EITN90 Radar and Remote Sensing
Lecture 6: Target fluctuation models

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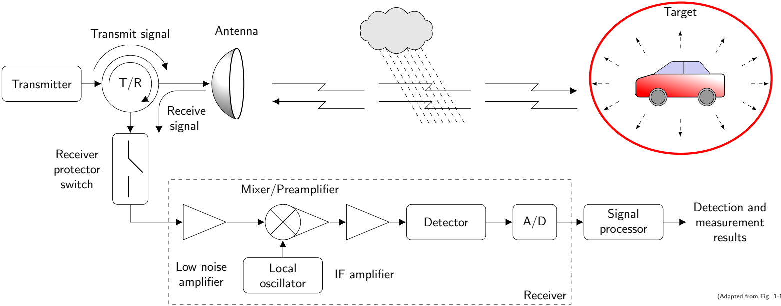
Outline

1. **Radar cross section of simple targets**
2. **Radar cross section of complex targets**
3. **Statistical characteristics of the RCS of complex targets**
4. **Target fluctuation models**
5. **Doppler spectrum of fluctuating targets**
6. **Conclusions**

Learning outcomes of this lecture

In this lecture we will

- Learn the origin of fluctuations in target RCS
- Describe the statistics of the fluctuations
- Characterize RCS decorrelation lengths
- Understand the basic assumptions of the Swerling fluctuation models



(Adapted from Fig. 1-1)

Outline

1. **Radar cross section of simple targets**
2. Radar cross section of complex targets
3. Statistical characteristics of the RCS of complex targets
4. Target fluctuation models
5. Doppler spectrum of fluctuating targets
6. Conclusions

Sphere scattering

The scattering from a sphere can be computed analytically. This gives very precise results, and spheres are often used as calibrating targets at measurement sites.

FIGURE 7-1 ▪
Examples of RCS calibration spheres. (Courtesy of Professor Nadav Levanon, Tel-Aviv University.)

Sphere scattering

A sphere has the same backscattering in all directions due to symmetry.

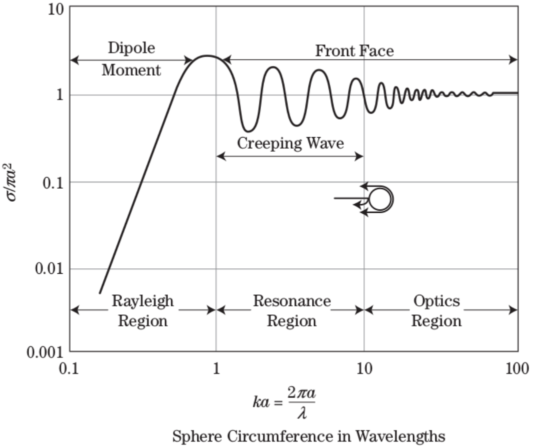
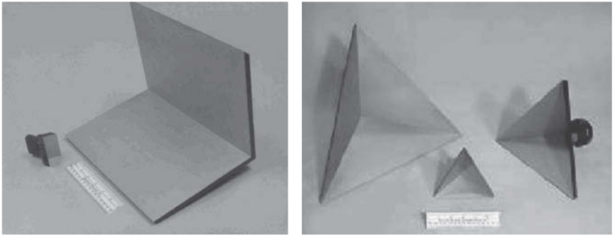


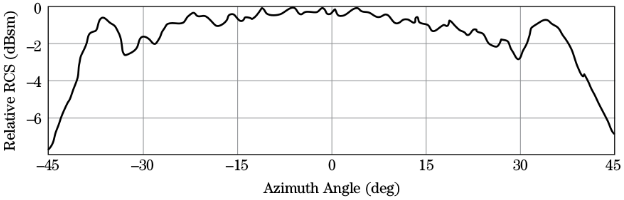
FIGURE 6-12 Sphere scattering from Rayleigh, resonance, and optics regions, [1].

Corner reflectors

A corner reflector has a strong backscattering in a large angular interval. Dihedral = two walls, trihedral = three walls.



(a)

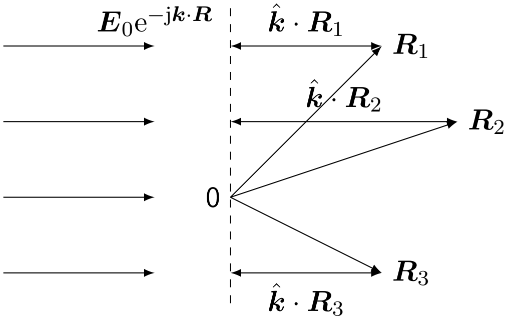


(b)

FIGURE 7-3 ■ Corner reflectors. (a) Dihedrals. (b) Trihedrals. (Courtesy of Professor Nadav Levanon, Tel-Aviv University.)

▸ Discussion

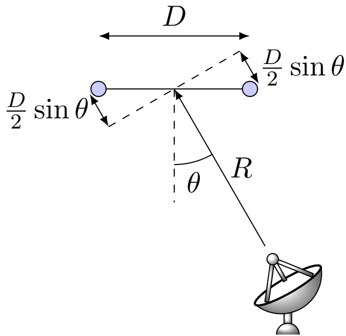
Multiple targets



When several scatterers are subjected to an incident wave $\boldsymbol{E}_0 \mathrm{e}^{-\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{R}}$, the backscattering is (complex addition)

$$\sigma_{\mathrm{tot}} = \left| \sum_{i=1}^{N} \sqrt{\sigma_i}\, \mathrm{e}^{-\mathrm{j}2\boldsymbol{k}\cdot\boldsymbol{R}_i} \right|^2$$

Two scatterers

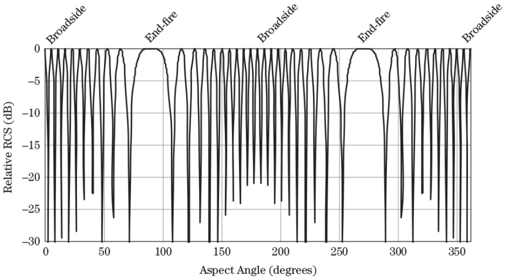


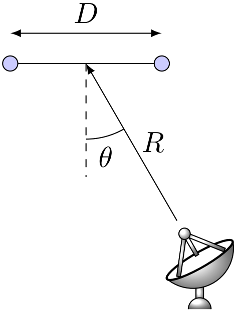
$$\sigma_{\text{tot}} = \left| \sum_{i=1}^{N} \sqrt{\sigma_i}\, \mathrm{e}^{-\mathrm{j}2\boldsymbol{k}\cdot\boldsymbol{R}_i} \right|^2 = \left| \sqrt{\sigma_0}\mathrm{e}^{-\mathrm{j}2k\frac{D}{2}\sin\theta} + \sqrt{\sigma_0}\mathrm{e}^{-\mathrm{j}2k(-\frac{D}{2}\sin\theta)} \right|^2$$

$$= 4\sigma_0 \cos^2\left(\frac{2\pi D \sin\theta}{\lambda}\right)$$

Two scatterers, RCS as function of angle

FIGURE 7-5 ■ Relative radar cross section of the dumbbell target of Figure 7-4 when $D = 5\lambda$ and $R = 10{,}000\,D$.





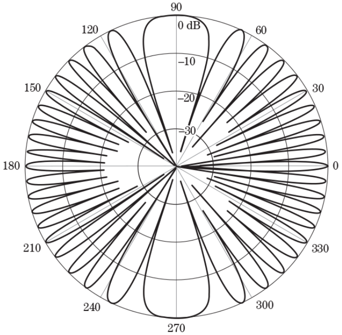


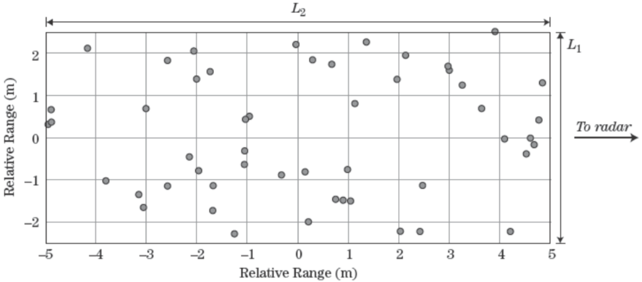
FIGURE 7-6 ■ Polar plot of the data of Figure 7-5.

Outline

Randomly distributed targets

Put out a number of randomly distributed targets.

FIGURE 7-7 ■ Random distribution of 50 scatterers used to obtain Figure 7-8. See text for additional details.



With $\boldsymbol{R}_i$ a random variable, the RCS

$$\sigma = \left| \sum_{i=1}^{N} \sqrt{\sigma_i} \mathrm{e}^{-\mathrm{j}2\boldsymbol{k}\cdot\boldsymbol{R}_\mathrm{i}} \right|^2$$

also becomes a random variable, with some probability density function as angle or frequency varies. However, once the scatterers are fixed the model (including σ) is completely deterministic.

Randomly distributed targets

The RCS as function of angle appears as a random variable, with some probability density.

FIGURE 7-8 Relative RCS of the complex target of Figure 7-7 at a range of 10 km and radar frequency of 10 GHz.

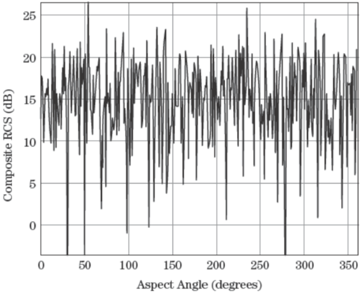
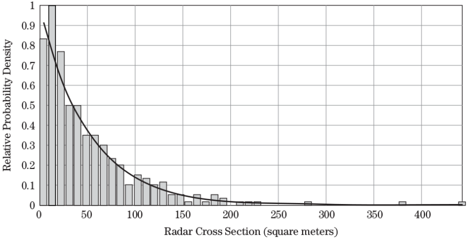


FIGURE 7-9 Histogram of linear-scale RCS data of Figure 7-8.



Outline

Equal scatterers

A large number of approximately equal scatterers with uniformly distributed phase have exponentially distributed RCS:

$$p(\sigma) = \begin{cases} \frac{1}{\bar{\sigma}} \exp\left[-\frac{\sigma}{\bar{\sigma}}\right] & \sigma \geq 0 \\ 0 & \sigma < 0 \end{cases}$$

where $\bar{\sigma}$ is the mean. The amplitude voltage ($\varsigma = \sqrt{\sigma}$) is Rayleigh distributed:

$$p(\varsigma) = \begin{cases} \frac{2\varsigma}{\bar{\sigma}} \exp\left[-\frac{\varsigma^2}{\bar{\sigma}}\right] & \varsigma \geq 0 \\ 0 & \varsigma < 0 \end{cases}$$

For any RCS distribution $p_\sigma(\sigma)$, the corresponding amplitude distribution $p_\varsigma(\varsigma)$ is given by $p_\varsigma(\varsigma) = 2\varsigma p_\sigma(\varsigma^2)$. This is seen from

$$1 = \int p_\sigma(\sigma)\, \mathrm{d}\sigma \overset{\sigma \to \varsigma^2}{=} \int \underbrace{p_\sigma(\varsigma^2) 2\varsigma}_{= p_\varsigma(\varsigma)}\, \mathrm{d}\varsigma$$

where $\mathrm{d}\sigma = \mathrm{d}(\varsigma^2) = 2\varsigma\, \mathrm{d}\varsigma$.

One dominating scatterer

When the situation has one dominating and many small scatterers, the RCS can be modeled by a fourth-degree chi-square distribution (easier to handle analytically than the true Rician distribution):

$$p(\sigma) = \begin{cases} \frac{4\sigma}{\bar{\sigma}^2} \exp\left[-\frac{2\sigma}{\bar{\sigma}}\right] & \sigma \geq 0 \\ 0 & \sigma < 0 \end{cases}$$

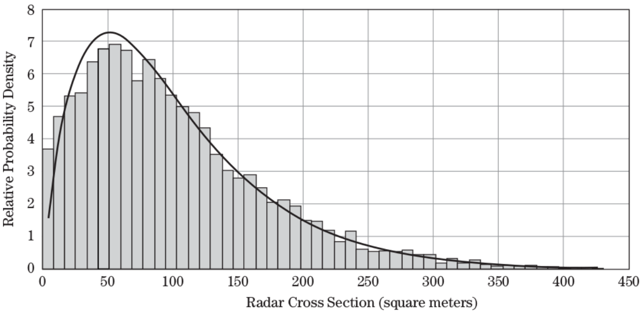


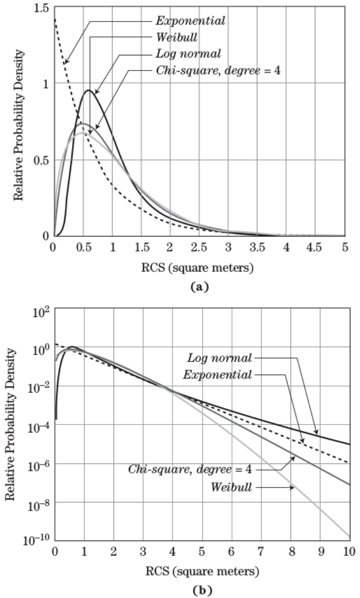
FIGURE 7-10 ■ Comparison of a fourth-degree chi-square PDF and the histogram of linear-scale RCS data for one dominant scatterer with many small scatterers. See text for details.

TABLE 7-1 ■ Common Statistical Models for Radar Cross Section

Model Name	PDF for RCS σ	Comment
Nonfluctuating, Marcum, Swerling 0, or Swerling 5	$p(\sigma) = \delta_D(\sigma - \bar{\sigma})$ $\text{var}(\sigma) = 0$	Constant echo power, e.g. calibration sphere or perfectly stationary reflector with no radar or target motion.
Exponential (chi-square of degree 2)	$p(\sigma) = \frac{1}{\bar{\sigma}} \exp\left[\frac{-\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma) = \bar{\sigma}^2$	Many scatterers, randomly distributed, none dominant. Used in Swerling case 1 and 2 models.
Chi-square of degree 4	$p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp\left[\frac{-2\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma) = \bar{\sigma}^2/2$	Approximation to case of many small scatterers + one dominant, with RCS of dominant equal to $1+\sqrt{2}$ times the sum of RCS of others. Used in Swerling case 3 and 4 models.
Chi-square of degree $2m$, Weinstock	$p(\sigma) = \frac{m}{\Gamma(m)\bar{\sigma}} \left[\frac{m\sigma}{\bar{\sigma}}\right]^{m-1} \exp\left[\frac{-m\sigma}{\bar{\sigma}}\right]$ $\text{var}(\sigma) = \bar{\sigma}^2/m$	Generalization of the two preceding cases. Weinstock cases correspond to $0.6 \le 2m \le 4$. Higher degrees correspond to presence of a more dominant single scatterer.
Weibull	$p(\sigma) = CB\sigma^{C-1} \exp\left[-B\sigma^C\right]$ $\bar{\sigma} = \Gamma\left(1+1/C\right) B^{-1/C}$ $\text{var}(\sigma) = B^{-2/C}\left[\Gamma\left(1+2/C\right) - \Gamma^2\left(1+1/C\right)\right]$	Empirical fit to many measured target and clutter distributions. Can have longer "tail" than previous cases.
Log-normal	$p(\sigma) = \frac{1}{\sqrt{2\pi}\, s\sigma} \exp\left[-\ln^2\left(\sigma/\sigma_m\right)/2s^2\right]$ $\bar{\sigma} = \sigma_m \exp\left(s^2/2\right)$ $\text{var}(\sigma) = \sigma_m^2 \exp\left(s^2\right)\left[\exp\left(s^2\right) - 1\right]$	Empirical fit to many measured target and clutter distributions. "Tail" is longest of previous cases. σ_m is the median value of σ.

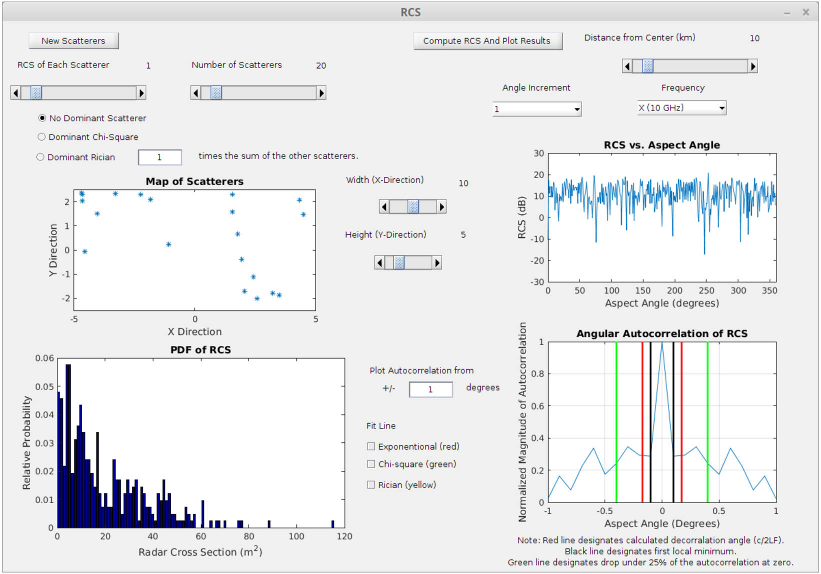
Graphical comparison of the distributions

FIGURE 7-12 ■ Comparison of five models for the probability density function of radar cross section with the same mean (except for the exponential) and variance. See text for additional details. (a) Linear scale. (b) Log scale, showing detail of the PDF "tails."



Discussion

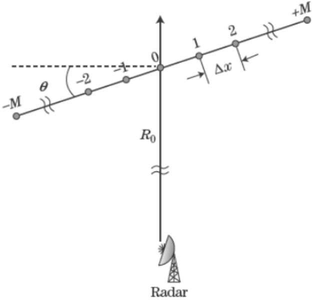
Matlab demo (see http://radarsp.com)



RCS decorrelation

Decorrelation occurs when the observation of RCS is significantly changed by an alteration of time, frequency, or angle. To estimate when this occurs, consider the situation below.

FIGURE 7-13 ■ Geometry for calculation of RCS correlation length in frequency and aspect angle.



$$\sigma = \left| \sum_{i=-M}^{M} \sqrt{\sigma_i} \mathrm{e}^{-\mathrm{j}2\boldsymbol{k}\cdot\boldsymbol{R}_\mathrm{i}} \right|^2 = \sigma_0 \left| \sum_{i=-M}^{M} \mathrm{e}^{-\mathrm{j}2ki\Delta x \sin\theta} \right|^2$$

$$= \sigma_0 \frac{\sin^2[(2M+1)k\Delta x \sin\theta]}{\sin^2(k\Delta x \sin\theta)}$$

RCS decorrelation

Introduce the total length $L = (2M+1)\Delta x$ of the collection of scatterers:

$$\sigma = \sigma_0 \frac{\sin^2(kL\sin\theta)}{\sin^2(k\Delta x \sin\theta)}$$

The first zero corresponds to $kL\sin\theta = \pi$, implying the correlation lengths (using $k = 2\pi/\lambda = 2\pi f/c$ and $\sin\theta \approx \theta$ for small θ)

$$\Delta f = \frac{c}{2L\sin\theta} \quad \text{(frequency decorrelation)}$$
$$\Delta\theta = \frac{\lambda}{2L} \quad \text{(angle decorrelation)}$$

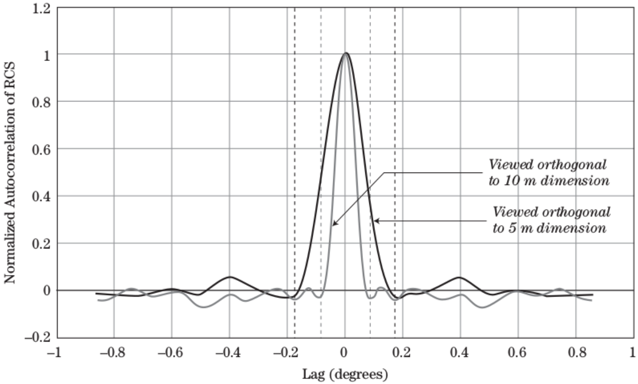
$L\sin\theta$ is the projected length of the target in boresight.

The book derives these values differently (from an autocorrelation function), but the results are the same. However, different definitions of decorrelation may apply in different applications.

RCS correlation properties, width of target

Autocorrelation in angle for a rectangular target $5\,\mathrm{m} \times 10\,\mathrm{m}$. Note that a wider target has shorter correlation length.

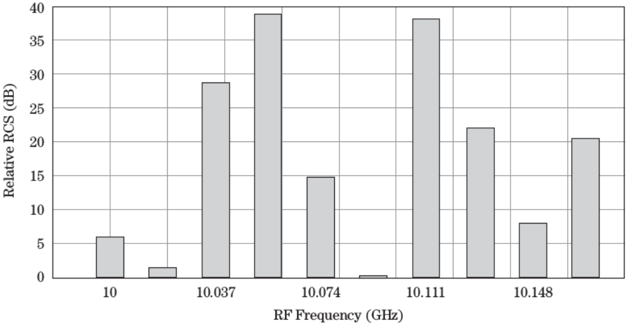
FIGURE 7-14 ■ Decorrelation in angle of RCS of target from Figure 7-8. See text for details.



RCS correlation properties, frequency agility

Variation in RCS due to frequency agility for $5\,\mathrm{m} \times 10\,\mathrm{m}$ target.

FIGURE 7-15 ■ Variation in RCS due to frequency agility for a constant viewing angle. See text for details.



Frequency agility can be used to decorrelate contributions from clutter and atmospheric effects.

▸ Discussion

Outline

Detection framework

A radar with azimuth beamwidth θ_3 and pulse repetition frequency PRF is rotating at angular speed $\Omega\,\mathrm{rad/s}$. This provides

$$N = \frac{\theta_3}{\Omega}\mathrm{PRF} \quad \text{main beam samples per } 360^\circ \text{ sweep}$$

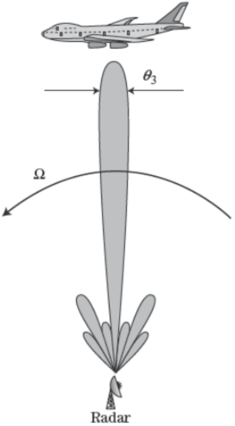


FIGURE 7-16 Rotating antenna rationale for Swerling model decorrelation assumptions. Echoes from a given target are collected in blocks. Each rotation of the antenna results in a new block, and each block contains multiple pulse returns.

Swerling models

Extreme cases for decorrelation. The non-fluctuating case is sometimes listed as the Swerling 0 or Swerling 5 case.

TABLE 7-3 ■ Swerling Models

Probability Density Function of RCS	Decorrelation	
	Scan-to-Scan	Pulse-to-Pulse
Exponential	Case 1	Case 2
Chi-square, degree 4	Case 3	Case 4

- The exponential distribution models many equal scatterers.
- Chi-square degree 4 models one dominant scatterer surrounded by many small.
- Scan-to-scan decorrelation may occur for long scan times.
- Pulse-to-pulse decorrelation may occur for frequency agile systems.

www.radartutorial.eu ▸ Discussion

Different decorrelations, scan or pulse

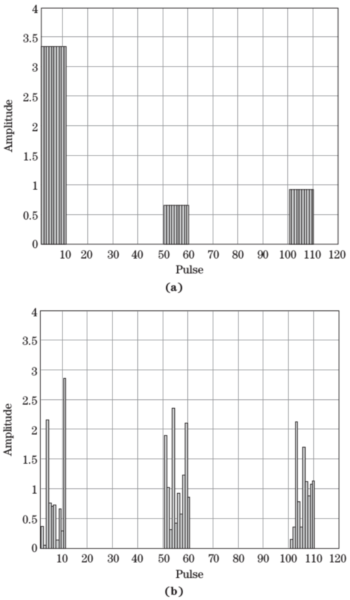


FIGURE 7-17 ■ Notional sequences of Swerling target samples. Results from three scans with 10 pulses per scan are shown. (a) Swerling case 1. (b) Swerling case 4.

Example: choice of model

Consider the following scenario:

- Target: $10\,\mathrm{m}$ long, complex aircraft.
- Frequency: $10\,\mathrm{GHz}$.
- Range: $30\,\mathrm{km}$.
- Cross-range velocity: $200\,\mathrm{m/s}$.

Choice of model:

- The complex nature of the aircraft with no clear dominating scatterer suggests an exponential PDF.
- Decorrelation angle: $\Delta\theta = \frac{\lambda}{2L} = 1.5\,\mathrm{mrad} = 0.86^\circ$. This corresponds to a flight time $\Delta t = R\Delta\theta/v = 225\,\mathrm{ms}$.
- If dwell time $T_\mathrm{d} < 225\,\mathrm{ms}$ we expect coherence during pulse-to-pulse and choose Swerling 1.
- If dwell time $T_\mathrm{d} > 225\,\mathrm{ms}$ the signal is decorrelated within the PRI, and we choose Swerling 2.

Outline

Doppler shift

A target traveling at radial velocity $v\cos\psi$ towards the radar causes a frequency shift $f \to f + f_\mathrm{d}$. After demodulation (removal of carrier frequency f), the baseband signal will ideally be

$$y[m] = A\mathrm{e}^{\mathrm{j}2\pi f_\mathrm{d} mT}$$

where f_d is easily detected by a Fourier transform of the data $y[m]$. However, amplitude fluctuations may change this ($A \to A[m]$).

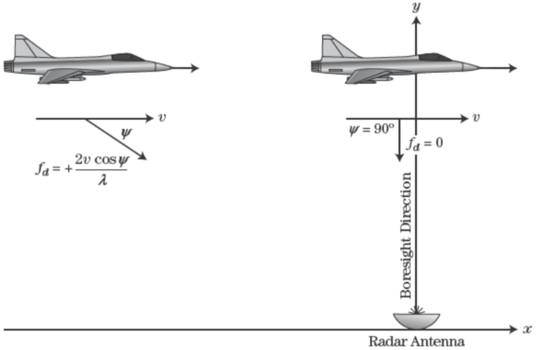
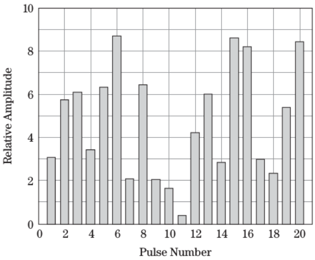


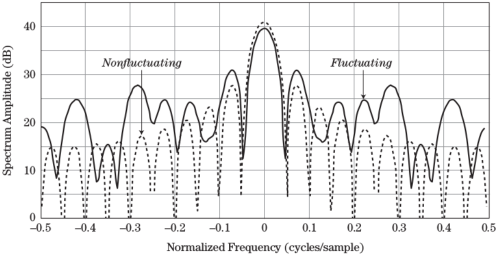
FIGURE 8-1 ■ Doppler shift is determined by the radial component of relative velocity between the target and radar.

Doppler spectrum of fluctuating targets

FIGURE 7-18 ■
Effect of amplitude fluctuations on target Doppler spectrum.
(a) 20-pulse Rayleigh fluctuating amplitude sequence.
(b) Spectrum of the fluctuating and nonfluctuating data.



(a)



(b)

More on Doppler processing in next lecture!

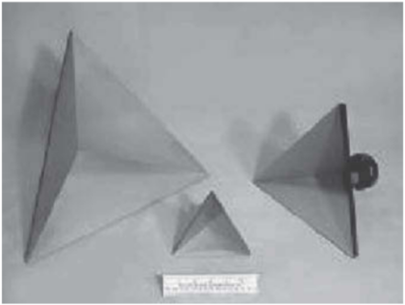
Outline

Conclusions

- Interference causes fluctuations in RCS with viewing angle and frequency, which may be modeled as random.
- Typical probability densities have been reviewed.
- An interactive matlab program is available for investigating effects of number of scatterers, frequency, range etc.
- Decorrelations in angle and frequency have been quantified.
- The bounding cases of Swerling models have been presented.

Discussion

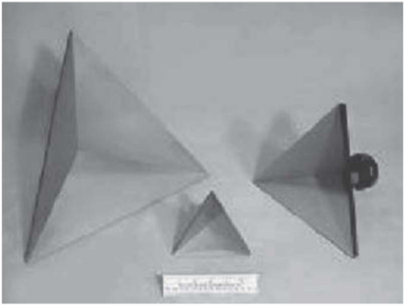
Why do you think there are different sizes of the trihedrals in the picture?



◂ Go back

Discussion

Why do you think there are different sizes of the trihedrals in the picture?



Answer: What matters is the size of the scatterer in terms of wavelengths. The different sizes target different wavelengths.

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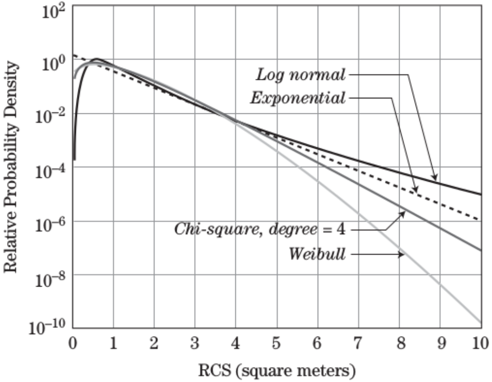
Discussion

What is the implication of a long tail in the probability distribution?



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Discussion

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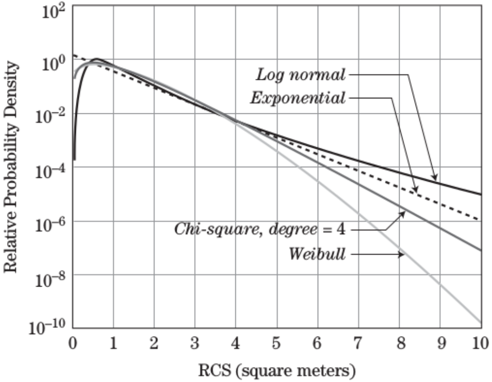

Answer: with a long tail (slow decrease), there is a higher probability for extreme events.

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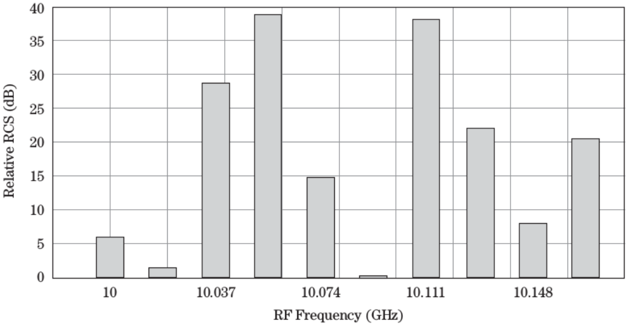
Discussion

With the frequency response below, what approximate mean (dB) RCS would a frequency agile system detect, switching between the different frequency bins? 0 dB, 10 dB, 20 dB, 30 dB, or 40 dB?

FIGURE 7-15 ■ Variation in RCS due to frequency agility for a constant viewing angle. See text for details.



◂ Go back

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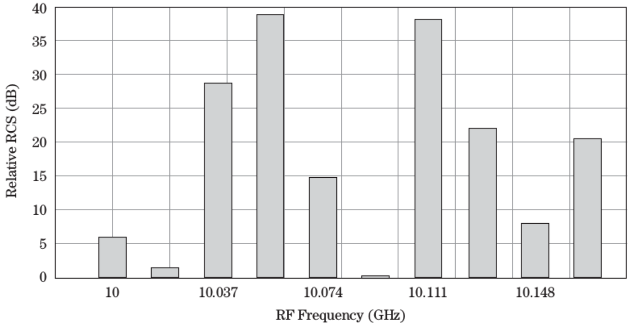

Answer: Around 20 dB (more exactly 18 dB).

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Discussion

With a decorrelation time of $\varDelta T$, pulse repetition interval PRI, and dwell time $T_\mathrm{d} = n_\mathrm{p}\mathrm{PRI}$, what is the condition for a Swerling 1 situation (scan-to-scan decorrelation), and a Swerling 2 situation (pulse-to-pulse decorrelation)?

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Discussion

With a decorrelation time of ΔT, pulse repetition interval PRI, and dwell time $T_\mathrm{d} = n_\mathrm{p}\mathrm{PRI}$, what is the condition for a Swerling 1 situation (scan-to-scan decorrelation), and a Swerling 2 situation (pulse-to-pulse decorrelation)?

Answer:

- Swerling 1: $T_\mathrm{d} > \Delta T$ (or $NT_\mathrm{d} > \Delta T$, with N being the number of scan positions for a search radar)
- Swerling 2: $\mathrm{PRI} > \Delta T$

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