# EITN90 Radar and Remote Sensing
# Lecture 7: Doppler phenomenology and data acquisition

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# Outline

1. **Doppler shift**
2. **The Fourier transform**
3. **Spectrum of a pulsed radar signal**
4. **Pulsed radar data acquisition**
5. **Doppler signal model**
6. **Range-Doppler spectrums**
   - Stationary radar
   - Moving radar
7. **FMCW radar**
8. **Conclusions**

# Learning outcomes of this lecture

In this lecture we will

- See how relative motion induces the Doppler frequency shift
- Introduce the Fourier transform to describe signals
- Study the spectrum of pulsed radar signals
- Understand I/Q channels for data acquisition
- See examples of range-Doppler spectra

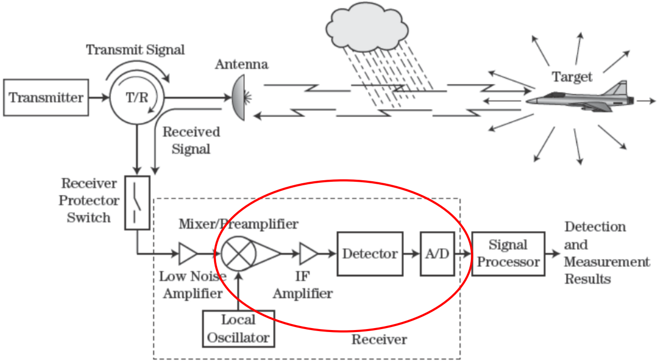



**FIGURE 1-1** ■ Major elements of the radar transmission/ reception process.

# Outline

# Doppler shift

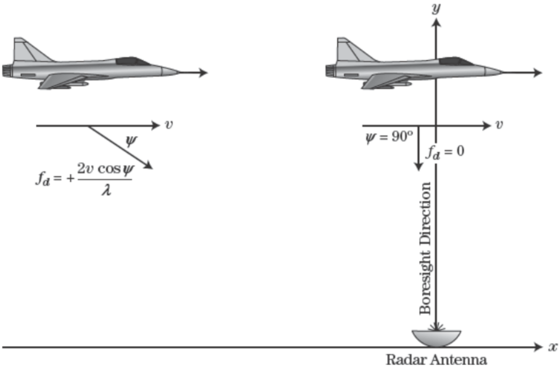



**FIGURE 8-1** Doppler shift is determined by the radial component of relative velocity between the target and radar.

$$f_{\mathrm{d}} = \frac{2v_{\mathrm{r}}}{c} f = \frac{2v}{\lambda} \cos\psi$$

Only the radial motion (towards/from the radar) matters.

# Doppler shift, examples

**TABLE 8-1** ■ Doppler Shift as a Function of Velocity and Frequency

| Radiofrequency $f$ | | Doppler Shift $f_d$ (Hz) | | |
|---|---|---|---|---|
| Band | Frequency (GHz) | 1 m/s | 1 knot | 1 mph |
| L | 1 | 6.67 | 3.43 | 2.98 |
| S | 3 | 20.0 | 10.3 | 8.94 |
| C | 5 | 33.3 | 17.1 | 14.9 |
| X | 10 | 66.7 | 34.3 | 29.8 |
| $K_u$ | 16 | 107 | 54.9 | 47.7 |
| $K_a$ | 35 | 233 | 120 | 104 |
| W | 95 | 633 | 326 | 283 |

$$f_{\mathrm{d}} = \frac{2v_{\mathrm{r}}}{c} f = \frac{2v}{\lambda} \cos \psi$$

Since most speeds $v$ are very small compared to speed of light $c$, the Doppler shift is small compared to the carrier frequency.

# Outline

# The Fourier transform

The Fourier transform is the archetypical method to consider a time domain function in frequency domain:

$$X(f) = \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-\mathrm{j}2\pi f t} \, \mathrm{d}t$$
$$x(t) = \int_{-\infty}^{\infty} X(f) \mathrm{e}^{\mathrm{j}2\pi f t} \, \mathrm{d}f$$

Often, the transform is instead expressed in terms of angular frequency $\omega = 2\pi f$:

$$\hat{X}(\omega) = \int_{-\infty}^{\infty} x(t) \mathrm{e}^{-\mathrm{j}\omega t} \, \mathrm{d}t$$
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{X}(\omega) \mathrm{e}^{\mathrm{j}\omega t} \, \mathrm{d}\omega$$

▸ Discussion

# Some explicit Fourier transforms

| $x(t)$ | $X(f)$ |
|---|---|
| $1$ | $\delta(f)$ |
| $\mathrm{e}^{\mathrm{j}2\pi f_0 t}$ | $\delta(f - f_0)$ |
| $\cos(2\pi f_0 t)$ | $\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$ |
| $\mathrm{rect}(t/\tau)$ | $\tau \frac{\sin(\pi f \tau)}{\pi f \tau} = \tau\,\mathrm{sinc}(\pi f \tau)$ |
| $\mathrm{e}^{-(t/\tau)^2/2}$ | $\tau\sqrt{2\pi}\mathrm{e}^{-(2\pi f \tau)^2/2}$ |

# Some properties of the Fourier transform

- Linearity:

$$x(t) = ax_1(t) + bx_2(t) \quad \Leftrightarrow \quad X(f) = aX_1(f) + bX_2(f)$$

- Time shifting:

$$x(t) = y(t - t_0) \quad \Leftrightarrow \quad X(f) = \mathrm{e}^{-\mathrm{j}2\pi f t_0} Y(f)$$

- Frequency shifting:

$$x(t) = \mathrm{e}^{\mathrm{j}2\pi f_0 t} y(t) \quad \Leftrightarrow \quad X(f) = Y(f - f_0)$$

- Scaling ($a > 0$ is a real number):

$$x(t) = y(at) \quad \Leftrightarrow \quad X(f) = \frac{1}{a} Y(f/a)$$

- Convolution vs product ($[x_1 * x_2](t) = \int x_1(t-\tau) x_2(\tau)\,\mathrm{d}\tau$):

$$\begin{aligned} x(t) &= [x_1 * x_2](t) \quad &\Leftrightarrow \quad X(f) &= X_1(f) X_2(f) \\ x(t) &= x_1(t) x_2(t) \quad &\Leftrightarrow \quad X(f) &= [X_1 * X_2](f) \end{aligned}$$

# Discrete Fourier transform and FFT

The Discrete Fourier Transform (DFT) is a discretization of the continuous transform in time and frequency:

$$X_k = \sum_{n=0}^{N-1} x_n \mathrm{e}^{-\mathrm{j}2\pi kn/N}$$

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \mathrm{e}^{\mathrm{j}2\pi kn/N}$$

The Fast Fourier Transform (FFT) is any implementation of the DFT that can be considered "fast".

The most known is the Cooley-Tukey radix-2 algorithm, requiring the number of samples to be $N = 2^r$ for some integer $r$. If this is not the case, zero-padding can be applied with little penalty.

# The effect of zero-padding a DFT

Assume a sampled signal $x = \{x_n\}_{n=0}^{N-1}$ is augmented by a number of zeros,

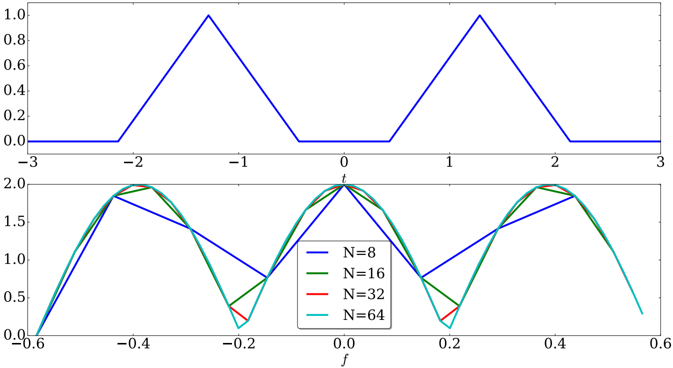
$$y = \{y_n\}_{n=0}^{M-1}, \quad y_n = \begin{cases} x_n & n = 0, 1, \ldots, N-1 \\ 0 & n = N, N+1, \ldots, M-1 > N \end{cases}$$

The corresponding DFT is then

$$Y_k = \sum_{n=0}^{M-1} y_n \mathrm{e}^{-\mathrm{j}2\pi kn/M} = \sum_{n=0}^{N-1} x_n \mathrm{e}^{-\mathrm{j}2\pi kn/M}$$

which can be seen as an interpolation in frequency since the step length $1/M$ is smaller than the original $1/N$.

# The effect of zero-padding a DFT

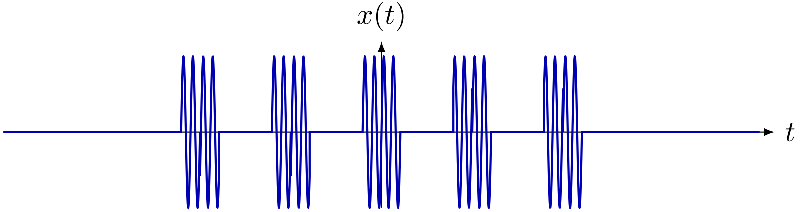



Original time-domain function (top graph) sampled at 8 points. Augmented with zeros to 16, 32, and 64 points, makes the DFT interpolate between the original points.

# Outline

# Pulsed radar signals

We intend to find out the spectrum of a pulsed radar signal:



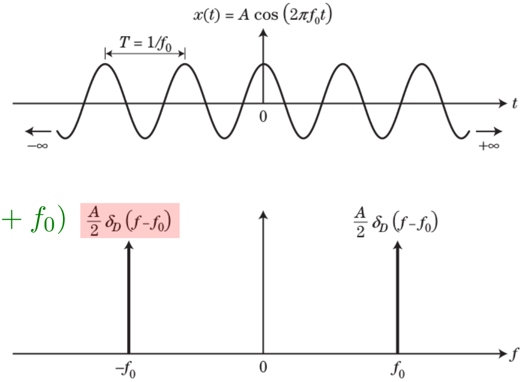

The strategy is to express the signal as a modulated carrier:

$$x(t) = \mathrm{e}^{\mathrm{j}2\pi ft} p(t)$$

where $\mathrm{e}^{\mathrm{j}2\pi ft}$ is the carrier wave, and $p(t)$ is the modulation (change in amplitude).

# Infinite length continuous wave

**FIGURE 8-3** ■ Infinite-length continuous wave (CW) signal of frequency $f_0$ and its frequency spectrum.




$\frac{A}{2}\delta(f + f_0)$

$$x(t) = A\cos(2\pi f_0 t) = \frac{A}{2}(\mathrm{e}^{\mathrm{j}2\pi f_0 t} + \mathrm{e}^{-\mathrm{j}2\pi f_0 t})$$

$$X(f) = \frac{A}{2}\delta(f - f_0) + \frac{A}{2}\delta(f + f_0)$$

# Single rectangular pulse

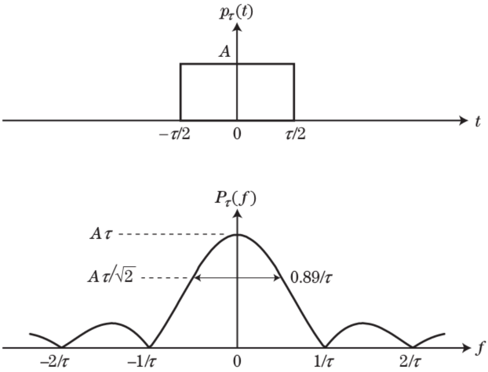



**FIGURE 8-4** ■ A single simple pulse and its spectrum.

$$p_\tau(t) = \begin{cases} A, & -\tau/2 < t < \tau/2 \\ 0, & \text{otherwise} \end{cases}$$

$$P_\tau(f) = \int_{-\tau/2}^{\tau/2} A e^{-j2\pi f t}\, dt = A\tau \frac{\sin(\pi f \tau)}{\pi f \tau} = A\tau\, \mathrm{sinc}(\pi f \tau)$$

Note the contradictory definition of $\mathrm{sinc}(z)$ in the book's (8.15).

# Infinite pulse train

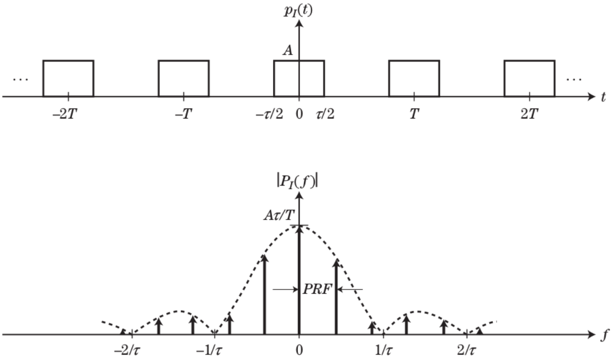



**FIGURE 8-5** Infinite pulse train signal and its spectrum.

$$p_{\mathrm{I}}(t) = \sum_{n=-\infty}^{\infty} p_\tau(t - nT) = \left[ p_\tau(\cdot) * \sum_{n=-\infty}^{\infty} \delta(\cdot - nT) \right](t)$$

$$P_{\mathrm{I}}(f) = \underbrace{\{A\tau \operatorname{sinc}(\pi f \tau)\}}_{=P_\tau(f)} \left\{ \frac{1}{T} \sum_{k=-\infty}^{\infty} \delta(f - k \cdot \underbrace{\mathrm{PRF}}_{=1/T}) \right\}$$

# Finite pulse train

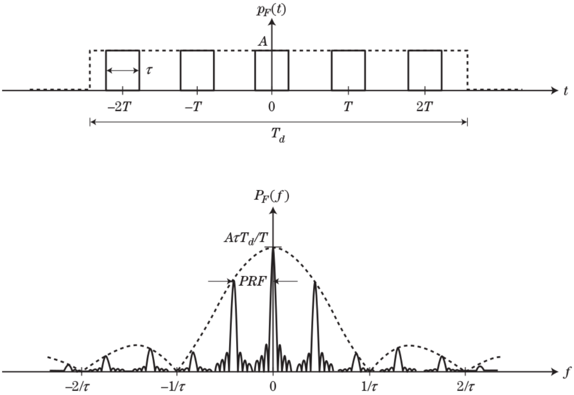


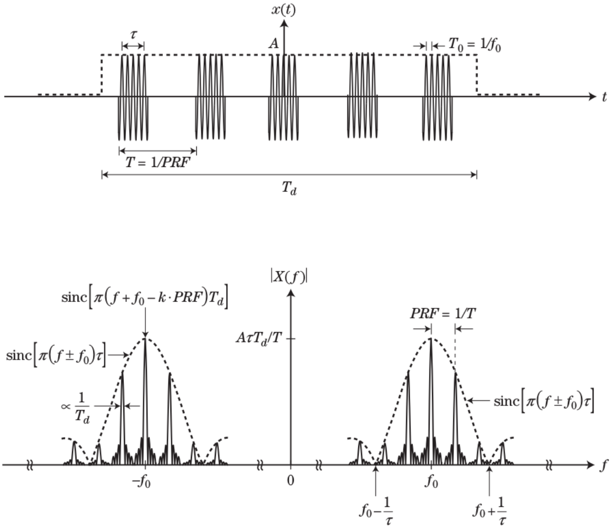

**FIGURE 8-6** Finite pulse train signal and its spectrum.

$$p_F(t) = p_I(t) \cdot p_{T_d}(t), \quad p_{T_d}(t) = \begin{cases} 1, & -T_d/2 < t < T_d/2 \\ 0, & \text{otherwise} \end{cases}$$

$$P_F(f) = [P_I(\cdot) * P_{T_d}(\cdot)](f) = \frac{AT_d\tau}{T} \sum_{k=-\infty}^{\infty} \mathrm{sinc}(\pi\tau k \cdot \mathrm{PRF})\, \mathrm{sinc}[\pi(f - k \cdot \mathrm{PRF})T_d]$$

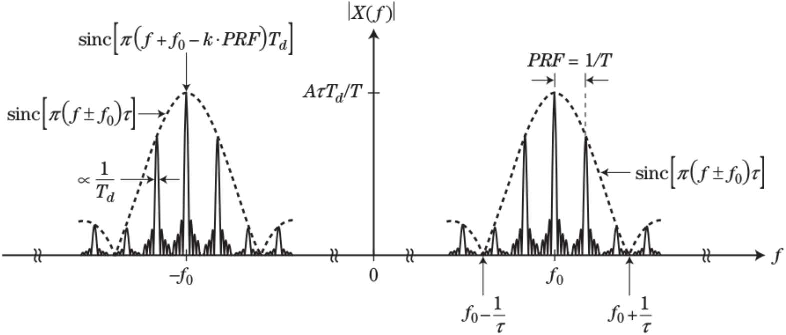
# End result: modulated finite pulse train

**FIGURE 8-7** ■ Finite duration modulated pulse train signal and its Fourier transform.




The carrier wave $\cos(2\pi f_0 t)$ shifts the spectrum to $\pm f_0$.

# Frequency scales




Four frequency scales:

- Bandwidth of spectral lines ($1/T_\mathrm{d}$)
- Spacing of spectral lines ($1/T$)
- Rayleigh bandwidth of single pulse envelopes ($1/\tau$)
- Center frequencies ($\pm f_0$)

Discussion

# Pulsed waveform spectrum with moving targets

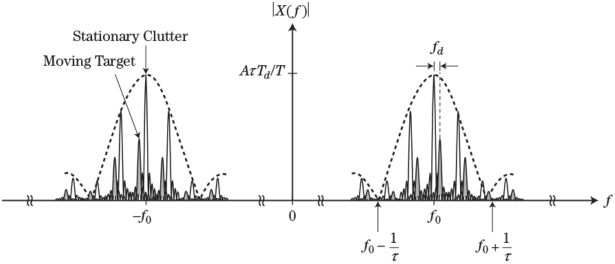



**FIGURE 8-8** Spectrum of the received signal from a moving target and stationary clutter.

The stationary clutter stays centered at $\pm f_0$, whereas a moving target signal is shifted by $f_\mathrm{d}$. The stationary clutter can be filtered out, allowing detection of the weaker target.

# Doppler resolution

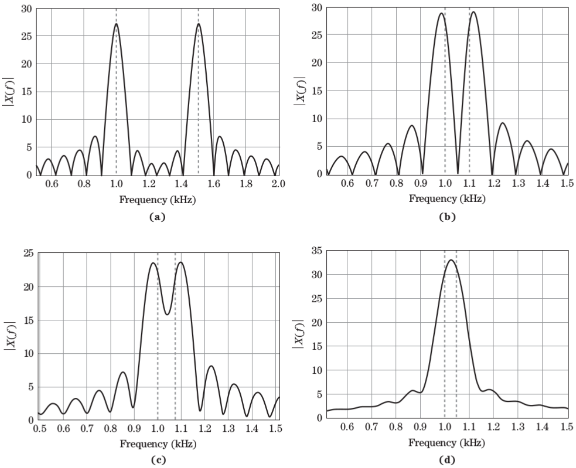
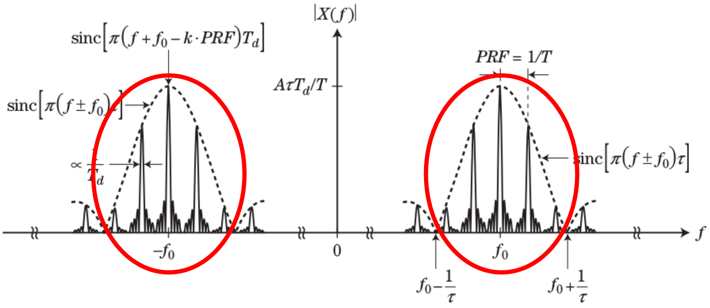



**FIGURE 8-9** ■ Illustration of the concept of Doppler resolution. Individual spectral lines have 100 Hz Rayleigh bandwidth and zero relative phase. (a) 500 Hz spacing. (b) 100 Hz spacing. (c) 75 Hz spacing. (d) 50 Hz spacing.

Since linewidth is proportional to $1/T_{\mathrm{d}}$, Doppler resolution improves with increasing dwell time $T_{\mathrm{d}}$.

# Receiver bandwidth effects



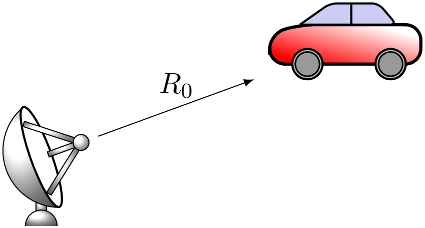

- In principle, the signal carries power at all frequencies.
- About 91% of the total energy is inside the main lobe.
- Extending the bandwidth outside the main lobe increases signal with at most additional 9%, but noise increases proportionally to bandwidth.
- To maximize SNR, the receiver bandwidth should match expected signal.

# Why multiple pulses?

- Consider an X-band radar ($10\,\mathrm{GHz}$) with $10\,\mu\mathrm{s}$ pulses. A target moving at Mach 1 ($340\,\mathrm{m/s}$), implies a Doppler shift of $22.7\,\mathrm{kHz}$.
- Without Doppler shift, one pulse has $(10 \cdot 10^{-6}) \cdot (10 \cdot 10^{9}) = 100\,000$ cycles.
- With Doppler shift, one pulse has $(10 \cdot 10^{-6}) \cdot (10 \cdot 10^{9} + 22.7 \cdot 10^{3}) = 100\,000.227$ cycles, only about a quarter of a cycle more.
- The Doppler resolution from one pulse is $1/\tau = 1/(10 \cdot 10^{-6})\,\mathrm{Hz} = 100\,\mathrm{kHz}$, not sufficient to resolve the Doppler peak at $22.7\,\mathrm{kHz}$ shift.
- Using multiple pulses, the resolution becomes $1/T_{\mathrm{d}} = 1/(N \cdot \mathrm{PRI}) = \mathrm{PRF}/N$, which can be made small enough.

# Outline

# Received signal




Transmitted signal:

$$x(t) = A\cos(2\pi f_0 t + \theta) = \mathrm{Re}\{A\mathrm{e}^{\mathrm{j}(2\pi f_0 t + \theta)}\} = \mathrm{Re}\{(A\mathrm{e}^{\mathrm{j}\theta})\mathrm{e}^{\mathrm{j}2\pi f_0 t}\}$$

Received signal:

$$y(t) \sim x\left(t - \frac{2R_0}{c}\right) = \mathrm{Re}\left\{\left(A\mathrm{e}^{\mathrm{j}(\theta - \frac{4\pi R_0}{\lambda})}\right)\mathrm{e}^{\mathrm{j}2\pi f_0 t}\right\}$$

Complex amplitude of received signal: $A\exp\left[\mathrm{j}\left(\theta - \frac{4\pi}{\lambda}R_0\right)\right]$

▸ Discussion

# Video detector

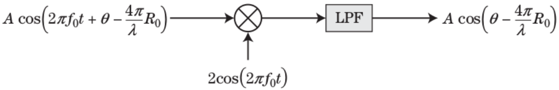



**FIGURE 8-11** Single-channel detector.

By mixing the received signal with a reference signal $2\cos(2\pi f_0 t)$, we obtain (writing $\theta - 4\pi R_0/\lambda = \theta'$)

$$A\cos(2\pi f_0 t + \theta') \cdot 2\cos(2\pi f_0 t)$$
$$= \frac{A}{2}\left[\mathrm{e}^{\mathrm{j}(2\pi f_0 t+\theta')} + \mathrm{e}^{-\mathrm{j}(2\pi f_0 t+\theta')}\right]\left[\mathrm{e}^{\mathrm{j}2\pi f_0 t} + \mathrm{e}^{-\mathrm{j}2\pi f_0 t}\right]$$
$$= \frac{A}{2}\left[\mathrm{e}^{\mathrm{j}(4\pi f_0 t+\theta')} + \mathrm{e}^{-\mathrm{j}(4\pi f_0 t+\theta')} + \mathrm{e}^{\mathrm{j}\theta'} + \mathrm{e}^{-\mathrm{j}\theta'}\right]$$

After low pass filtering: $A\cos(\theta') = A\cos(\theta - 4\pi R_0/\lambda)$

Not enough to determine **both** amplitude $A$ and phase $\theta'$!

# Coherent detector

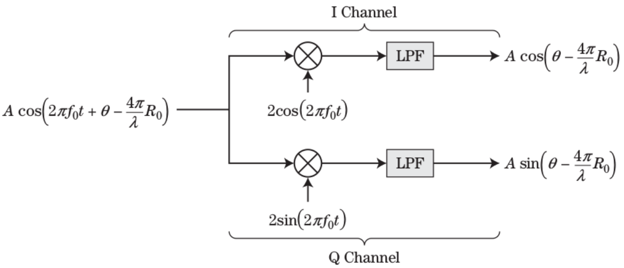



**FIGURE 8-12** ■ Coherent or I/Q detector.

- In-phase (I) channel, reference signal $\cos(2\pi f_0 t)$.
- Quadrature (Q) channel, reference signal $\cos(2\pi f_0 t + \pi/2) = -\sin(2\pi f_0 t)$.

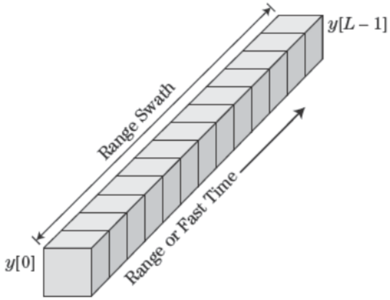
The I/Q channels can be combined to form the **analytic signal**

$$a = I + \mathrm{j}Q = A\cos\theta' + \mathrm{j}A\sin\theta' = A\mathrm{e}^{\mathrm{j}\theta'}$$

from which both amplitude $A$ and phase $\theta'$ can be determined.
Requires an accurate phase difference between I and Q references.

# Range bins, fast time

**FIGURE 8-13** ■ Range bins and range swath. Each cube represents a single complex voltage measurement.




- Sample the receiver output (down-converted frequency, almost constant during sample time).
- Each reflected pulse contributes to only one sample.
- Store the samples as a vector, with elements being called range bins, range gates, range cells, or fast-time samples.

# Slow time, datacube

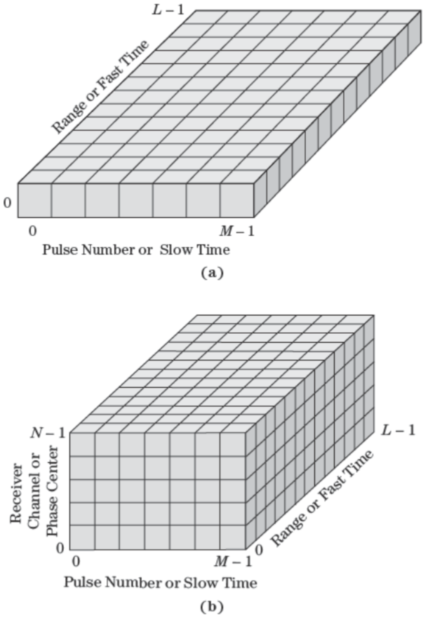



**FIGURE 8-14** ■ (a) Fast-time/slow-time CPI data matrix. (b) Datacube.

# Outline

# Measuring Doppler with multiple pulses

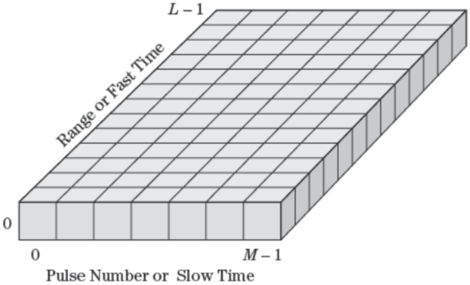



**FIGURE 8-14** ■ (a) Fast-time/slow-time CPI data matrix. (b) Datacube.

$$y[m] = A\exp\{\mathrm{j}[\theta - (4\pi/\lambda)(R_0 - vmT)]\}$$
$$= A\exp\left\{\mathrm{j}\left[2\pi\left(\frac{2v}{\lambda}\right)(mT) + \theta - \left(\frac{4\pi R_0}{\lambda}\right)\right]\right\}$$
$$= A\exp[\mathrm{j}(2\pi f_\mathrm{d} t_m + \theta')], \quad 0 \le m \le M-1$$

The Doppler shift $f_\mathrm{d}$ can be found from a frequency analysis of the received analytic signal!

# Coherent pulses

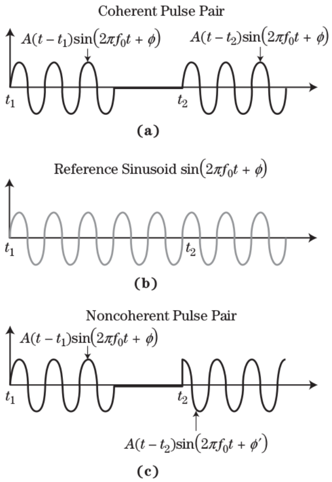



**FIGURE 8-15** (a) Coherent pulse pair. (b) Reference oscillator. (c) Noncoherent pulse pair.

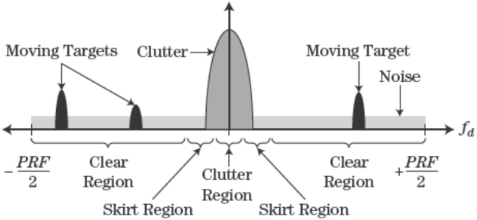
If the pulses are non-coherent, the phase in consecutive pulses is uncorrelated, and frequency analysis cannot be used.

# Outline

# Outline

# Doppler spectrum in one range bin

**FIGURE 8-16** ■ Notional Doppler spectrum for one range bin, viewed by a stationary radar.




The signal is sampled every PRI seconds, hence the Doppler spectrum (DFT in slow time) is contained in $[-\mathrm{PRF}/2, \mathrm{PRF}/2]$, where $\mathrm{PRF} = 1/\mathrm{PRI}$.

Clutter and stationary targets are centered at $f_\mathrm{d} = 0$, moving targets and noise appear throughout the spectrum.

# Range-Doppler spectrum

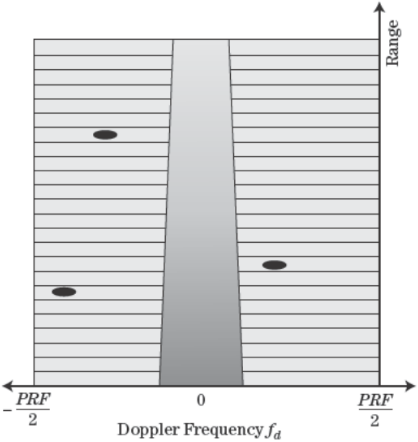



**FIGURE 8-17** ■ Notional range-Doppler distribution viewed by a stationary radar.

The range-Doppler spectrum is obtained by plotting a radar signal as function of both range (fast-time) and Doppler frequency (Fourier transformed slow-time).

# Range-Doppler spectrum, realistic data

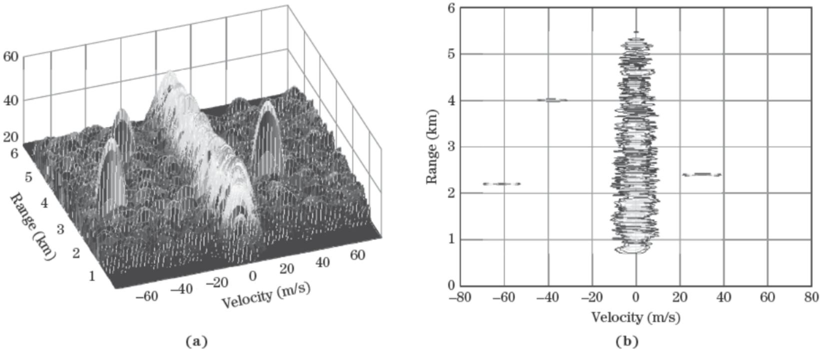
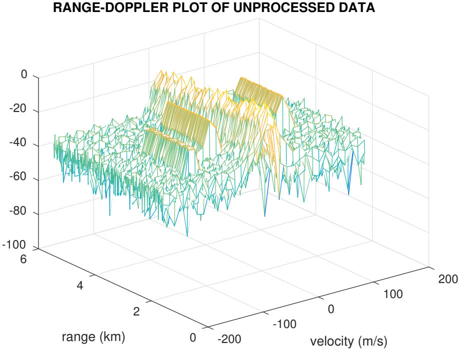


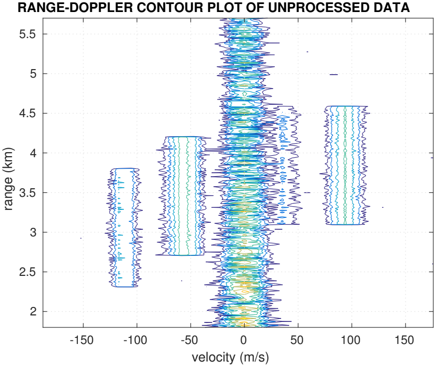

**FIGURE 8-18** ■ Simulated range-Doppler distribution for a stationary radar with clutter and three moving targets. (a) Three-dimensional display. (b) Contour plot.

Note the clutter ridge around $f_\mathrm{d} \approx 0$, and targets in different range bins.

# Simulated data, http://radarsp.com

FRSP Demos/FRSP Non-GUI demos/Pulse Doppler/

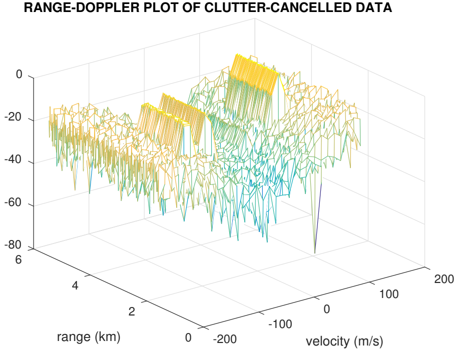


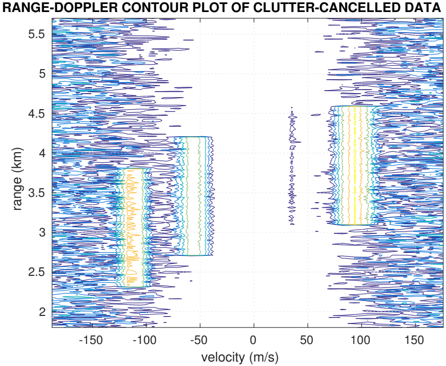




Target extent in range corresponds to uncompressed pulse length $10\,\mu\text{s} \cdot c/2 = 1.5\,\text{km}.$

# Simulated data, http://radarsp.com

Clutter cancellation through high-pass filtering.

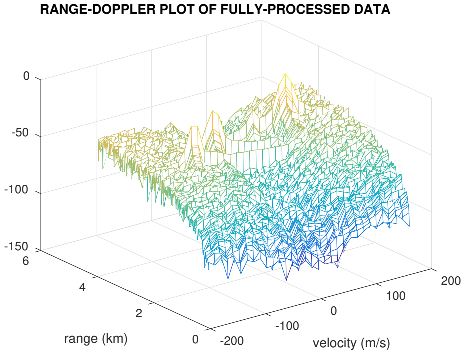


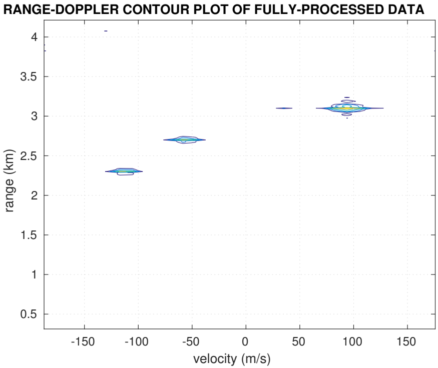




Note target on clutter ridge edge is almost deleted.

# Simulated data, http://radarsp.com

Pulse compression (a chirped pulse is used, better range resolution)



# Outline

# Main lobe clutter spreading

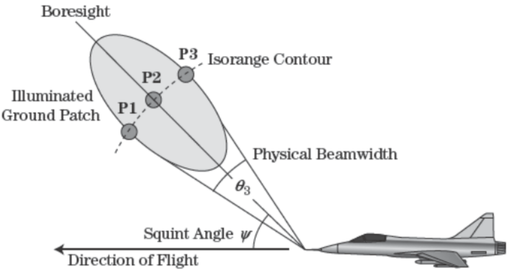



**FIGURE 8-19** ■ Geometry for computing Doppler spread induced by radar platform motion.

The Doppler shift depends on squint angle $\psi$, which implies a frequency broadening due to the beam width $\theta_3$:

$$f_{\mathrm{d}} = \frac{2v}{\lambda}\cos\psi \quad \Rightarrow \quad B_{\mathrm{MLC}} = \frac{4v}{\lambda}\sin\left(\frac{\theta_3}{2}\right)\sin\psi \approx \frac{2v\theta_3}{\lambda}\sin\psi$$

# Clutter spectrum elements

**FIGURE 8-20** ■ Geometry for computing Doppler spread induced by radar platform motion.

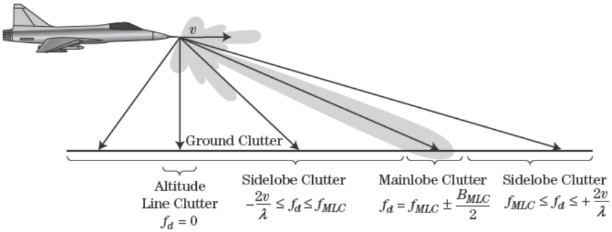
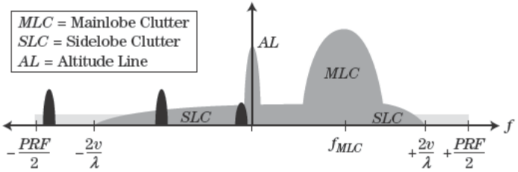


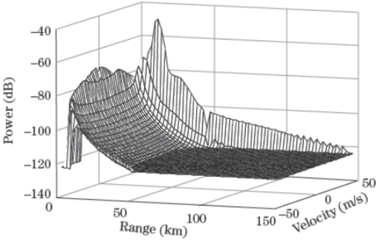

**FIGURE 8-21** ■ Notional Doppler spectrum for moving radar platform (see text for details).



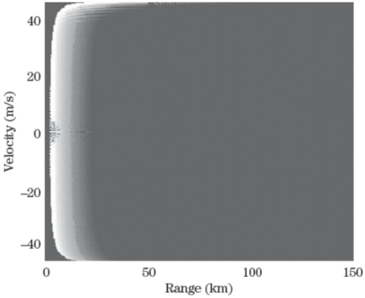

The clutter spectrum of a moving platform has many elements, induced by antenna sidelobes and different velocity components.

# Example of range-Doppler clutter distribution

**FIGURE 8-23** ■ Simulated range-velocity distribution for a stationary clutter observed from a moving radar (see text for details). (a) Three-dimensional display. (b) Plan view.




(a)

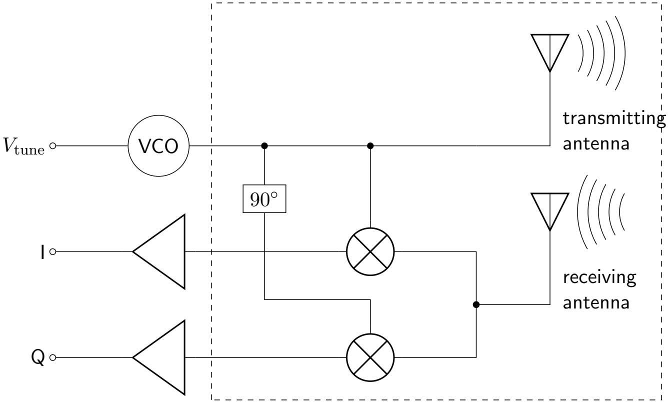



(b)

# Outline

# Frequency Modulated Continuous Wave (FMCW)

In a continuous wave system, the transmitter is always on. The frequency can be tuned by a control signal.




This makes for a cheap system, where the range information is obtained by frequency synthesis, rather than time-of-flight measurements.

Discussion

# Signal processing I

With a target at range $R$, the received signal is

$$r(t) = A\mathrm{e}^{\mathrm{j}2\pi f(t-2R/c)}$$

After downconversion (removing $\mathrm{e}^{\mathrm{j}2\pi ft}$) the analytic signal is

$$a = I + \mathrm{j}Q = A\mathrm{e}^{-\mathrm{j}4\pi fR/c}$$

Measure at discrete times $t_n$ with changing frequency $f_n$:

$$\begin{aligned} t_n &= n\Delta t & n &= 0, 1, \ldots, N-1 \\ f_n &= f_0 + nB/N & B &= \text{bandwidth} \\ R_n &= R_0 - nv\Delta t & v &= \text{radial velocity} \end{aligned}$$

The sampled signal is then

$$a_n = \underbrace{A\mathrm{e}^{-\mathrm{j}4\pi f_0 R_0/c}}_{=A'} \mathrm{e}^{\mathrm{j}n2\pi\left(-\frac{2BR_0}{Nc} + \frac{2vf_0\Delta t}{c}\right)} \mathrm{e}^{\mathrm{j}2\pi\frac{n^2 2Bv\Delta t}{Nc}}$$

The last term is a chirping effect, which can be neglected if the target's movement in one pulse $(Nv\Delta t)$ is much less than the spatial resolution $c/(2B)$: $\frac{n^2 2Bv\Delta t}{Nc} < \frac{Nv\Delta t}{c/(2B)} \ll 1$.

# Signal processing II

Using both up-chirped and down-chirped signals,

$$a_n = A' \mathrm{e}^{\mathrm{j}n2\pi\left(-\frac{2BR_0}{Nc}+\frac{2vf_0\varDelta t}{c}\right)} \quad \text{(up-chirp, } f_n = f_0 + nB/N)$$

$$a_n = A' \mathrm{e}^{\mathrm{j}n2\pi\left(\frac{2BR_0}{Nc}+\frac{2vf_0\varDelta t}{c}\right)} \quad \text{(down-chirp, } f_n = f_0 + (N-n)B/N)$$

we can use FFT to compute both $\frac{2BR_0}{Nc}$ and $\frac{2vf_0\varDelta t}{c}$.

**Conclusion:**

- Both range $R$ and velocity $v$ can be detected by FMCW.
- Necessary to use $I$ and $Q$ signals, sweeping the frequency over bandwidth $B$ in $N$ steps up and down. Dwell time is $2N\varDelta t$.
- The technique will be used in the lab on Friday.

# Outline

# Conclusions

- The Doppler shift is due to relative motion between radar and target.
- The Fourier transform can be used to compute frequency domain data.
- The spectrum of a pulsed radar has been characterized, having several frequency scales.
- The I/Q channels and analytic signal have been introduced.
- The range-Doppler spectrum provide information on both range and velocity.
- Range and velocity can be extracted from FMCW data using Fourier analysis.

# Discussion

The Fourier transform is defined by

$$X(f) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}2\pi f t}\,\mathrm{d}t$$
$$x(t) = \int_{-\infty}^{\infty} X(f)\mathrm{e}^{\mathrm{j}2\pi f t}\,\mathrm{d}f$$

If $x(t)$ has units of volts (V), what unit does $X(f)$ have?

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# Discussion

The Fourier transform is defined by

$$X(f) = \int_{-\infty}^{\infty} x(t)\mathrm{e}^{-\mathrm{j}2\pi ft}\,\mathrm{d}t$$
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If $x(t)$ has units of volts (V), what unit does $X(f)$ have?

Answer: $[X(f)] = \mathrm{Vs}$ ($x(t)$ is multiplied by $\mathrm{d}t$ with units of s).

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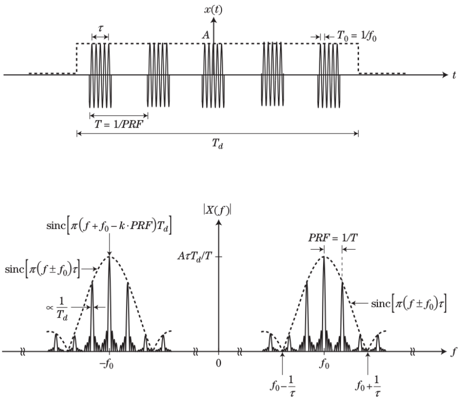
# Discussion

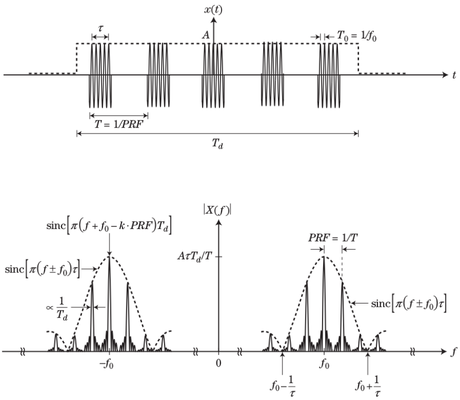
**FIGURE 8-7** ■ Finite duration modulated pulse train signal and its Fourier transform.




How does the spectrum look if 1) $T_{\mathrm{d}} = T = \tau$, 2) $T_{\mathrm{d}} > T = \tau$?

# Discussion

**FIGURE 8-7** ■
Finite duration modulated pulse train signal and its Fourier transform.




How does the spectrum look if 1) $T_\mathrm{d} = T = \tau$, 2) $T_\mathrm{d} > T = \tau$?

Answer: 1) Equal to the dashed sinc envelope. This corresponds to a single rectangular pulse of length $T_\mathrm{d} = T = \tau$ with carrier frequency $f_0$. 2) With $T_\mathrm{d} > T = \tau$, there is a single peak inside the dashed sinc envelope (one long rectangular pulse).

# Discussion

The complex amplitude of the received signal is

$$A_{\mathrm{c}} = A \exp\left[\mathrm{j}\left(\theta - \frac{4\pi}{\lambda} R_0\right)\right]$$

If we could measure $A_{\mathrm{c}}$, how can we extract information on the range $R_0$?

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# Discussion

The complex amplitude of the received signal is

$$A_c = A \exp\left[j\left(\theta - \frac{4\pi}{\lambda}R_0\right)\right]$$

If we could measure $A_c$, how can we extract information on the range $R_0$?

Answer: By varying the wavelength $\lambda$, we control the phase due to $R_0$. For instance, using two wavelengths $\lambda_1$ and $\lambda_2$ we get

$$\frac{A_{c1}}{A_{c2}} = \exp\left[-j4\pi R_0\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\right] \quad \Rightarrow \quad R_0 = \frac{\arg(A_{c1}/A_{c2})}{-j4\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)}$$

(under the requirement that $|4\pi R_0(\frac{1}{\lambda_1} - \frac{1}{\lambda_2})| < \pi$).

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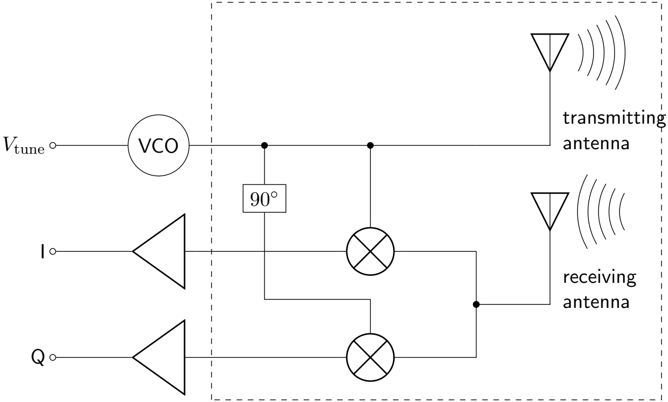
# Discussion

Would a continuous wave radar be best for long or short ranges?

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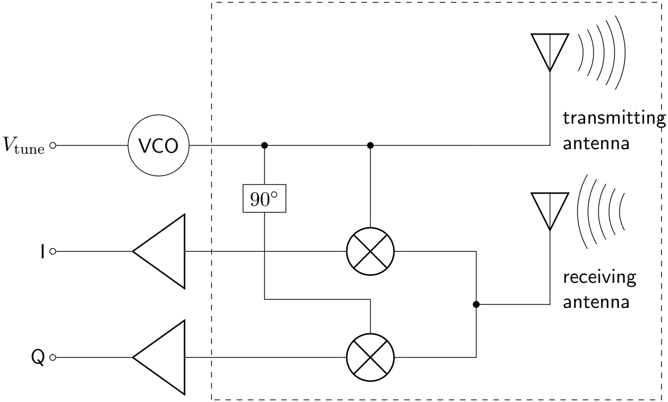
# Discussion

Would a continuous wave radar be best for long or short ranges?




Answer: High duty cycle implies low power and short range.

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