EITN90 Radar and Remote Sensing
Lecture 8: Radar antennas

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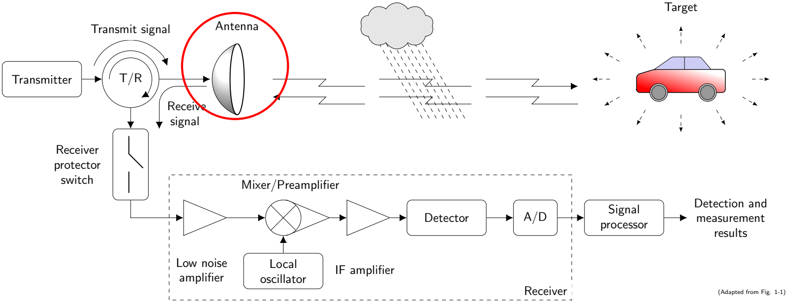
Outline

Learning outcomes of this lecture

In this lecture we will

- Review basic antenna concepts, particularly gain
- How the antenna affects the radar application
- Study the two main high-gain antenna solutions:
  - Reflector antennas
  - Phased array antennas



Outline

The isotropic antenna

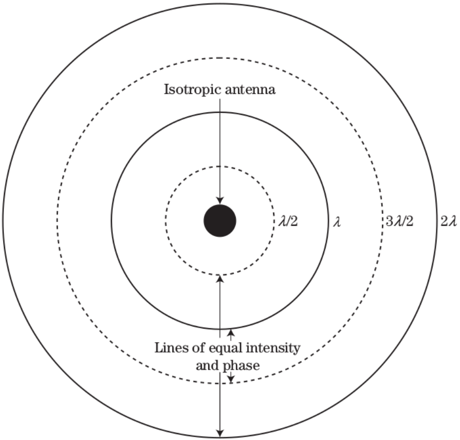


FIGURE 9-2 ■ The isotropic antenna radiates equally in all directions. The concentric rings indicate spheres of equal phase and radiation intensity.

Radiation intensity: $I = \dfrac{P_{\mathrm{t}}}{4\pi}$ [W/steradians]

Power density: $Q_{\mathrm{t}} = \dfrac{P_{\mathrm{t}}}{4\pi R^2}$ [W/m^2]

Angular selectivity of an array

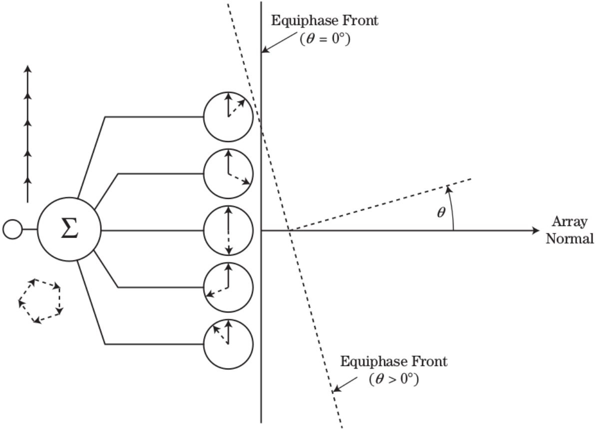
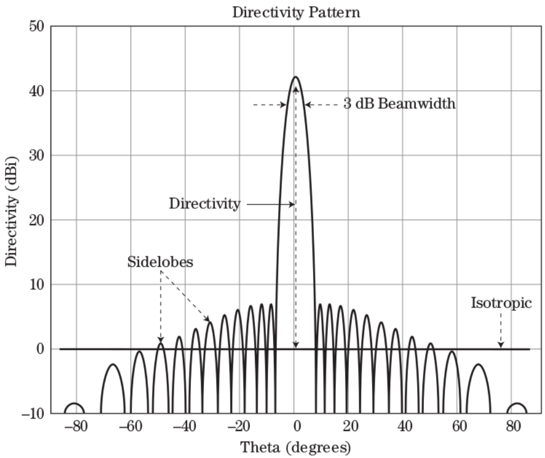


FIGURE 9-3 ■ The response of an array to an incoming plane wave is the sum of the element excitation vectors, which will combine constructively or destructively depending upon the incidence angle.

The received complex amplitudes may sum up **constructively** (normal incidence $\theta = 0$, solid arrows), or **destructively** (oblique incidence $\theta > 0$, dashed arrows). The result is angular selectivity.

Radiation pattern

FIGURE 9-4 ■ The directivity pattern of a radar antenna and some related parameters.



$$\text{Beam width:} \quad \theta_3 = \frac{\alpha\lambda}{L}, \quad \alpha \approx 1$$

Note that many results in the book are valid for large antennas, but not necessarily for small (where antenna dimension $L \ll \lambda$).

Directivity, gain, and efficiency

The directivity and gain of an antenna are both measures of radiation intensity I, but are differently normalized:

$$D(\theta,\phi) \stackrel{\text{def}}{=} \frac{I(\theta,\phi)}{P_{\text{rad}}/(4\pi)}, \quad P_{\text{rad}} = \text{power radiated from antenna}$$

$$G(\theta,\phi) \stackrel{\text{def}}{=} \frac{I(\theta,\phi)}{P_{\text{acc}}/(4\pi)}, \quad P_{\text{acc}} = \text{power accepted by the antenna}$$

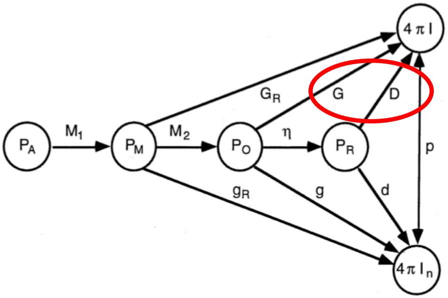
The powers are related by $P_{\text{rad}} = P_{\text{acc}} - P_{\text{loss}}$, where P_{loss} is the power lost in the antenna, for instance resistive losses. Thus, $G \leq D$, and the antenna radiation efficiency is $\eta = G/D$.

$$D_{\text{max}} = \frac{\eta_{\text{a}} 4\pi A}{\lambda^2} \approx \frac{4\pi (0.88)^2}{\theta_3 \phi_3} = \frac{32\,000}{\theta_3[\text{degrees}]\,\phi_3[\text{degrees}]}$$

The above applies to a uniformly illuminated rectangular antenna. The aperture efficiency is η_{a}.

IEEE standard for definition of terms for antennas

IEEE Std 145-2013
IEEE Standard for Definitions of Terms for Antennas



P_A = power available from the generator
P_M = power to matched transmission line
P_O = power accepted by the antenna
P_R = power radiated by the antenna
I = radiation intensity
I_n = partial radiation intensity†
M_1 = impedance mismatch factor 1
M_2 = impedance mismatch factor 2

η = radiation efficiency
G_R = realized gain
G = gain
D = directivity
g_R = partial realized gain
g = partial gain
d = partial directivity
p = polarization efficiency

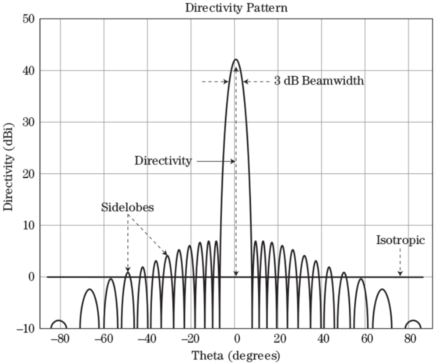
†All partial quantities correspond to a specified polarization, n.

Figure 1—Gain and directivity flow chart

Sidelobes

Radiation in sidelobes is usually undesired. It is characterized by the maximum sidelobe level relative the main beam (dB) or relative an isotropic antenna (dBi).

FIGURE 9-4 ■ The directivity pattern of a radar antenna and some related parameters.



$D_{\max} = 43\,\mathrm{dBi}$, $\mathrm{SLL} = -36\,\mathrm{dB}$, or $\mathrm{SLL} = 7\,\mathrm{dBi}$.

Average sidelobe level

Another characterization of sidelobe radiation is the average ratio of sidelobe power to that of an isotropic antenna with the same input power:

$$\mathrm{SLL}_{\mathrm{ave}} = \frac{\frac{P_{\mathrm{SL}}}{\Omega_{\mathrm{SL}}}}{\frac{P_{\mathrm{t}}}{4\pi}} = \frac{P_{\mathrm{t}} - P_{\mathrm{MB}}}{4\pi - \Omega_{\mathrm{MB}}} \frac{4\pi}{P_{\mathrm{t}}} = \frac{1 - \frac{P_{\mathrm{MB}}}{P_{\mathrm{t}}}}{1 - \frac{\Omega_{\mathrm{MB}}}{4\pi}} \approx 1 - \frac{P_{\mathrm{MB}}}{P_{\mathrm{t}}}$$

- $P_{\mathrm{t}} =$ total radiated power
- $P_{\mathrm{MB}} =$ power radiated in main beam
- $P_{\mathrm{SL}} =$ power radiated in sidelobes
- $\Omega_{\mathrm{MB}} =$ main beam solid angle (typically $\Omega_{\mathrm{MB}} \ll 4\pi$)
- $\Omega_{\mathrm{SL}} =$ sidelobe solid angle

The simplified version can be interpreted as power conservation.

Discussion

Aperture tapers

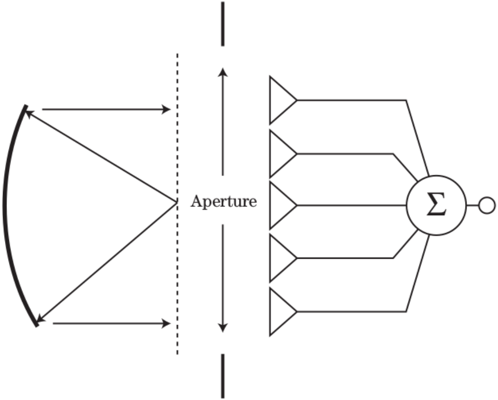


FIGURE 9-5 ■ The aperture for a reflector and array antenna.

The sidelobe structure can be controlled by the spatial distribution of the electric field across the aperture, either by shaping a reflector or controlling the elements of an array.

Example: Taylor tapering

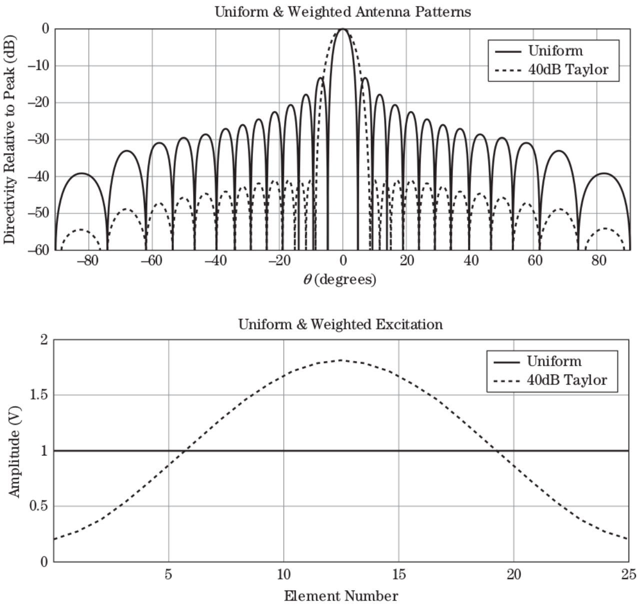


FIGURE 9-6 Radiation patterns resulting from uniform and low sidelobe aperture tapers.

Example: Taylor tapering

TABLE 9-1 ■ Aperture Efficiency and Beamwidth Factor Values for Different Taylor Distributions

Sidelobe Level (dB)	Beamwidth Factor (α)	Aperture Efficiency (η_a) dB
−13	0.88	0.0
−20	0.98	−0.22
−25	1.05	−0.46
−30	1.12	−0.70
−35	1.18	−0.95
−40	1.25	−1.18
−45	1.30	−1.39

For lower side lobe levels, the beamwidth is increased.

Very low SLL:s, below $-40\,\mathrm{dB}$, are very difficult to realize, due to finite tolerances in antenna components, reflector shape, thermal effects, antenna alignment etc.

Phase errors

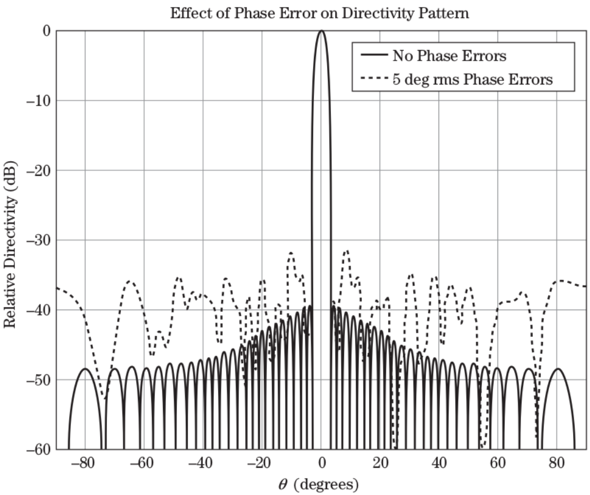


FIGURE 9-7 Radiation pattern with and without random phase errors.

$$\frac{D_e}{D_0} = \exp(-\delta_{\rm rms}^2)$$

$\delta_{\rm rms} = 5^\circ \Leftrightarrow \frac{5}{360} = \frac{1}{72}$ fraction of λ, requiring control on scale 0.42 mm at 10 GHz.

Outline

Applications

The antenna has a very direct effect on the radar system.

- The radar can only see within the antenna's field of view (FOV).
- The maximum range is limited by the gain of the antenna:

$$R_{\mathrm{max}} = \left[\frac{P_{\mathrm{t}} G^2 \lambda^2 \sigma}{(4\pi)^3 L_{\mathrm{s}} P_{\mathrm{min}}} \right]^{1/4}$$

- The spatial resolution $\Delta R = c/(2B)$ depends on bandwidth B, which is often restricted by the antenna function.
- Different combinations of average transmit power and effective area may be important, $P_{\mathrm{ave}} A_{\mathrm{e}}^2$ and $P_{\mathrm{ave}} A_{\mathrm{e}}$:

Track
$$\frac{P_{\mathrm{ave}} A_{\mathrm{e}}^2}{\lambda^2} = \frac{\mathrm{SNR} \cdot 4\pi k T_0 F L_{\mathrm{s}} R^4 \cdot \mathrm{PRF}}{\sigma}$$

Search
$$P_{\mathrm{avg}} A_{\mathrm{e}} = \frac{\mathrm{SNR} \cdot 4\pi k T_0 F L_{\mathrm{s}} R^4}{\sigma} \left(\frac{\Omega}{T_{\mathrm{fs}}} \right)$$

Constant track/search performance

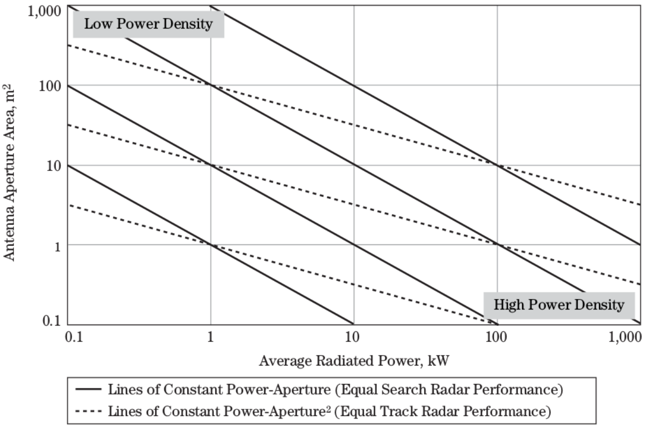


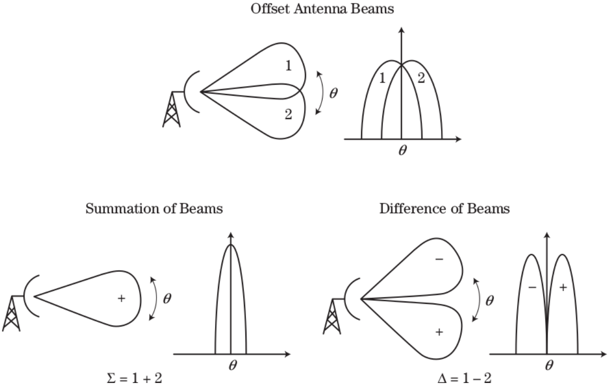
FIGURE 9-8 ■ Lines of constant track and search performance mapped onto the power-aperture space.

Discussion

Monopulse

A target's angular location can be accurately determined in one pulse (no sweep) by using two closely spaced beams in the antenna.

FIGURE 9-9 ■ A sum and delta beam can be formed by adding and subtracting overlapping antenna beams.



$$v_{\text{error}}(\theta) = \frac{|\Delta(\theta)|}{|\Sigma(\theta)|} \cos\beta \qquad \beta = \arg(\Delta/\Sigma)$$

Monopulse — increased angular sensitivity

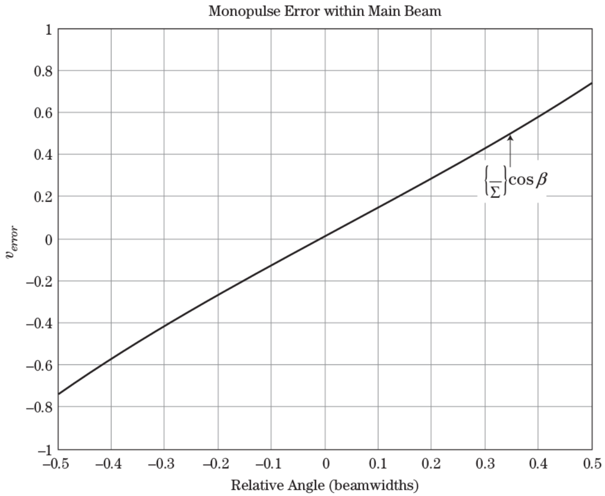


FIGURE 9-10 ■ The monopulse error signal is a ratio of the delta beam over the sum beam and is linear within the 3 dB beamwidth.

$$\Delta\theta = \frac{\theta_3}{k_{\mathrm{m}}\sqrt{2 \cdot \mathrm{SNR}}}\sqrt{1 + \left(\frac{k_{\mathrm{m}}\theta}{\theta_3}\right)^2}$$

$k_{\mathrm{m}} =$ slope of curve

Monopulse

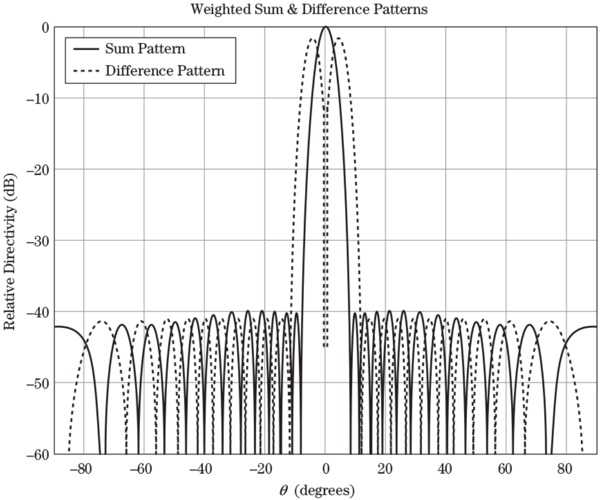


FIGURE 9-11 Low sidelobe sum and delta patterns created with Taylor and Bayliss distributions.

Transmit with Σ pattern, receive in both Σ and Δ. Can add the same functionality in elevation at the cost of more antenna ports.

Outline

Reflector antennas

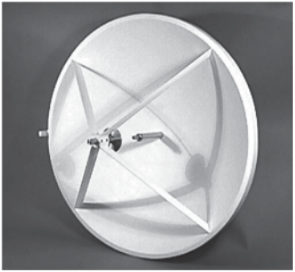


FIGURE 9-12 ■ Cassegrain reflector antenna developed by Quinstar Technology, Inc. (Used with permission.)

Often rotational symmetric. Note the subreflector (or antenna feed) needs to be supported in the aperture.

Parabolic reflector

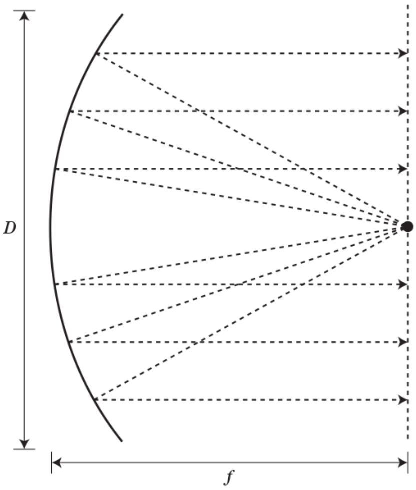


FIGURE 9-13 ■ For a parabolic reflector, all rays travel the same distance from the feed to the aperture plane.

The reflector is often characterized by the focal length to diameter ratio, f/D.

Feed

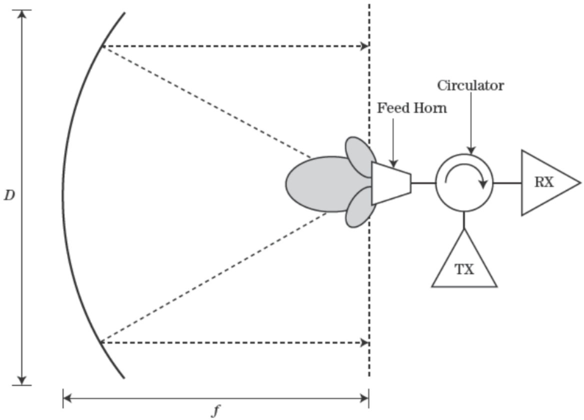
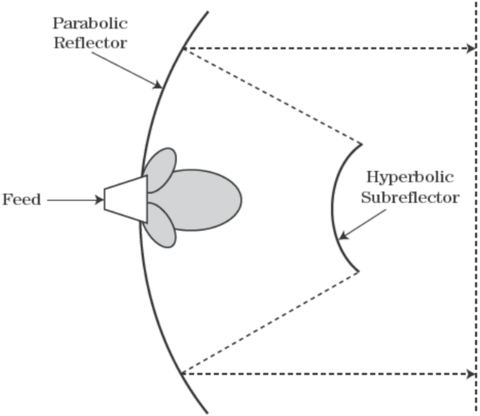


FIGURE 9-14 ■ The feed horn for a reflector antenna is sized to provide the desired aperture illumination on the reflector surface.

The higher f/D, the more directive feed horn is necessary to avoid spillover (power that misses the reflector).

Subreflector (Cassegrain configuration)

FIGURE 9-15 ■ Using a subreflector creates a long effective f/D in much less space.



The size requirements can be relaxed by using a subreflector.

Offset feed

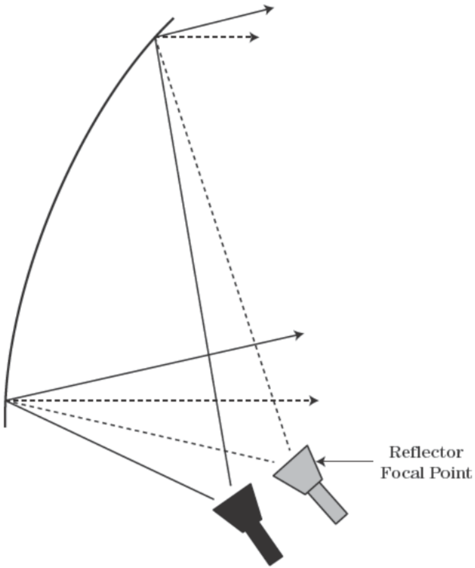


FIGURE 9-16 ■ The feed horn can be offset to eliminate blockage. Moving the horn from the focal point results in a limited amount of beam scanning.

Outline

Phased array

FIGURE 9-17 ■ AN/APG-81 – F-35 active electronically scanned array (AESA) radar. (Courtesy of Northrop Grumman Electronic Systems. Used with permission.)

With a phased array, the antenna beam can be scanned electronically. Much faster than mechanical steering, but more complex electronic implementation.

Array factor

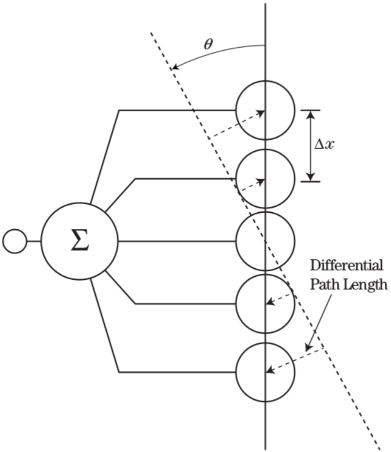


FIGURE 9-18 ■ A plane wave from angle θ intersecting with a five-element array.

$$\mathrm{AF}(\theta) = \frac{1}{N}\sum_{n=1}^{N} \exp\left[-\mathrm{j}\left(\frac{2\pi}{\lambda} n \Delta x \sin\theta - \phi_n\right)\right]$$

▸ Discussion

Uniform phase shift: $\phi_n = \frac{2\pi}{\lambda} n \Delta x \sin\theta_s$

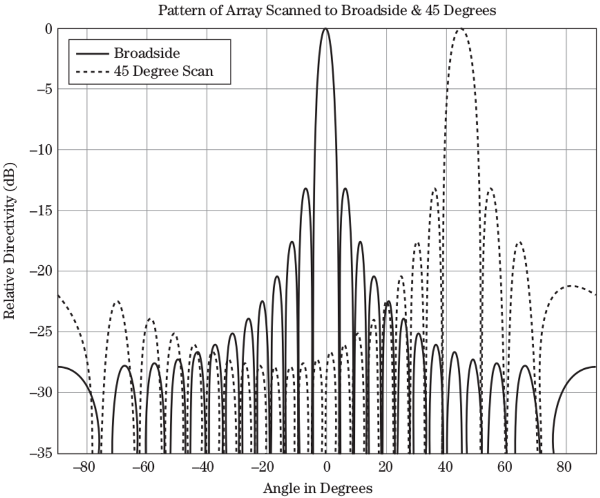


FIGURE 9-19 ■ Radiation pattern for an unsteered array and an array electronically scanned to 45°.

$$\mathrm{AF}(\theta) = \frac{1}{N}\sum_{n=1}^{N} \exp\left[-\mathrm{j}\left(\frac{2\pi}{\lambda} n \Delta x (\sin\theta - \sin\theta_s)\right)\right]$$

Phase shifters

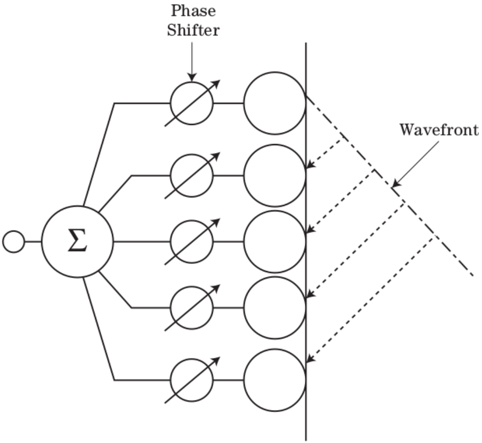


FIGURE 9-20 Phase shifters can be inserted after the radiating elements to electronically scan the beam.

The phase shift in each antenna element can be controlled by a phase shifter.

Phase shifters, quantization error

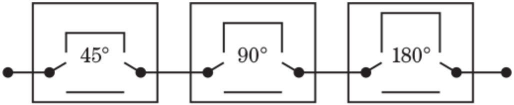


FIGURE 9-21 ■ Example of a 3-bit switched line length phase shifter.

TABLE 9-2 ■ Relationship among Phase-Shifter Bits, Phase Error, and Gain Loss

Number of Bits (N)	Least Significant Bit	RMS Phase Error	Gain Loss (dB)
2	90°	26°	0.65
3	45°	13°	0.15
4	22.5°	6.5°	0.04
5	11.25°	3.2°	0.01
6	5.625°	1.6°	0.00

More bits give higher resolution and less gain loss, but also higher insertion loss (there are losses in each stage of the phase shifter). Phase shifters are typically narrow-band.

Grating lobes

From the array factor

$$\mathrm{AF}(\theta) = \frac{1}{N}\sum_{n=1}^{N} \exp\left[-\mathrm{j}\left(\frac{2\pi}{\lambda} n \Delta x (\sin\theta - \sin\theta_\mathrm{s})\right)\right]$$

it is seen that it is maximized when

$$\frac{\Delta x}{\lambda}(\sin\theta - \sin\theta_\mathrm{s}) = 0, \pm 1, \pm 2, \ldots$$

The zero is the intended main beam $\theta = \theta_\mathrm{s}$, but with large enough spacing Δx other angles θ can correspond to the non-zero integers. These are called **grating lobes**.

If scanning is restricted to the region $\theta \in [-\theta_\mathrm{s}, \theta_\mathrm{s}]$, grating lobes are absent if

$$\Delta x \leq \frac{\lambda}{1 + |\sin\theta_\mathrm{s}|}$$

For $\Delta x < \lambda/2$, there are no grating lobes regardless of θ_s.

Grating lobes, example for $\Delta x = \lambda$

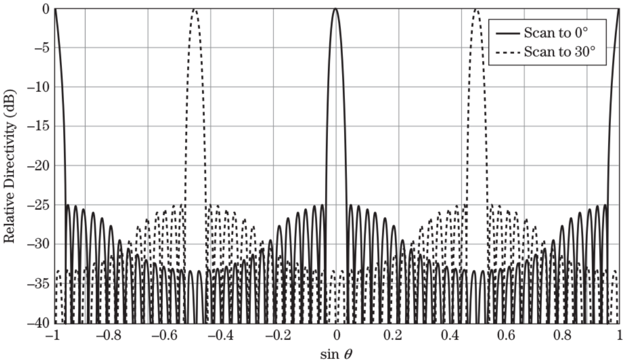
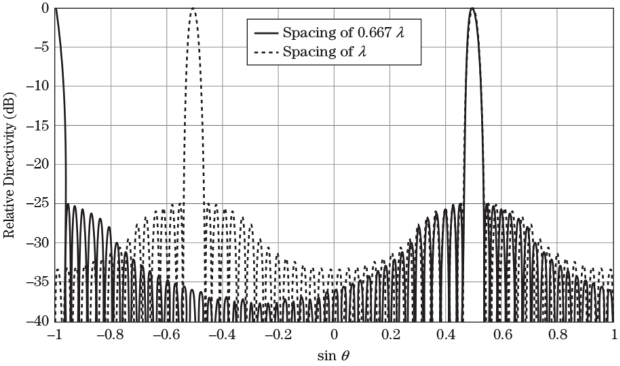


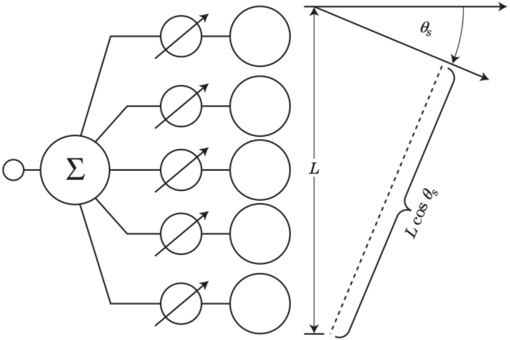
FIGURE 9-22 Antenna pattern with grating lobes for a scanned and unscanned array with one wavelength element spacing.

FIGURE 9-23 ■ Comparison between two arrays showing that grating lobes become less significant when the element spacing decreases.



Gain loss

FIGURE 9-24 ■ The projected aperture of a phased array decreases with scan angle, resulting in beam broadening and a directivity loss.



When scanning electrically, the physical aperture has a fixed orientation. The width of the main beam (directivity) is determined by the projected aperture. Typically scanning with a planar array would be constrained to $\pm 60^{\circ}$ or less.

Array element pattern

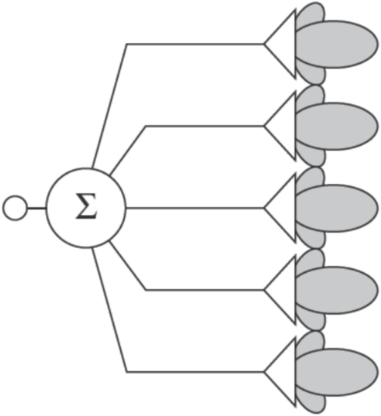
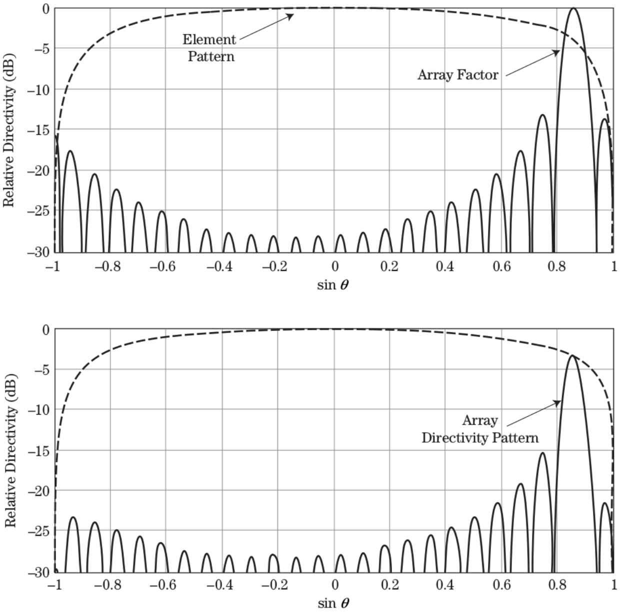


FIGURE 9-25 Array elements are not isotropic and therefore have directivity.

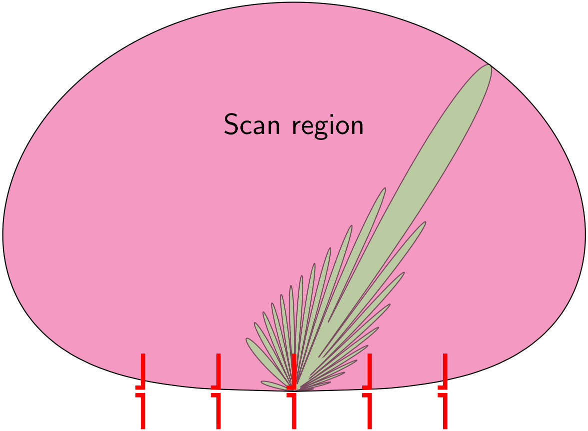
$$E(\theta) = E_{\mathrm{e}}(\theta)\,\mathrm{AF}(\theta) = \frac{E_{\mathrm{e}}(\theta)}{N}\sum_{n=1}^{N}\exp\left[-\mathrm{j}\frac{2\pi}{\lambda}n\varDelta x(\sin\theta - \sin\theta_{\mathrm{s}})\right]$$

Array element influence on beam steering

FIGURE 9-26 ■ The total antenna pattern of an array (lower plot) is the product of the array factor and the element pattern (upper plot).

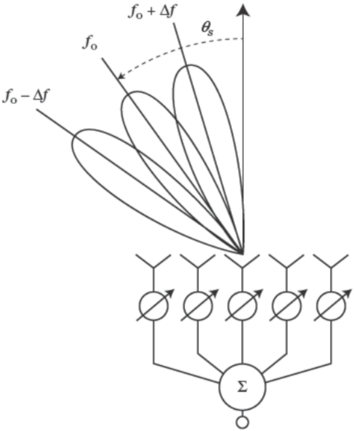


Typical effect when scanning



Wideband phased arrays

FIGURE 9-27 ■ A phase-shifter-based phased array will mispoint the beam during wideband operation for off-broadside scan angles.



The narrow bandwidth of phase shifters may cause the beam to point in different angles depending on frequency. The tolerable fractional instantaneous bandwidth is $B_{\mathrm{i}} = \frac{\theta_3}{2\sin\theta_{\mathrm{s}}}$. Problem for wideband radars with narrow beamwidth.

Wideband phased arrays

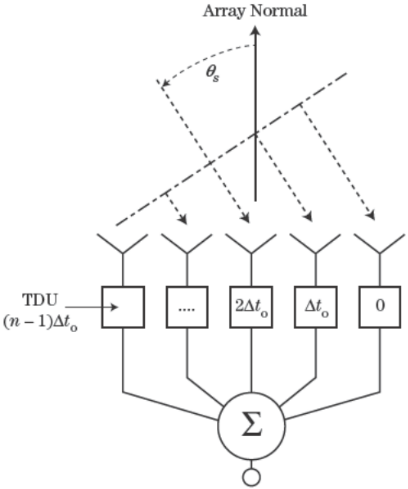


FIGURE 9-28 ■ The phase front of an off-axis plane wave will strike each element at a different time. The time of arrival difference is frequency independent. A TDU-based phased array is ideal for wideband operation.

Instead of phase shifting, the time delay unit (TDU) implements a fixed time delay in each chain. However, they have long been considered bulky and costly.

Wideband phased arrays: subarrays

FIGURE 9-29 ■ The subarray architecture is often used for wideband operation with TDUs inserted behind each subarray.

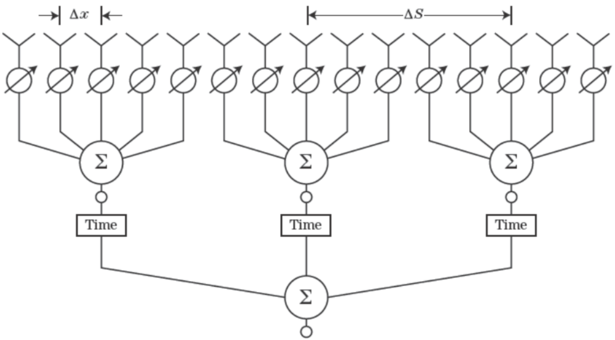
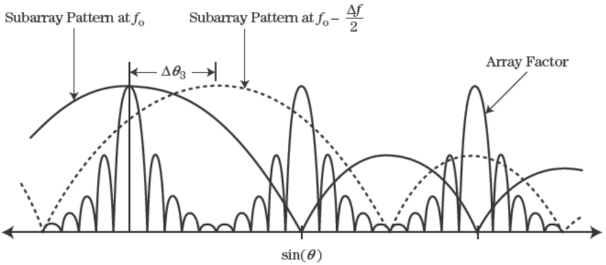


FIGURE 9-30 ■ For wideband operation the subarray pattern will squint, but the subarray AF will not.



Outline

Passive array architecture

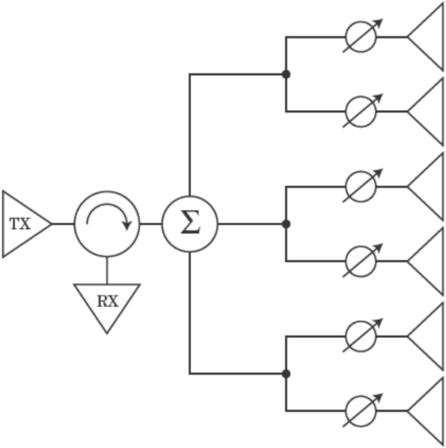
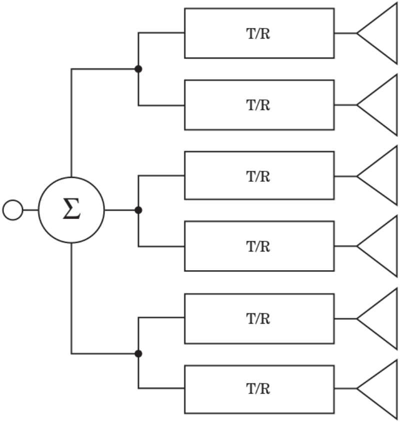


FIGURE 9-31 ■ The passive array uses one PA to drive the entire array and one LNA to set the noise figure.

High power handling in all array components.

Active array architecture

FIGURE 9-32 ■ An AESA places a PA and LNA behind each radiator. These electronics are usually packaged into a T/R module.



Individual T/R units behind each antenna element, phase and amplitude can be controlled. Costly, but extremely flexible.

Trade-off power-aperture

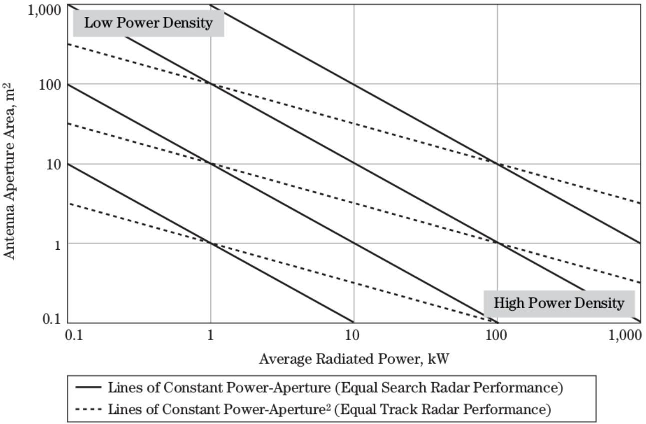


FIGURE 9-8 Lines of constant track and search performance mapped onto the power-aperture space.

Power requirements in each element can be traded with aperture size: low power requires larger area and more elements, but each element may be cheap and there is less need for cooling.

Subarray architecture

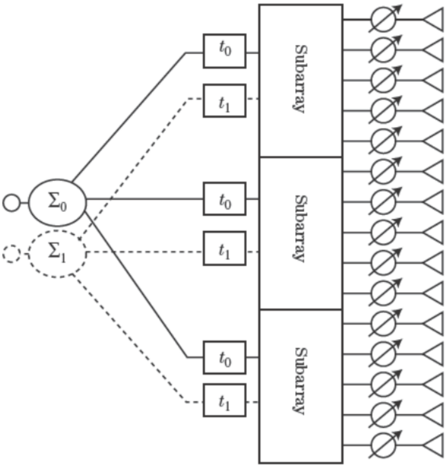


FIGURE 9-33 ■ An array divided into three subarrays. Different time delays are applied at the subarray level to form two simultaneous receive beams.

Breaking down the array into subarrays may simplify manufacturing and maintainance. Digitizing the signal at subarray level allows many simultaneous beams (achieved by processing).

Simultaneous beam operation

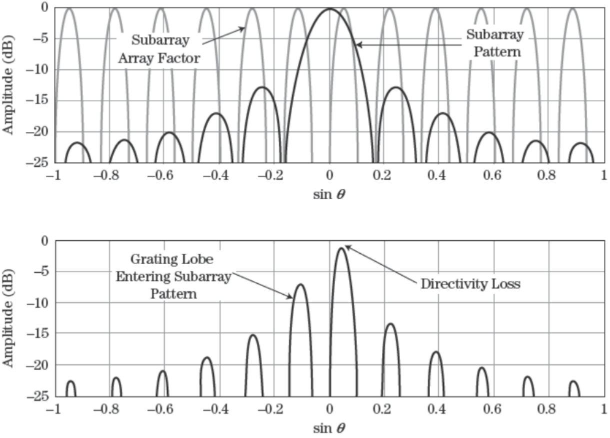


FIGURE 9-34 ■ During simultaneous beam operation the subarray pattern remains fixed, and the subarray AF will electronically scan. Grating lobes become significant when subarray AF lobes enter the subarray pattern mainlobe. Upper plot: Subarray pattern and subarray array factor. Lower: Combined antenna pattern.

Since the centers of the subarrays are far apart, grating lobes appear when scanning the subarray AF, but are suppressed by the fixed subarray pattern.

Overlapped subarrays

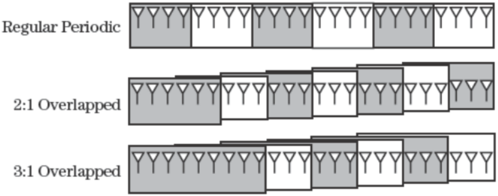


FIGURE 9-35 ■ Options for implementing the overlapped subarray architecture.

The subarray distance can be reduced by overlapping subarrays, at the price of a more complicated feeding network (since each element will be connected to more than one subarray).

Outline

Conclusions

- Basic antenna concepts: directive radiation pattern, side lobes, aperture tapering.
- Antenna effects on radar function: FOV, gain and range, power-aperture tradeoff.
- Improved angular localization with sum and difference patterns.
- Reflector antennas provide high gain in different configurations, but little scan possibility.
- Phased arrays are extremely flexible, but also costly. Various levels of analog/digital solutions.

Discussion

Consider an antenna where all sidelobes have similar levels around $\mathrm{SLL} = -30\,\mathrm{dB}$. Using the relation

$$\mathrm{SLL}_{\mathrm{ave}} \approx 1 - \frac{P_{\mathrm{MB}}}{P_{\mathrm{t}}}$$

provide an estimate of how much of the total power P_{t} is contained in the sidelobes.

◂ Go back

Discussion

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$$\mathrm{SLL}_{\mathrm{ave}} \approx 1 - \frac{P_{\mathrm{MB}}}{P_{\mathrm{t}}}$$

provide an estimate of how much of the total power P_{t} is contained in the sidelobes.

Answer: $\frac{P_{\mathrm{SL}}}{P_{\mathrm{t}}} = \frac{P_{\mathrm{t}} - P_{\mathrm{MB}}}{P_{\mathrm{t}}} = 1 - \frac{P_{\mathrm{MB}}}{P_{\mathrm{t}}} \approx -30\,\mathrm{dB} = 10^{-3}$.

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Discussion

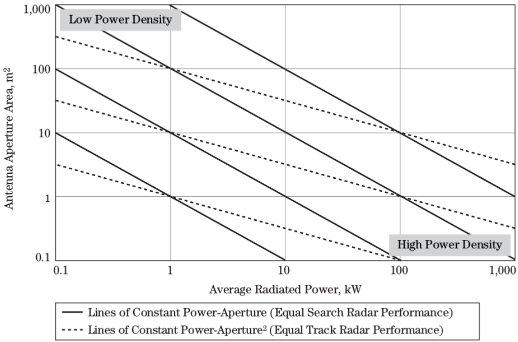


FIGURE 9-8 ■ Lines of constant track and search performance mapped onto the power-aperture space.

With a linear cost model (unrealistic!) of $1\,000\,\mathrm{SEK/m^2}$ and $1\,000\,\mathrm{SEK/kW}$, what would you recommend for a search radar application, large aperture area, high power, or something in between?

Discussion

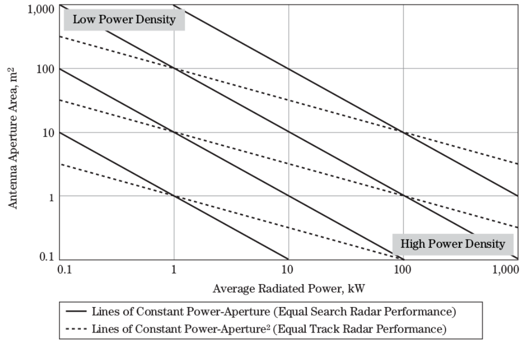


FIGURE 9-8 ■ Lines of constant track and search performance mapped onto the power-aperture space.

With a linear cost model (unrealistic!) of $1\,000\,\mathrm{SEK}/\mathrm{m}^2$ and $1\,000\,\mathrm{SEK}/\mathrm{kW}$, what would you recommend for a search radar application, large aperture area, high power, or something in between?

Answer: Cost $= (\frac{A}{\mathrm{m}^2} + \frac{P}{\mathrm{kW}}) \cdot 1\,000\,\mathrm{SEK}$. For fixed performance, the cost is dominated by either aperture area or power when one becomes large. The lowest cost is in between.

Discussion

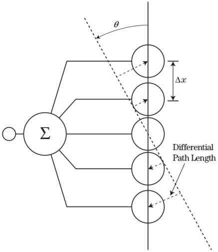


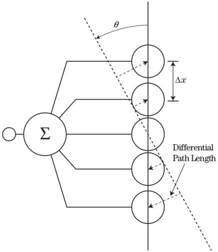
FIGURE 9-18 ■ A plane wave from angle θ intersecting with a five-element array.

$$\mathrm{AF}(\theta) = \frac{1}{N}\sum_{n=1}^{N} \exp\left[-\mathrm{j}\left(\frac{2\pi}{\lambda} n \Delta x \sin\theta - \phi_n\right)\right]$$

How should the phase in each element, ϕ_n, be chosen to maximize radiation in direction $\theta = \pi/4$?

◂ Go back

Discussion



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How should the phase in each element, ϕ_n, be chosen to maximize radiation in direction $\theta = \pi/4$?

Answer: The exponentials add in phase (argument is zero) at $\theta = \pi/4$ if $\phi_n = \frac{2\pi}{\lambda} n \Delta x \sin(\pi/4)$. For a more general result, replace $\pi/4$ by an arbitrary scan angle θ_s.

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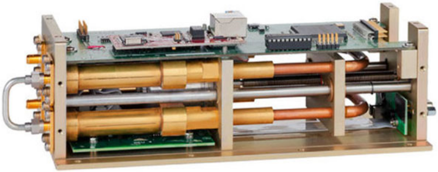
Discussion

A possible implementation of time delay units is to change length of a transmission line leading up to the element, for instance with a mechanical trombone structure as below.



Why would this be considered bulky?

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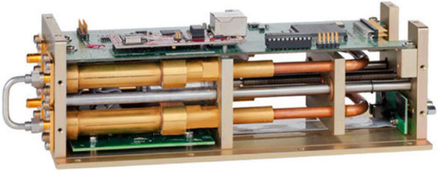
Discussion

A possible implementation of time delay units is to change length of a transmission line leading up to the element, for instance with a mechanical trombone structure as below.



Why would this be considered bulky?

Answer: The time delay is proportional to the length of transmission line used, which has to be significantly meandered to fit in a small space if a long delay is required.

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