g_m}(1 + (\omega RC)^2)\right)$$

$$\overline{i_n^2} = \frac{4kT\gamma}{g_m}(\omega C)^2$$

$$\overline{v_n i_n^*} = 4kT\gamma g_m\left[\frac{1 + j\omega CR}{g_m}\right]\left[\frac{-j\omega C}{g_m}\right]$$

$$F_{min} \approx 1 + 2\sqrt{\gamma}\frac{\omega}{\omega_T}$$

where $\omega << \omega_T$ and $\omega_T = \frac{g_m}{C_{gs}}$.

$$X_{opt} = \frac{1}{\omega C}$$

$$R_{opt} = \sqrt{R^2 + \frac{R}{\gamma g_m}\left(\frac{\omega_T}{\omega}\right)^2} \neq R_i + R_G$$

Noise Resistance:

$$R_n = \frac{\overline{v_n^2}}{4kT} = R + \frac{\gamma}{g_m}(1 + (\omega RC)^2)$$

$$F = F_{min} + \frac{4R_n}{Z_0}\frac{|\Gamma_s - \Gamma_{s,opt}|^2}{(1 - |\Gamma_s|^2)|1 + \Gamma_{s,opt}|^2}$$

11 Transmission Lines

Velocity:

$$v = \frac{c}{\epsilon_{eff}} = \frac{1}{\sqrt{LC}}$$

Characteristic Impedance:

$$Z_0 = \sqrt{\frac{L}{C}}$$

Voltage traveling wave (lossless):

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

Phase propagation constant:

$$\beta = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

11.1 Reflection Coefficients

Reflection at load:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reflection at source:

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}$$

Input impedance:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

Power reflection:

$$|\Gamma|^2 = \frac{|Z_{in} - Z_0|^2}{|Z_{in} + Z_0|^2}|$$

$$P_r = |\Gamma|^2 P_{AVS}$$

11.2 Bounce Diagram

$$v_n^- = v_n^+ \cdot \Gamma_L$$

$$v_{n+1}^+ = v_n^- \cdot \Gamma_S$$

Steady state:

$$V_{ss} = V_0 \frac{R_L}{R_L + R_S}$$

Total voltage:

$$V_{tot} = \sum v_i$$

11.3 Lossy Transmission Line

$$V(z) = V_0^+ e^{-\gamma z} = V_0^+ e^{-\alpha z} e^{-j\beta z}$$

12 S-Parameters and Smith Chart

12.1 S-Parameters

$$s_{11} = \frac{b_1}{a_1}\bigg|_{a_2=0} = \Gamma_{in}$$
$$s_{12} = \frac{b_1}{a_2}\bigg|_{a_1=0} = \frac{2v_1}{v_s} = reverse\ gain$$
$$s_{21} = \frac{b_2}{a_1}\bigg|_{a_2=0} = \frac{2v_2}{v_s} = gain$$
$$s_{22} = \frac{b_2}{a_2}\bigg|_{a_1=0} = \Gamma_{out}$$

13 Low Noise Amplifier

13.1 Conjugate Match

$$y_{in} = y_{11} - \frac{y_{12} \cdot y_{21}}{y_{22} + y_L} = \left[y_{12} = 0\right] = y_{11}$$
$$y_{out} = y_{22} - \frac{y_{12} \cdot y_{21}}{y_{11} + y_S} = \left[y_{12} = 0\right] = y_{22}$$

Optimal power transfer from source to load:

$$y_S = y_{11}^*$$
$$y_L = y_{22}^*$$

13.2 Stubs

Phase propagation constant:

$$\beta = \frac{\omega}{v_p} = \frac{2\pi f\sqrt{\epsilon_r}}{c}$$