EITN90 Radar and Remote Sensing
Lecture 2: The Radar Range Equation

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Spring 2020

Outline

Learning outcomes of this lecture

In this lecture we will

- Develop a physical model for the received power of a radar from a target at a distance
- Interpret the result in user terms and designer terms for different applications
- Investigate the requirements and methods of search and detection

Outline

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1. **Radar Range Equation**
   - Received power
   - Signal to noise ratio
   - Losses
   - Multiple pulses
   - Application oriented RRE:s
2. **Radar Search and Detection**
   - Search mode fundamentals
   - Detection fundamentals
3. **Conclusions**

Isotropic radiation pattern

Equal radiation in all directions.

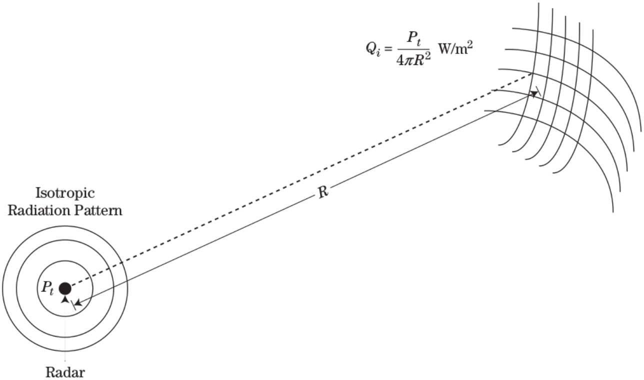


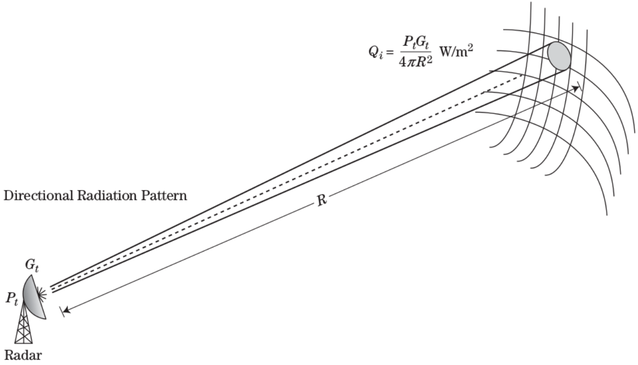
FIGURE 2-1 Power density at range R from the radar transmitter, for an isotropic (omnidirectional) antenna.

- P_t = transmitted power $[\mathrm{W}]$
- R = distance from source $[\mathrm{m}]$
- Q_i = power density $[\mathrm{W/m^2}]$

Directional radiation pattern

Stronger radiation in some directions.

FIGURE 2-2 ■
Power density at range R given transmit antenna gain G_t.



Transmitting antenna gain $G_\mathrm{t}(\theta, \phi) \stackrel{\mathrm{def}}{=} \lim_{R\to\infty} \frac{Q_\mathrm{i}(R,\theta,\phi)}{P_\mathrm{t}/(4\pi R^2)} \stackrel{\mathrm{def}}{=} \frac{4\pi A_\mathrm{e}}{\lambda^2}$.

Effective area A_e and gain G_t represent the same physical concept, just a scaling by $4\pi/\lambda^2$.

Scattered power from a target

Target is hit by power density Q_i, and scatters the power.

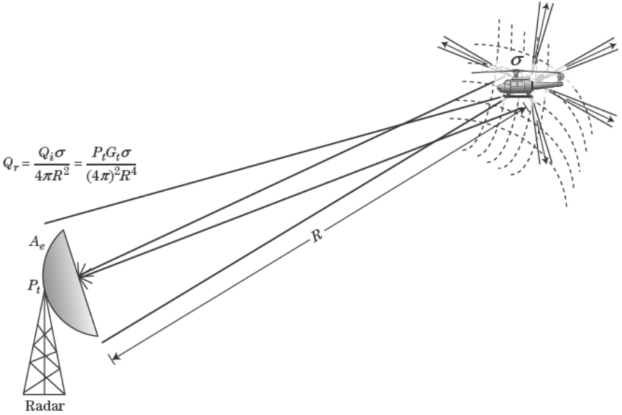


FIGURE 2-3 ■ Power density, Q_r, back at the radar receive antenna.

Radar cross section (RCS) $\sigma \overset{\text{def}}{=} \lim_{R\to\infty} \frac{4\pi R^2 Q_\mathrm{r}}{Q_\mathrm{i}}$ (Q_i held constant).

Received power

Received power is $P_\mathrm{r} = A_\mathrm{e} Q_\mathrm{r}$, effective area $A_\mathrm{e} \stackrel{\mathrm{def}}{=} \frac{\lambda^2}{4\pi} G_\mathrm{r}$.

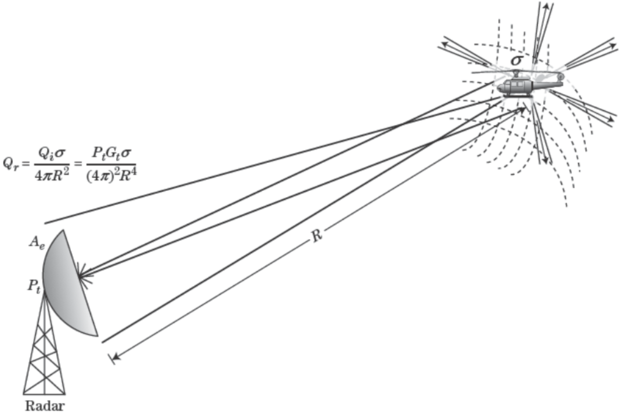


FIGURE 2-3 Power density, Q_r, back at the radar receive antenna.

Putting everything together implies the Radar Range Equation

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

Radar Range Equation

The radar range equation is the fundamental model for estimating the received power in a given scenario.

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

- P_t = peak transmitted power $[\mathrm{W}]$
- G_t = gain of transmit antenna (unitless)
- G_r = gain of receive antenna (unitless)
- λ = carrier wavelength $[\mathrm{m}]$
- σ = mean RCS of target $[\mathrm{m}^2]$
- R = range from radar to target $[\mathrm{m}]$

▸ Discussion question

Radar Range Equation, dB scale

The decibel (dB) scale is defined as

$$P_\mathrm{r}\,[\mathrm{dB}] \stackrel{\mathrm{def}}{=} 10\log_{10}(P_\mathrm{r})$$

The logarithm function has the properties $\log_{10}(ab) = \log_{10}(a) + \log_{10}(b)$, $\log_{10}(a/b) = \log_{10}(a) - \log_{10}(b)$, and $\log_{10}(a^b) = b\log_{10}(a)$. The RRE is then

$$P_\mathrm{r}\,[\mathrm{dB}] = P_\mathrm{t}\,[\mathrm{dB}] + G_\mathrm{t}\,[\mathrm{dB}] + G_\mathrm{r}\,[\mathrm{dB}] + 2\cdot\lambda\,[\mathrm{dB}] + \sigma\,[\mathrm{dB}] \underbrace{-30\log_{10}(4\pi)}_{=-33\,\mathrm{dB}} - 4\cdot R\,[\mathrm{dB}]$$

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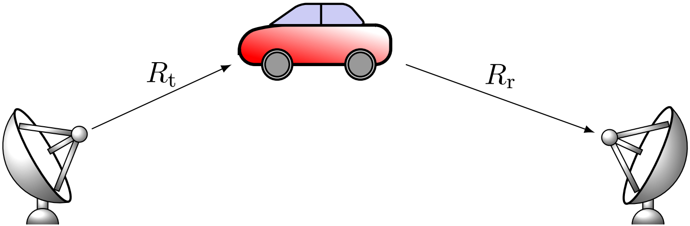
For quantities with physical units, it is common to introduce a reference level (note context sensitivity for dBm):

- $10\log_{10}\left(\frac{P_{\mathrm{r}}}{1\,\mathrm{W}}\right) \stackrel{\mathrm{def}}{=} P_{\mathrm{r}}\,[\mathrm{dBW}]$ (W for Watt)
- $10\log_{10}\left(\frac{P_{\mathrm{r}}}{1\,\mathrm{mW}}\right) \stackrel{\mathrm{def}}{=} P_{\mathrm{r}}\,[\mathrm{dBm}]$ (m for milli-Watt)
- $10\log_{10}\left(\frac{\lambda}{1\,\mathrm{m}}\right) \stackrel{\mathrm{def}}{=} \lambda\,[\mathrm{dBm}]$ (m for meter)
- $10\log_{10}\left(\frac{\sigma}{1\,\mathrm{m}^2}\right) \stackrel{\mathrm{def}}{=} \sigma\,[\mathrm{dBsm}]$ (sm for square meters)

Bistatic scenario

In a bistatic scenario, with two antennas separated in space, the transmit and receive distances R_t and R_r are usually different:

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R_\mathrm{t}^2 R_\mathrm{r}^2}$$



We will focus on the monostatic scenario.

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Thermal noise

The power of the thermal noise in the radar receiver is

$$P_\mathrm{n} = kT_\mathrm{s}B = kT_0FB$$

where the different factors are

- k is Boltzmann's constant $(1.38 \cdot 10^{-23}\,\mathrm{Ws/K})$
- T_0 is the standard temperature (290 K)
- T_s is the system noise temperature $(T_\mathrm{s} = T_0F)$
- B is the instantaneous receiver bandwidth in Hz
- $F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$ is the noise figure of the receiver subsystem (unitless)

SNR version of RRE

The thermal noise of the receiver can be combined with the RRE to yield the signal to noise ratio

$$\mathrm{SNR} = \frac{P_\mathrm{r}}{P_\mathrm{n}} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F B}$$

SNR version of RRE

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The final radar performance is determined by the signal to interference ratio, where

$$\mathrm{SIR} = \frac{S}{N + C + J} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4} \frac{1}{k T_0 F B + C + J}$$

- S = signal power
- N = noise power
- C = clutter power
- J = jammer power

Often only one of S/N, S/C or S/J is dominating.

Clutter

The radar signal can be scattered against many other things in the background. These interfering signals are called *clutter*.

Since the clutter scatterers are typically located close to the scatterer we want to detect, all terms in the radar equation cancel and the target signal to clutter ratio is

$$\mathrm{SCR} = \frac{\sigma}{\sigma_{\mathrm{c}}}$$

The clutter RCS σ_{c} can be significant, depending on how much is being illuminated by the radar. There are two typical kinds of clutter:

- Surface clutter
- Volume clutter

More on clutter in Chapter 5.

Discussion question

Surface clutter

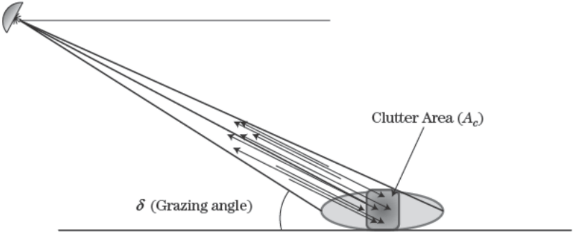


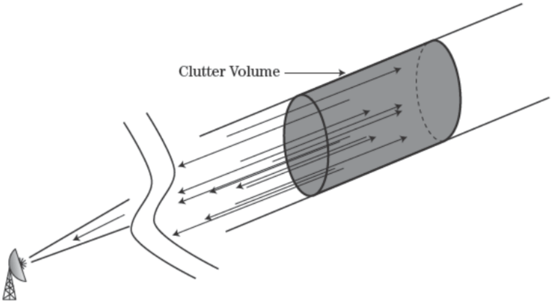
FIGURE 2-7 ■ Area (surface) clutter.

$$\sigma_{\text{cs}} = A_{\text{c}}\sigma^0$$

- σ_{cs} is the surface clutter radar cross section (square meters)
- A_{c} is the area of the illuminated (ground or sea surface) clutter cell (square meters)
- σ^0 is the surface backscatter coefficient (average reflectivity per unit area) (square meters per square meters, or unitless)

Volume clutter

FIGURE 2-8 ■ Volumetric (atmospheric) clutter.



$$\sigma_{\mathrm{cv}} = V_{\mathrm{c}}\eta$$

- σ_{cv} is the volume clutter radar cross section (square meters)
- V_{c} is the volume of the illuminated clutter cell (cubic meters)
- η is the volumetric backscatter coefficient (average reflectivity per unit volume) (square meters per cubic meters, or reciprocal meters)

Jamming

Jamming is a method of disabling a radar system by sending a strong interfering signal, saturating the receiver. The received power from this signal is calculated by the one-way equation

$$P_{\mathrm{rj}} = \frac{P_{\mathrm{j}} G_{\mathrm{j}} G_{\mathrm{rj}} \lambda^2}{(4\pi)^2 R_{\mathrm{jr}}^2 L_{\mathrm{s}}}$$

- P_{rj} received power from the jammer
- P_{j} transmitted power from the jammer
- G_{j} gain of the jammer antenna
- G_{rj} gain of the receive antenna (in direction of jammer)
- R_{jr} distance between jammer and receiver
- L_{s} system losses

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Losses

We have neglected a number of real-life losses in the RRE so far. The typical system loss would be the combination of several:

$$L_\mathrm{s} = L_\mathrm{t} L_\mathrm{a} L_\mathrm{r} L_\mathrm{sp}$$

where the different factors are

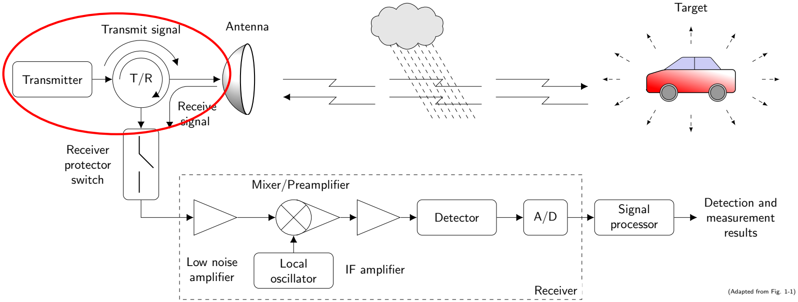
- L_s is the system loss
- L_t is the transmit loss
- L_a is the atmospheric loss
- L_r is the receiver loss
- L_sp is the signal processing loss

with the resulting system-loss SNR (an additional factor n_p can account for multiple pulses signal processing gain, see later slides)

$$\mathrm{SNR} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F B L_\mathrm{s}}$$

The various factors are discussed in the following slides.

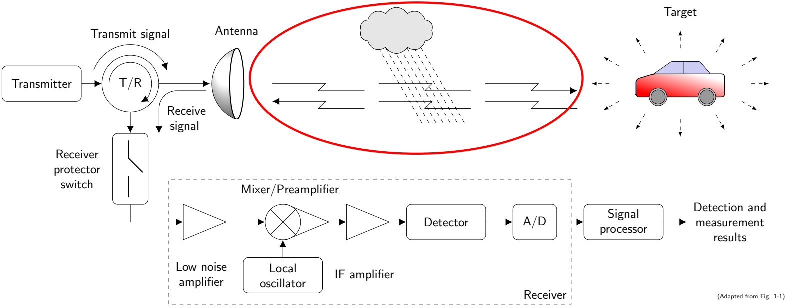
Transmit loss



Typically waveguides, cables, circulator, directional coupler, and switch add losses on the order of $L_{\mathrm{t}} \approx 3 - 4\,\mathrm{dB}$.

The antenna gain G may include some losses, depending on definition of its interface. Always consult datasheets!

Atmospheric loss



Atmospheric losses depend on frequency, weather conditions, altitude, etc. Typically measured in dB/km, and limits the range of the radar.

Atmospheric loss

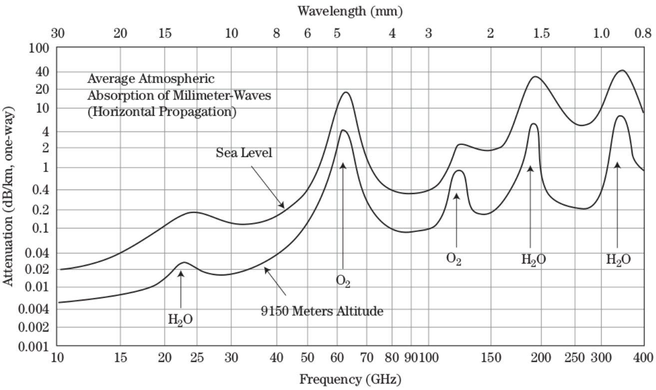
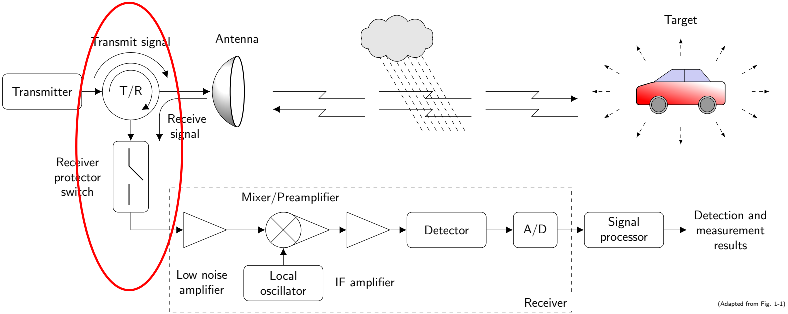


FIGURE 1-13 One-way atmospheric attenuation as a function of frequency at sea level and at 9150 meters altitude. (From U. S. Government work.)

Typical atmospheric losses as function of frequency, at two different altitudes. Note the peaks corresponding to resonant interaction with atmosphere molecules. Further losses are due to rain, fog etc.

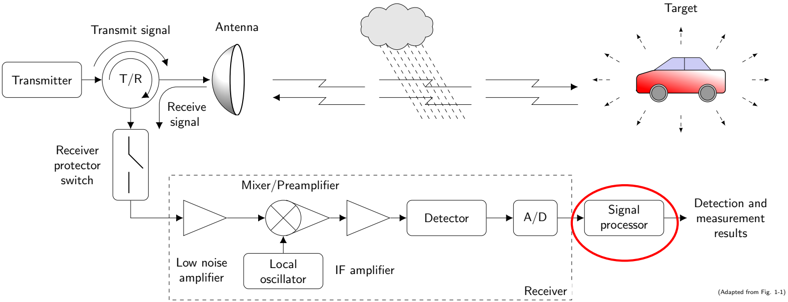
Long range radar systems tend to operate in frequency regions with low loss, but short-range systems may use losses for isolation.

Receive loss



Similar to transmit losses: waveguides, cables, circulator, switch, filters etc. Include losses up to the point where the noise figure F is specified ($F = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$).

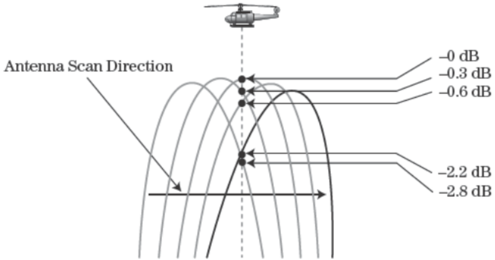
Signal processing loss



Even though the signal processor usually provides gain (typically on the order of n_{p}), the imperfections also provide some loss.

Signal processing loss: beam scanning

FIGURE 2-4 ■
Target signal loss due to beam scan.



Loss due to the target not being intercepted by the maximum gain of the beam. While tracking, beam can be kept on target.

Signal processing loss: straddle loss

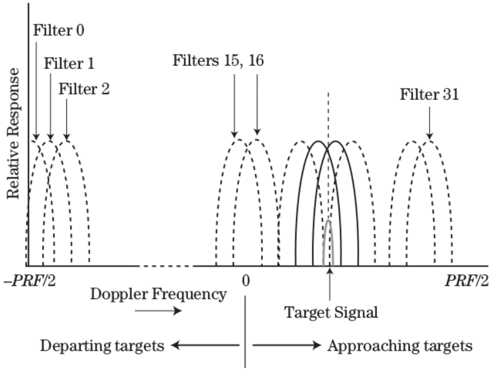


FIGURE 2-5 ■ Doppler filter bank, showing a target straddling two filters.

Discretization of range and Doppler frequencies in different processing bins may introduce loss around $1\,\mathrm{dB}$ in both range and Doppler. The dips can be reduced by increasing bin overlap (oversampling).

Outline

Multiple pulses

The SNR can be improved by using data from several pulses. The signal processing gain from this can be estimated as (assuming white noise)

- Coherent processing (both phase and amplitude):

$$\mathrm{SNR}(n_\mathrm{p}) = n_\mathrm{p}\,\mathrm{SNR}(1)$$

- Noncoherent processing (only amplitude):

$$\mathrm{SNR}(n_\mathrm{p}) \approx \sqrt{n_\mathrm{p}}\,\mathrm{SNR}(1)$$

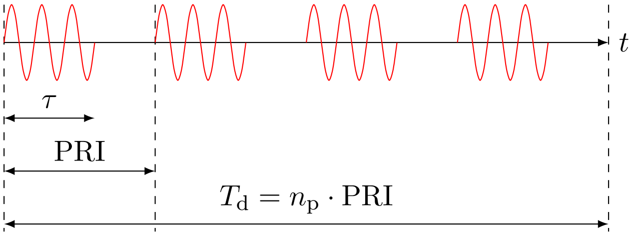
Typically, the processing gain by using multiple pulses can be estimated as

$$\sqrt{n_\mathrm{p}}\,\mathrm{SNR}(1) \leq \mathrm{SNR}(n_\mathrm{p}) \leq n_\mathrm{p}\,\mathrm{SNR}(1)$$

Using many pulses increases the measurement time.

Average power, coherent processing

We use n_p pulses, each with duration τ and repeated at Pulse Repetition Frequency (PRF=1/PRI, Pulse Repetition Interval).



- Dwell time $T_\mathrm{d} = n_\mathrm{p} \cdot \mathrm{PRI} = n_\mathrm{p}/\mathrm{PRF}$.
- Peak pulse power $P_\mathrm{t} = \frac{P_\mathrm{avg} T_\mathrm{d}}{n_\mathrm{p}\tau} = \frac{P_\mathrm{avg} T_\mathrm{d} B}{n_\mathrm{p}}$

The coherent processing SNR is then

$$\mathrm{SNR_c} = \underbrace{\left(\frac{P_\mathrm{avg} T_\mathrm{d} B}{n_\mathrm{p}}\right)}_{=\text{ peak } P_\mathrm{t} \text{ per pulse}} \frac{G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma n_\mathrm{p}}{(4\pi)^3 R^4 k T_0 F B L_\mathrm{s}} = \frac{P_\mathrm{avg} T_\mathrm{d} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F L_\mathrm{s}}$$

Pulse compression

There seems to be two conflicting requirements:

- High resolution requires short pulse time τ (or rather, high bandwidth)
- High SNR requires long pulse time τ

These requirements can be combined using pulse compression, explained in Chapter 20. The average power form of the RRE remains the same,

$$\mathrm{SNR}_{\mathrm{pc}} = \frac{P_{\mathrm{avg}} T_{\mathrm{d}} G_{\mathrm{t}} G_{\mathrm{r}} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F L_{\mathrm{s}}}$$

where

- $P_{\mathrm{avg}} T_{\mathrm{d}}$ is the energy in one pulse train
- $k T_0 F$ is the noise energy from the receiver

▸ Discussion question

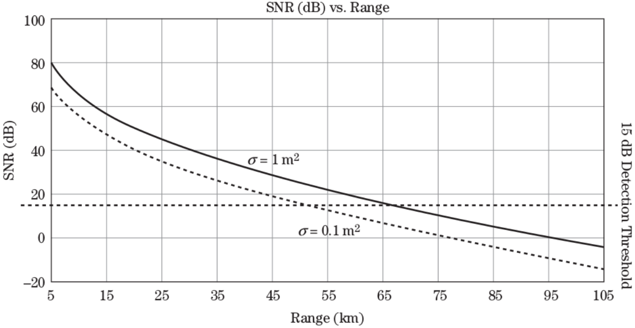
Outline

Case study: hypothetical radar system SNR

Transmitter:	150 kilowatt peak power
Frequency:	9.4 GHz
Pulse width:	1.2 microseconds
PRF:	2 kilohertz
Antenna:	2.5 meter diameter circular antenna (an efficiency $\eta = 0.6$ is used to determine antenna gain.)
Processing dwell time:	18.3 milliseconds
Receiver noise figure:	2.5 dB
Transmit losses:	3.1 dB
Receive losses:	2.4 dB
Signal processing losses:	3.2 dB
Atmospheric losses:	0.16 dB/km (one way)
Target RCS:	0 dBsm, –10 dBsm (1.0 and 0.1 m^2)
Target range:	5 to 105 km

Case study, graphical form

FIGURE 2-6 ■ Graphical solution to radar range equation.



Different detection ranges for the two different targets.

Search application

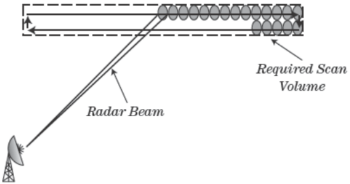


FIGURE 3-1 ■ ESA radar antenna beam scanning in the search mode.

A solid angle Ω is being scanned for targets at M beam positions with dwell time T_{d}. The total time to scan is then

$$T_{\mathrm{fs}} = MT_{\mathrm{d}} \approx \frac{\Omega}{\theta_3 \phi_3} T_{\mathrm{d}}$$

where θ_3 and ϕ_3 are the azimuth and elevation $3\,\mathrm{dB}$ beamwidths. Using $\theta_3 \phi_3 \approx \lambda^2 / A_{\mathrm{e}}$ and $G = 4\pi A_{\mathrm{e}}/\lambda^2$, the average power RRE can be written

$$\frac{P_{\mathrm{avg}} A_{\mathrm{e}}}{4\pi k T_0 F L_{\mathrm{s}}} \geq \mathrm{SNR}_{\mathrm{min}} \left(\frac{R^4}{\sigma} \right) \left(\frac{\Omega}{T_{\mathrm{fs}}} \right)$$

where "user terms" are on the right and "system designer terms" on the left. This shows that the power-aperture product $P_{\mathrm{avg}} A_{\mathrm{e}}$ has to be maximized in order to search a big solid angle Ω at small time T_{fs}.

Track application

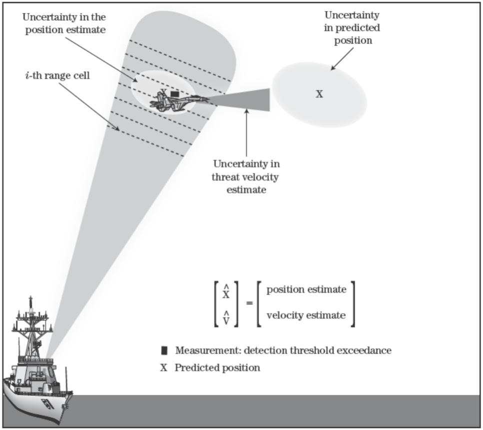


FIGURE 19-1 Tracking and prediction for a phased array radar.

When tracking one or several targets, important parameters are

- Tracking precision $\sigma_\theta \sim 1/\sqrt{\mathrm{SNR}}$
- Number of tracked targets N_t
- Updates per second r

RRE for track application

The RRE can be rewritten in terms of the tracking parameters as (see derivation in the book, Section 2.16)

$$\frac{P_{\mathrm{avg}} A_{\mathrm{e}}^3 k_{\mathrm{m}}^2}{\lambda^4 k T_0 F L_{\mathrm{s}}} = \left(\frac{\pi^2}{2}\right)\left(\frac{r N_{\mathrm{t}} R^4}{\sigma \cdot \sigma_\theta^2}\right)\left(\frac{1}{\cos^5(\theta_{\mathrm{scan}})}\right)$$

where $k_{\mathrm{m}} \in [1, 2]$ is a tracking system parameter, and the factor $\cos^5(\theta_{\mathrm{scan}})$ accounts for gain loss and beam broadening when scanning a phased array. This shows the strong dependence on antenna aperture for efficient tracking.

With known SNR rather than σ_θ, we could also write

$$\frac{P_{\mathrm{avg}} A_{\mathrm{e}}^2}{L_{\mathrm{s}} F \lambda^2} = \frac{\mathrm{SNR} \cdot 4\pi R^4 k T_0 \cdot \mathrm{PRF}}{\sigma}$$

which also demonstrates a strong dependence on A_{e}.

Some trade-offs

$$\mathrm{SNR} = \frac{P_{\mathrm{avg}} T_{\mathrm{d}} G_{\mathrm{t}} G_{\mathrm{r}} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F L_{\mathrm{s}}} = \frac{P_{\mathrm{avg}} T_{\mathrm{d}} A_{\mathrm{et}} A_{\mathrm{er}} \sigma / \lambda^2}{4\pi R^4 k T_0 F L_{\mathrm{s}}}$$

- Stealth technology: $\mathrm{SNR} \sim \sigma / R^4$ shows that σ needs to be reduced significantly in order to affect detection range R. This implies high costs.
- SNR increases with increased dwell time T_{d}, at the expense of longer measurement times.
- For fixed A_{e} and σ (antenna and scatterer large compared to wavelength), smaller wavelength increases SNR.

Discussion question

Outline

Outline

Task of the search mode

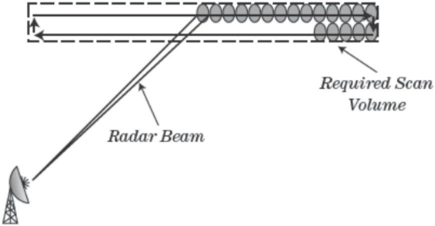


FIGURE 3-1 ■ ESA radar antenna beam scanning in the search mode.

The task of the search mode is to scan through a certain volume, and detect the presence of targets with no *a priori* knowledge of their existence.

The radar beam is directed at different angles, mechanically or electrically, and measurements are taken at each position. The scan has to be fast enough, so targets do not have too much time to move.

Mechanical vs electrical scanning

Mechanical

- Rotating turret.
- Typically scans in azimuth.
- Continuous movement 360° one direction or finite sector back-and-forth.
- Rotation speed needs to align with dwell time and range delay.

Electrical

- Phased array.
- Scans quickly in all directions.
- Beam width changes with angle.
- Scan loss can be compensated by increasing dwell time at large angles.

Search can be combined with track either by tracking-while-scanning (slow update), or search-and-track (interleaving track function, only ESA).

Outline

Threshold concept

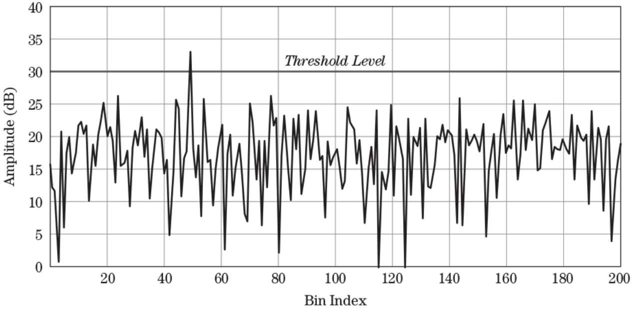


FIGURE 3-2 Concept of threshold detection. In this example, a target would be declared at bin #50.

A detection is registered when a signal is registered above a threshold, giving some margin to the noise floor. The signal needs to be considered as a random variable.

Probability

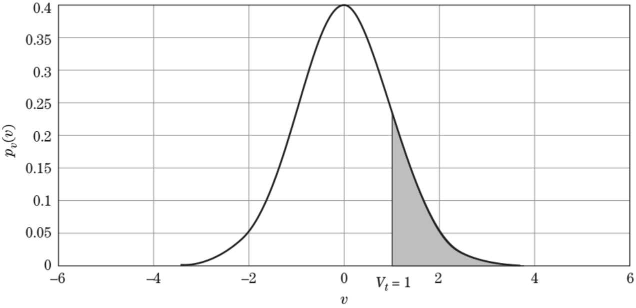


FIGURE 3-3 ■ Gaussian PDF for a voltage, v.

Probability of False Alarm: $$P_{\mathrm{FA}} = \int_{V_{\mathrm{t}}}^{\infty} p_{\mathrm{i}}(v)\,\mathrm{d}v$$

Probability of Detection: $$P_{\mathrm{D}} = \int_{V_{\mathrm{t}}}^{\infty} p_{\mathrm{s+i}}(v)\,\mathrm{d}v$$

The probability density function (PDF) is denoted p, index “i” for interference and index “s+i” for signal in the presence of interference.

Noise probability distribution: Rayleigh distribution

When measuring both amplitude and phase, $v = I + \mathrm{j}Q$, the I and Q signals due to noise are zero mean Gaussian. This implies the amplitude $r = \sqrt{I^2 + Q^2}$ is Rayleigh distributed, that is,

$$p_{\mathrm{i}}(r) = \frac{r}{\sigma_{\mathrm{n}}^2} \exp\left(-\frac{r^2}{2\sigma_{\mathrm{n}}^2}\right)$$

where σ_{n}^2 is the mean square voltage, or variance of the noise, called noise power. In the figure below, $\sigma_{\mathrm{n}}^2 = 0.04$.

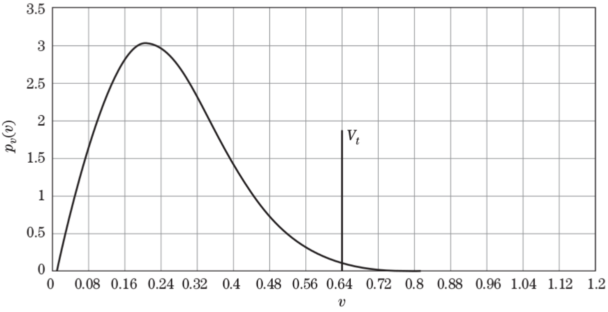


FIGURE 3-4 Rayleigh distribution with an arbitrary threshold.

Probability of false alarm

Using the Rayleigh distribution, the probability of false alarm can be computed explicitly (a truly rare case!):

$$P_{\mathrm{FA}} = \int_{V_{\mathrm{t}}}^{\infty} \frac{r}{\sigma_n^2} \exp\left(-\frac{r^2}{2\sigma_{\mathrm{n}}^2}\right) \mathrm{d}r = \exp\left(-\frac{V_{\mathrm{t}}^2}{2\sigma_{\mathrm{n}}^2}\right)$$

For a desired P_{FA}, this provides the required threshold:

$$V_{\mathrm{t}} = \sqrt{2\sigma_n^2 \ln(1/P_{\mathrm{FA}})}$$

To further reduce the P_{FA}, it is common to make confirmation measurements of a detection. With n confirmations, we get

$$P_{\mathrm{FA}}(n) = [P_{\mathrm{FA}}(1)]^n$$

for a false alarm in all dwells, which quickly reduces the P_{FA}.

Signal + noise PDF: Rician distribution

For a non-fluctuating target signal embedded in Gaussian noise, we obtain the Rice distribution:

$$p_{\mathrm{s+i}}(v) = \frac{v}{\sigma_{\mathrm{n}}^2} \exp\left(-\frac{v^2 + v_{\mathrm{s+i}}^2}{2\sigma_{\mathrm{n}}^2}\right) I_0(v v_{\mathrm{s+i}}/\sigma_{\mathrm{n}}^2)$$

where $v_{\mathrm{s+i}}$ is the mean amplitude, and I_0 is the modified Bessel function of the first kind and second order. For $v_{\mathrm{s+i}} = 0$ this is the Rayleigh distribution. The probability of detection is

$$P_{\mathrm{D}} = \int_{V_{\mathrm{t}}}^{\infty} p_{\mathrm{s+i}}(v)\,\mathrm{d}v = \int_{V_{\mathrm{t}}}^{\infty} \frac{v}{\sigma_{\mathrm{n}}^2} \exp\left(-\frac{v^2 + v_{\mathrm{s+i}}^2}{2\sigma_{\mathrm{n}}^2}\right) I_0(v v_{\mathrm{s+i}}/\sigma_{\mathrm{n}}^2)\,\mathrm{d}v$$

Not surprisingly, this does not have a closed solution, but can be implemented numerically (Marcum's Q-function).

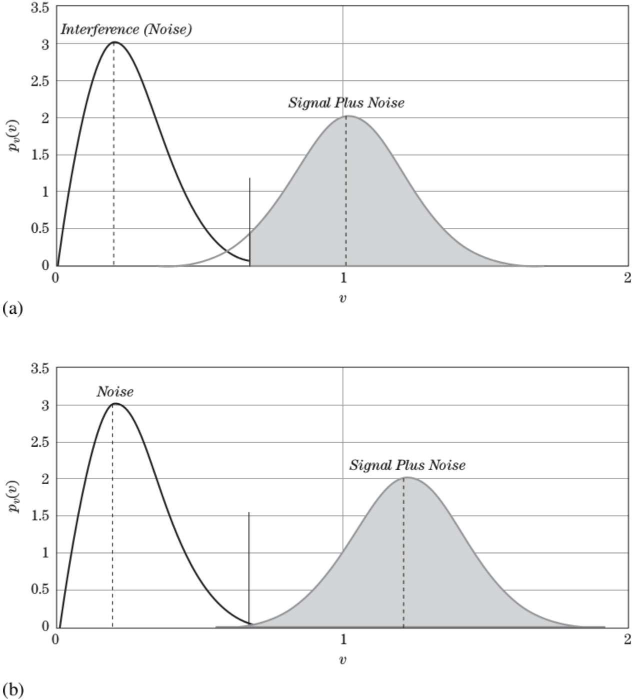
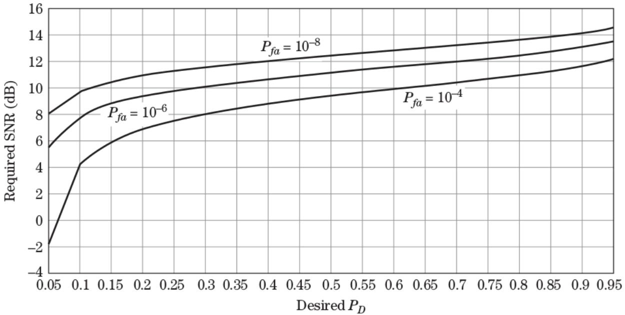


FIGURE 3-5 ■ (a) Noise-like distribution, with target-plus-noise distribution. (b) Noise-like distribution, with target-plus-noise distribution, demonstrating the higher P_D achieved with a higher SNR.

Receiver operating curves, ROC

To investigate the trade-off between P_{FA}, P_{D}, and SNR, a curve of two with varying values of the third can be plotted.

FIGURE 3-6 ■ SNR required to achieve a given P_D, for several P_{FA}'s, for a nonfluctuating (SW0) target in noise.



Higher level system requirements determine desired P_{FA} and P_{D} which can vary a lot. Typical values could be P_{D} around $50\% - 90\%$ for a P_{FA} in the order of $10^{-4} - 10^{-6}$. In the curves above, this requires SNR around $10\,\text{dB} - 13\,\text{dB}$.

Fluctuating targets: motivation

FIGURE 6-29 ■ A7C Corsair RCS measurement set up at the Navy Junction Ranch Range.

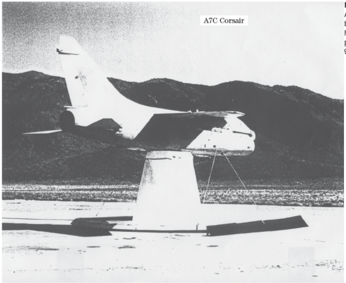
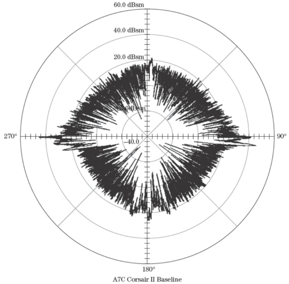


FIGURE 6-30 ■ A7 C measured backscatter for horizontal polarization at 9.5 GHz, 20 dB/div.



Big, real life targets have very complicated RCS, depending strongly on angle. A statistical description is necessary.

The Swerling models

Two different PDF:s (different target characteristics), combined with two different fluctuation rates: dwell-to-dwell or pulse-to-pulse.

TABLE 3-1 ■ Swerling Models

	Fluctuation Period	
Probability Density Function of RCS	**Dwell-to-Dwell**	**Pulse-to-Pulse**
Rayleigh	Case 1	Case 2
Chi-square, degree 4	Case 3	Case 4

TABLE 3-2 ■ Required SNR for Various Target Fluctuation Models

	P_D	**SW0**	**SW1**	**SW2**	**SW3**	**SW4**
$P_{FA} = 10^{-4}$	50	9.2	10.8	10.5	11	9.8
	90	11.6	19.2	19	16.5	15.2
$P_{FA} = 10^{-6}$	50	11.1	12.8	12.5	11.8	11.8
	90	13.2	21	21	17.2	17.1

For fixed P_{FA} and P_{D}, a fluctuating target (SW1-4) requires higher SNR than a non-fluctuating target (SW0).

Further details in Chapter 7.

Receiver operation curves

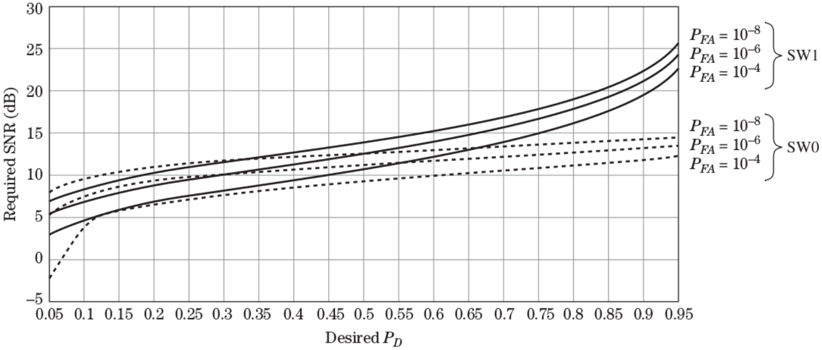


FIGURE 3-8 ■ SNR required to achieve a given P_D, several values of P_{FA}, for nonfluctuating (SW0) and fluctuating (SW1) target models.

For high P_{D} the required SNR for fluctuating targets is significantly higher than for non-fluctuating. For low P_{D} fluctuating targets may require lower SNR than fluctuating, but this is seldom an interesting region.

Closed-form solutions

Chapter 3.3.8 presents some closed form solutions for computing probabilities. These can be convenient in specific cases, but please **always check carefully the region of applicability of the formulas!**

Multiple dwells

The single-dwell detection probability can be improved using multiple dwells with only a small penalty in cumulative false alarm:

$$P_{\mathrm{D}}(n) = 1 - [1 - P_{\mathrm{D}}(1)]^n$$
$$P_{\mathrm{FA}}(n) = nP_{\mathrm{FA}}(1)$$

Requiring m-of-n detections can further improve both probabilities:

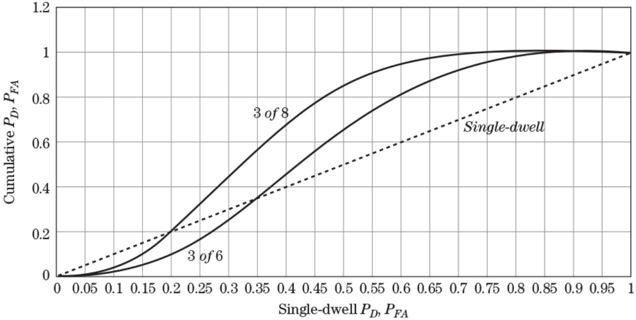


FIGURE 3-12 3-of-6 and 3-of-8 probability of threshold crossing versus single-dwell probability.

Outline

Conclusions

- The radar range equation estimates the received power or SNR.
- An overview of losses have been presented.
- The use of multiple pulses to strengthen SNR has been demonstrated.
- The RRE can be adapted to various specific applications, like search or track, with parameters specific for the application.
- Fundamentals of detection theory have been treated, introducing P_{D} and P_{FA}.

Discussion

Based on the radar range equation

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

should we go for long or short wavelengths to maximize P_r?

Go back

Discussion

Based on the radar range equation

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

should we go for long or short wavelengths to maximize P_r?
Answer:
With G_t and G_r fixed, long wavelengths are preferred.

Go back

Discussion

Based on the radar range equation

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

should we go for long or short wavelengths to maximize P_r?
Answer:
With G_t and G_r fixed, long wavelengths are preferred.
Using $G = \frac{4\pi A_\mathrm{e}}{\lambda^2}$, the RRE is

$$P_\mathrm{r} = \frac{P_\mathrm{t} A_\mathrm{te} A_\mathrm{re} \sigma}{4\pi \lambda^2 R^4}$$

Go back

Discussion

Based on the radar range equation

$$P_\mathrm{r} = \frac{P_\mathrm{t} G_\mathrm{t} G_\mathrm{r} \lambda^2 \sigma}{(4\pi)^3 R^4}$$

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$$P_\mathrm{r} = \frac{P_\mathrm{t} A_\mathrm{te} A_\mathrm{re} \sigma}{4\pi \lambda^2 R^4}$$

With A_te and A_re fixed, short wavelengths are preferred!

◂ Go back

Discussion

Give some examples of scatterers giving rise to clutter!

◂ Go back

Discussion

Give some examples of scatterers giving rise to clutter!

Answer:
Surface clutter: rocks, grass, sea waves, bushes, sand dunes...
Volume clutter: rain, hail, insect swarms, forest, sand storm...

Go back

Discussion

With the average power form of the radar range equation

$$\mathrm{SNR} = \frac{P_{\mathrm{avg}} T_{\mathrm{d}} G_{\mathrm{t}} G_{\mathrm{r}} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F L_{\mathrm{s}}}$$

things only get better with large dwell time T_{d}. Are there any disadvantages?

Go back

Discussion

With the average power form of the radar range equation

$$\mathrm{SNR} = \frac{P_{\mathrm{avg}} T_{\mathrm{d}} G_{\mathrm{t}} G_{\mathrm{r}} \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 F L_{\mathrm{s}}}$$

things only get better with large dwell time T_{d}. Are there any disadvantages?

Answer: yes, longer measurement time implies slower reaction.

Go back

Discussion

How do you think the radar cross section of a scatterer can be reduced?

◂ Go back

Discussion

How do you think the radar cross section of a scatterer can be reduced?

Answer:

1. Shaping the scatterer to divert reflection away from monostatic direction.
2. Absorbing the incoming wave.
3. Active cancellation: send out a signal that cancels the reflection signal (very difficult).

Go back