EITN90 Radar and Remote Sensing
Lecture 5: Target Reflectivity

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Outline

Learning outcomes of this lecture

In this lecture we will

- Study the properties of electromagnetic waves
- Define the radar cross section (RCS)
- Understand basic scattering and reflectivity physics
- Understand how two or more scattering centers interfere
- Illustrate some high-frequency scattering effects

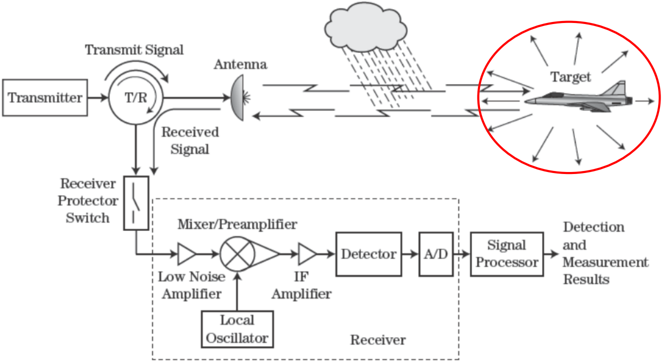


FIGURE 1-1 ■ Major elements of the radar transmission/reception process.

Outline

Electromagnetic waves

The time-harmonic Maxwell's equations in linear media are (where all fields have time dependence $\boldsymbol{E}(x,y,z,t) = \boldsymbol{E}(x,y,z)\mathrm{e}^{\mathrm{j}\omega t}$)

$$\boxed{\begin{cases} \nabla \times \boldsymbol{E} = -\mathrm{j}\omega \boldsymbol{B} = -\mathrm{j}\omega\mu \boldsymbol{H} \\ \nabla \times \boldsymbol{H} = \mathrm{j}\omega \boldsymbol{D} = \mathrm{j}\omega\epsilon \boldsymbol{E} \end{cases}}$$

where ϵ and μ are the permittivity and permeability of the material. A plane wave propagating in direction $\boldsymbol{k}$ is given by (where we use $\nabla \mathrm{e}^{-\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{R}} = -\mathrm{j}\boldsymbol{k}\mathrm{e}^{-\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{R}}$, or $\nabla \to -\mathrm{j}\boldsymbol{k}$)

$$\boxed{\begin{cases} \boldsymbol{E} = \boldsymbol{E}_0 \mathrm{e}^{\mathrm{j}(\omega t - \boldsymbol{k}\cdot\boldsymbol{R})} \\ \boldsymbol{H} = \boldsymbol{H}_0 \mathrm{e}^{\mathrm{j}(\omega t - \boldsymbol{k}\cdot\boldsymbol{R})} \end{cases} \quad \Rightarrow \quad \begin{cases} \boldsymbol{k} \times \boldsymbol{E}_0 = \omega\mu \boldsymbol{H}_0 \\ \boldsymbol{k} \times \boldsymbol{H}_0 = -\omega\epsilon \boldsymbol{E}_0 \end{cases}}$$

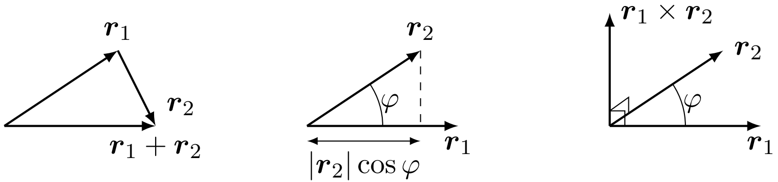
The cross products mean $(\boldsymbol{E}_0, \boldsymbol{H}_0, \boldsymbol{k})$ is a right-hand triple like $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}})$. In addition, the equations imply $\boxed{k = \omega\sqrt{\epsilon\mu}}$ and the ratio $\boxed{|\boldsymbol{E}_0|/|\boldsymbol{H}_0| = \sqrt{\mu/\epsilon} = \eta,}$ where η is the wave impedance.

Vector analysis, linear algebra

The vectors have three components, one for each spatial direction:

$$\boldsymbol{E} = E_x\hat{\boldsymbol{x}} + E_y\hat{\boldsymbol{y}} + E_z\hat{\boldsymbol{z}}$$

In particular, the position vector is $\boldsymbol{r} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$. Vector addition, scalar product and vector product are



- Addition: $\boldsymbol{r}_1 + \boldsymbol{r}_2 = (x_1 + x_2)\hat{\boldsymbol{x}} + (y_1 + y_2)\hat{\boldsymbol{y}} + (z_1 + z_2)\hat{\boldsymbol{z}}$.
- Scalar product: $\boldsymbol{r}_1 \cdot \boldsymbol{r}_2 = |\boldsymbol{r}_1|\,|\boldsymbol{r}_2|\cos\varphi = x_1x_2 + y_1y_2 + z_1z_2$.
- Vector product: orthogonal to both vectors, with length $|\boldsymbol{r}_1 \times \boldsymbol{r}_2| = |\boldsymbol{r}_1|\,|\boldsymbol{r}_2|\sin\varphi$, and $\boldsymbol{r}_1 \times \boldsymbol{r}_2 = -\boldsymbol{r}_2 \times \boldsymbol{r}_1$.

$$\hat{\boldsymbol{x}} \times \hat{\boldsymbol{y}} = \hat{\boldsymbol{z}}, \quad \hat{\boldsymbol{y}} \times \hat{\boldsymbol{z}} = \hat{\boldsymbol{x}}, \quad \hat{\boldsymbol{z}} \times \hat{\boldsymbol{x}} = \hat{\boldsymbol{y}}$$

Vector analysis, differentiation (not necessary to understand the book)

The nabla operator is

$$\boxed{\nabla = \hat{\boldsymbol{x}}\frac{\partial}{\partial x} + \hat{\boldsymbol{y}}\frac{\partial}{\partial y} + \hat{\boldsymbol{z}}\frac{\partial}{\partial z}}$$

The gradient, divergence, and curl operations are

$$\nabla g = \frac{\partial g}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial g}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial g}{\partial z}\hat{\boldsymbol{z}}$$

$$\nabla \cdot \boldsymbol{E} = \frac{\partial}{\partial x}E_x + \frac{\partial}{\partial y}E_y + \frac{\partial}{\partial z}E_z$$

$$\nabla \times \boldsymbol{E} = \frac{\partial}{\partial x}\hat{\boldsymbol{x}} \times \boldsymbol{E} + \frac{\partial}{\partial y}\hat{\boldsymbol{y}} \times \boldsymbol{E} + \frac{\partial}{\partial z}\hat{\boldsymbol{z}} \times \boldsymbol{E}$$

Cartesian representation (useful in numerics): $[\boldsymbol{E}] = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$ and

$$[\nabla \times \boldsymbol{E}] = \tfrac{\partial}{\partial x}\underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}}_{=[\hat{\boldsymbol{x}}\times\boldsymbol{E}]} + \tfrac{\partial}{\partial y}\underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}}_{=[\hat{\boldsymbol{y}}\times\boldsymbol{E}]} + \tfrac{\partial}{\partial z}\underbrace{\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}}_{=[\hat{\boldsymbol{z}}\times\boldsymbol{E}]}$$

Misprinted equations

Unfortunately, the vector cross product sign, $\times$, is sometimes not printed in the book. The correct versions of the affected equations are below:

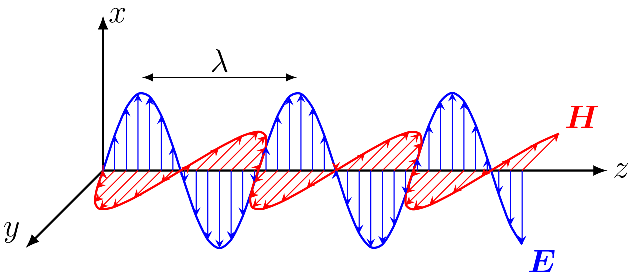
- (6.5) $\boldsymbol{P} = \frac{1}{2}\,\mathrm{Re}(\boldsymbol{E} \times \boldsymbol{H}^*) \quad \mathrm{W/m^2}$
- (6.6) $\boldsymbol{E}^{\mathrm{total}} \times \hat{\boldsymbol{n}} = (\boldsymbol{E}^{\mathrm{inc}} + \boldsymbol{E}^{\mathrm{scat}}) \times \hat{\boldsymbol{n}} = \boldsymbol{0}$
- Page 216, paragraph 2, line 3: $\hat{\boldsymbol{n}} \times \boldsymbol{E}^{\mathrm{scat}} = -\hat{\boldsymbol{n}} \times \boldsymbol{E}^{\mathrm{inc}}$

Comment: the vector product has amplitude

$$|\hat{\boldsymbol{n}} \times \boldsymbol{E}| = |\hat{\boldsymbol{n}}||\boldsymbol{E}| \sin\varphi = |\boldsymbol{E}| \sin\varphi$$

where φ is the angle between the unit vector $\hat{\boldsymbol{n}}$ and the vector $\boldsymbol{E}$. Hence, $\hat{\boldsymbol{n}} \times \boldsymbol{E}$ represents the part of $\boldsymbol{E}$ orthogonal to $\hat{\boldsymbol{n}}$ (tangential to the surface if $\hat{\boldsymbol{n}}$ is a surface normal).

Frequency and wavelength

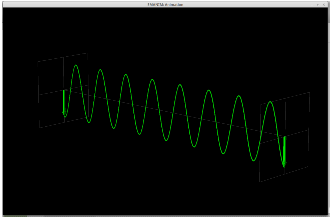


The electric field is $\boldsymbol{E}(x, y, z, t) = E_0 \cos(\omega t - kz)\hat{\boldsymbol{x}}$. The wavelength λ is the periodicity in z, determined by (using $\omega = 2\pi f$ and $\omega t - kz = \omega(t - kz/\omega) = \omega(t - z/v)$)

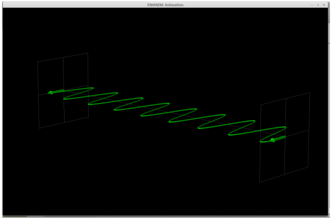
$$\boxed{\lambda f = v} \quad \Rightarrow \quad \boxed{k = \frac{\omega}{v} = \frac{2\pi}{\lambda}}$$

The polarization corresponds to the direction of the electric field. The wave depicted above is linearly polarized in the x-direction.

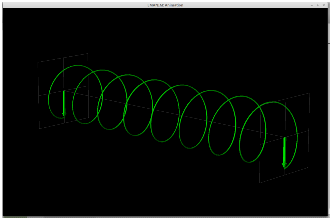
Polarization



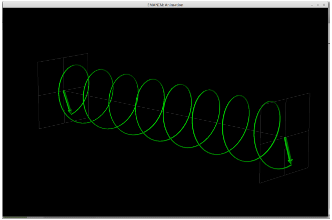
Vertical



Horizontal



Right hand



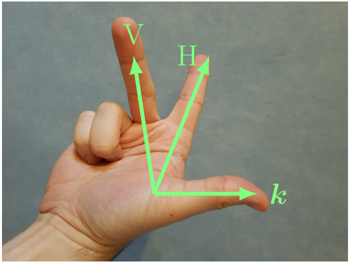
Left hand

See the animation program EMANIM.

Right-hand rules

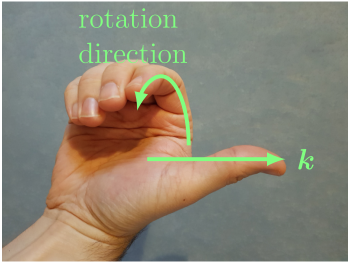
Let the propagation direction $\boldsymbol{k}$ be along the thumb. At any time, $\boldsymbol{E}$, $\boldsymbol{H}$, and $\boldsymbol{k}$ are orthogonal to each other.

Linear polarization



$\boldsymbol{E}$ oscillating along Horizontal or Vertical direction, $\boldsymbol{H}$ along the other.

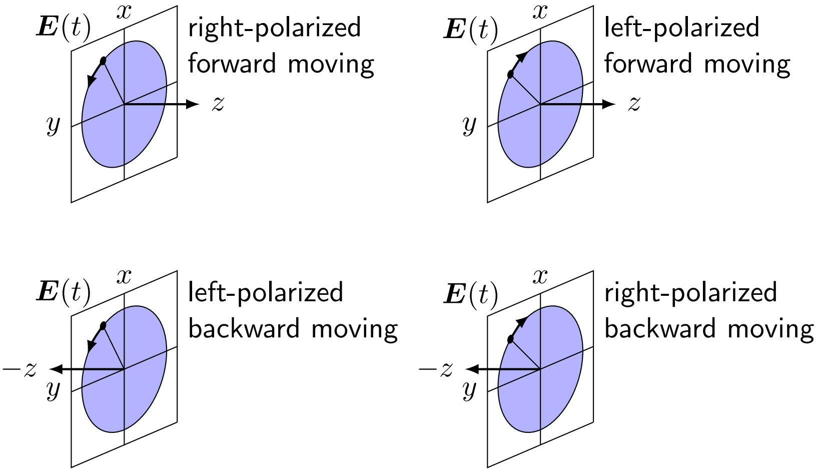
Circular polarization



$\boldsymbol{E}$ rotating along Right or Left hand fingers, $\boldsymbol{H}$ rotating the same but at right angle.

IEEE definition of left and right

With your right hand thumb in the propagation direction and fingers in rotation direction in a fixed plane: right hand circular.



Refractive index and wave impedance

From the expression of a plane wave

$$\boldsymbol{E} = \boldsymbol{E}_0 \mathrm{e}^{\mathrm{j}(\omega t - \boldsymbol{k}\cdot\boldsymbol{R})} = \boldsymbol{E}_0 \mathrm{e}^{\mathrm{j}\omega(t - \frac{k}{\omega}\frac{\boldsymbol{k}}{k}\cdot\boldsymbol{R})}$$

where $\boldsymbol{k}/k = \hat{\boldsymbol{k}}$ is the propagation direction, it is seen that only the projection of the position vector $\boldsymbol{R}$ in the propagation direction $\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{k}} \cdot \boldsymbol{R}$, matters, and the speed of propagation is

$$\boxed{v = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} = \frac{c}{n}, \quad \text{where} \quad n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}}$$

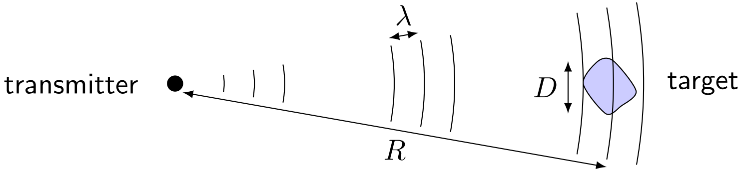
The refractive index n is the speed of an electromagnetic wave in a material (v), relative to the speed in vacuum (c). The wave impedance in the material is given by (denoted by η or Z)

$$\boxed{\frac{|\boldsymbol{E}|}{|\boldsymbol{H}|} = \sqrt{\frac{\mu}{\epsilon}} = \eta = Z}$$

Discussion

Plane waves vs spherical waves

The plane wave is a beautiful theoretical tool, but it typically only applies locally around the target or transmitter/receiver.



The wavefront is spherical close to the transmitter, but approximately plane at the target if the range R satisfies

$$\boxed{R \geq \frac{2D^2}{\lambda}}$$

where D is the diameter of the target and λ the wavelength.

Induced currents

Assume the target is made of metal. The electrons move around so as to cancel the field inside the metal, quantified through the boundary condition (zero tangential electric field)

$$\boldsymbol{E}^{\mathrm{tot}} \times \hat{\boldsymbol{n}} = (\boldsymbol{E}^{\mathrm{inc}} + \boldsymbol{E}^{\mathrm{scat}}) \times \hat{\boldsymbol{n}} = \boldsymbol{0}$$

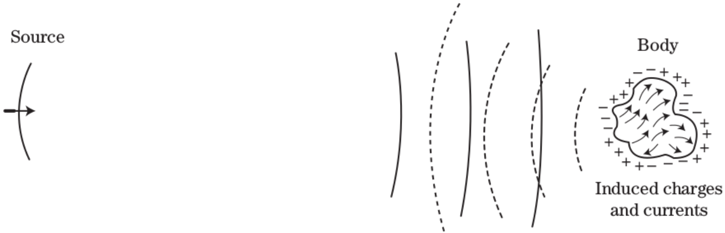


FIGURE 6-3 ■ Charges and currents: are induced on a PEC to satisfy the perfect conductor boundary conditions of zero tangential field (short circuit); and, consequently, re-radiate a scattered field $\boldsymbol{E}^{scat}$. From Knott [3].

A more general boundary condition is $\boldsymbol{E}^{\mathrm{tot}}_{\mathrm{tan}} = Z_{\mathrm{S}}\hat{\boldsymbol{n}} \times \boldsymbol{H}^{\mathrm{tot}}$, where Z_{S} is the surface impedance ($Z_{\mathrm{S}} = 0$ for perfect conductors). This does not change the general results in this lecture.

Radiation

Since the incident field is oscillating like $\mathrm{e}^{\mathrm{j}\omega t}$, the induced surface currents $\boldsymbol{J}_{\mathrm{S}}$ and surface charges ρ_{S} on the target will oscillate with the same frequency and radiate a scattered field $\boldsymbol{E}^{\mathrm{scat}}$.

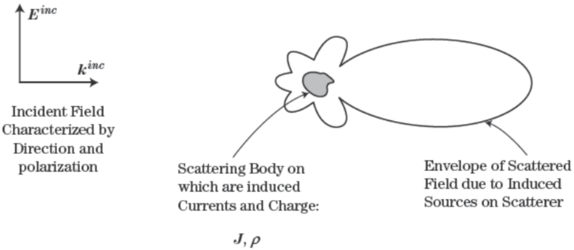


FIGURE 6-4 ■ PEC induced currents and charges reradiate a scattered field [1].

$$\boldsymbol{E}^{\mathrm{scat}}(\boldsymbol{R}_{\mathrm{f}}) = \int \left(-\mathrm{j}\omega\mu \boldsymbol{J}(\boldsymbol{R}_{\mathrm{s}}) g(\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{R}_{\mathrm{s}}) + \frac{\rho_{\mathrm{S}}(\boldsymbol{R}_{\mathrm{s}})}{\epsilon} \nabla g(\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{R}_{\mathrm{s}}) \right) \mathrm{d}S(\boldsymbol{R}_{\mathrm{s}})$$

The function $g(\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{R}_{\mathrm{s}}) = \frac{\mathrm{e}^{-\mathrm{j}k|\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{R}_{\mathrm{s}}|}}{4\pi|\boldsymbol{R}_{\mathrm{f}} - \boldsymbol{R}_{\mathrm{s}}|}$ is called the *Green's function*, corresponding to radiation at field point $\boldsymbol{R}_{\mathrm{f}}$ from a unit point source at $\boldsymbol{R}_{\mathrm{s}}$.

Scattering theory definitions

At large range from the scatterer, the scattered field can be written

$$\boldsymbol{E}^{\mathrm{scat}} = \frac{\mathrm{e}^{-\mathrm{j}kR}}{R}\boldsymbol{F}(\hat{\boldsymbol{k}}^{\mathrm{scat}})$$

where $\boldsymbol{F}(\hat{\boldsymbol{k}}^{\mathrm{scat}})$ is the far field amplitude in scattering direction $\hat{\boldsymbol{k}}^{\mathrm{scat}}$. Given knowledge of scattered electric and magnetic fields $\boldsymbol{E}^{\mathrm{s}}$ and $\boldsymbol{H}^{\mathrm{s}}$ on a surface S enclosing the scatterer, this is

$$\boldsymbol{F}(\hat{\boldsymbol{k}}) = \frac{\mathrm{j}k}{4\pi}\hat{\boldsymbol{k}} \times \int_S \left[\hat{\boldsymbol{k}} \times (\hat{\boldsymbol{n}} \times \eta_0 \boldsymbol{H}^{\mathrm{s}}) + \boldsymbol{E}^{\mathrm{s}} \times \hat{\boldsymbol{n}}\right] \mathrm{e}^{\mathrm{j}k\hat{\boldsymbol{k}}\cdot\boldsymbol{r}}\,\mathrm{d}S$$

The bistatic scattering cross section is

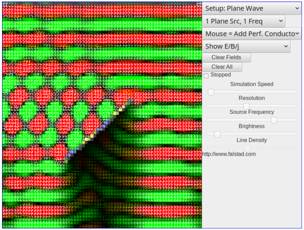
$$\sigma_*(\hat{\boldsymbol{k}}^{\mathrm{inc}}, \hat{\boldsymbol{k}}^{\mathrm{scat}}) = \lim_{R\to\infty} 4\pi R^2 \frac{\left|\boldsymbol{E}^{\mathrm{scat}}(\hat{\boldsymbol{k}}^{\mathrm{scat}})\right|^2}{\left|\boldsymbol{E}^{\mathrm{inc}}(\hat{\boldsymbol{k}}^{\mathrm{inc}})\right|^2} = 4\pi \frac{\left|\boldsymbol{F}(\hat{\boldsymbol{k}}^{\mathrm{scat}})\right|^2}{\left|\boldsymbol{E}^{\mathrm{inc}}\right|^2}$$

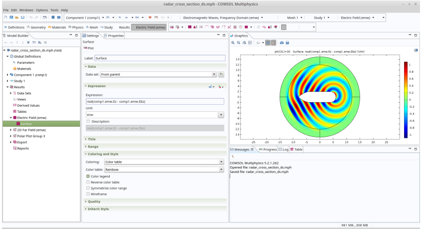
and the monostatic radar cross section at incident direction $\hat{\boldsymbol{k}}$ is

$$\sigma(\hat{\boldsymbol{k}}) = \sigma_*(\hat{\boldsymbol{k}}, -\hat{\boldsymbol{k}})$$

Simulations

The scattering theory can be implemented numerically. This provides excellent tools to compute and visualize the scattered field.





http://falstad.com/emwave2/ http://www.comsol.com

Outline

Radar cross section

In this chapter, power density is denoted by P instead of Q as in Chapter 1.

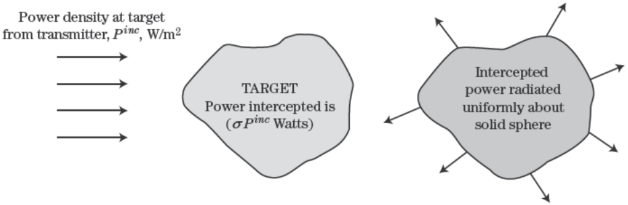


FIGURE 6-9 Intuitive derivation of radar cross section [1].

The IEEE definition of RCS is

$$\boxed{\sigma \stackrel{\text{def}}{=} \lim_{R\to\infty} 4\pi R^2 \frac{|\boldsymbol{E}^{\text{scat}}|^2}{|\boldsymbol{E}^{\text{inc}}|^2}}$$

which is motivated by intercepted power σP^{inc} and isotropically radiated scattered power density $P^{\text{scat}} = (\sigma P^{\text{inc}})/(4\pi R^2)$.

Factors affecting the RCS

The bistatic RCS of a target depends on the following factors:

- Target geometry and material composition.
- Direction of transmitter relative to target.
- Direction of receiver relative to target.
- Frequency or wavelength.
- Transmitter polarization.
- Receiver polarization.

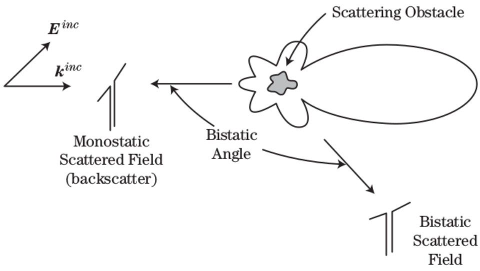


FIGURE 6-10 Monostatic or bistatic target cross section cases.

In a monostatic setting, transmitter and receiver are co-located.

Discussion

Polarization scattering matrix

It is sometimes necessary to keep track of polarization. In general, the polarization scattering matrix (PSM) $\mathbf{S}$ is defined by

$$\boldsymbol{E}^{\text{scat}}(\hat{\boldsymbol{k}}^{\text{scat}}) = \mathbf{S}(\hat{\boldsymbol{k}}^{\text{scat}}, \hat{\boldsymbol{k}}^{\text{inc}}) \cdot \boldsymbol{E}^{\text{inc}}(\hat{\boldsymbol{k}}^{\text{inc}})$$

where the arguments $\hat{\boldsymbol{k}}^{\text{scat}}$ and $\hat{\boldsymbol{k}}^{\text{inc}}$ are often left out to write $\boldsymbol{E}^{\text{scat}} = \mathbf{S} \cdot \boldsymbol{E}^{\text{inc}}$. The PSM can be represented in linear polarization

$$\begin{pmatrix} E_{\text{V}}^{\text{scat}} \\ E_{\text{H}}^{\text{scat}} \end{pmatrix} = \begin{pmatrix} S_{\text{VV}} & S_{\text{VH}} \\ S_{\text{HV}} & S_{\text{HH}} \end{pmatrix} \begin{pmatrix} E_{\text{V}}^{\text{inc}} \\ E_{\text{H}}^{\text{inc}} \end{pmatrix}$$

or circular polarization

$$\begin{pmatrix} E_{\text{R}}^{\text{scat}} \\ E_{\text{L}}^{\text{scat}} \end{pmatrix} = \begin{pmatrix} S_{\text{RR}} & S_{\text{RL}} \\ S_{\text{LH}} & S_{\text{LL}} \end{pmatrix} \begin{pmatrix} E_{\text{R}}^{\text{inc}} \\ E_{\text{L}}^{\text{inc}} \end{pmatrix}$$

Converting between polarizations

The relation between LP and CP in transmission is

$$\begin{pmatrix} E_{\mathrm{R}}^{\mathrm{t}} \\ E_{\mathrm{L}}^{\mathrm{t}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mathrm{j} \\ 1 & -\mathrm{j} \end{pmatrix} \begin{pmatrix} E_{\mathrm{H}}^{\mathrm{t}} \\ E_{\mathrm{V}}^{\mathrm{t}} \end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix} E_{\mathrm{H}}^{\mathrm{t}} \\ E_{\mathrm{V}}^{\mathrm{t}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -\mathrm{j} & \mathrm{j} \end{pmatrix} \begin{pmatrix} E_{\mathrm{R}}^{\mathrm{t}} \\ E_{\mathrm{L}}^{\mathrm{t}} \end{pmatrix}$$

and in reflection we have (due to different propagation direction)

$$\begin{pmatrix} E_{\mathrm{R}}^{\mathrm{r}} \\ E_{\mathrm{L}}^{\mathrm{r}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{j} \\ 1 & \mathrm{j} \end{pmatrix} \begin{pmatrix} E_{\mathrm{H}}^{\mathrm{r}} \\ E_{\mathrm{V}}^{\mathrm{r}} \end{pmatrix} \quad \Leftrightarrow \quad \begin{pmatrix} E_{\mathrm{H}}^{\mathrm{r}} \\ E_{\mathrm{V}}^{\mathrm{r}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ \mathrm{j} & -\mathrm{j} \end{pmatrix} \begin{pmatrix} E_{\mathrm{R}}^{\mathrm{r}} \\ E_{\mathrm{L}}^{\mathrm{r}} \end{pmatrix}$$

Hence, the polarization scattering matrix in LP and CP are related by

$$\begin{pmatrix} S_{\mathrm{RR}} & S_{\mathrm{RL}} \\ S_{\mathrm{LR}} & S_{\mathrm{LL}} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -\mathrm{j} \\ 1 & \mathrm{j} \end{pmatrix} \begin{pmatrix} S_{\mathrm{HH}} & S_{\mathrm{HV}} \\ S_{\mathrm{VH}} & S_{\mathrm{VV}} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -\mathrm{j} & \mathrm{j} \end{pmatrix}$$

Note that the sense of rotation of circular polarization changes with each reflection.

Outline

Sphere scattering

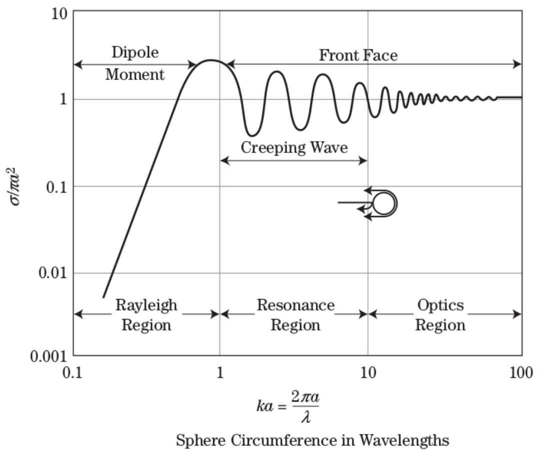


FIGURE 6-12 Sphere scattering from Rayleigh, resonance, and optics regions, [1].

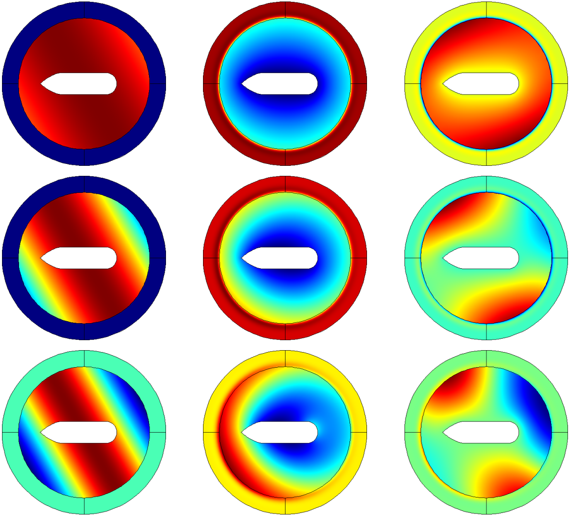
In the following slides, scattering from an oblong object is shown. Note the outmost spherical shell corresponds to a material layer absorbing outgoing radiation, and does not correspond to a physical region.

Low frequency: Rayleigh scattering

$f = f_0/8$

$f = f_0/4$

$f = f_0/2$



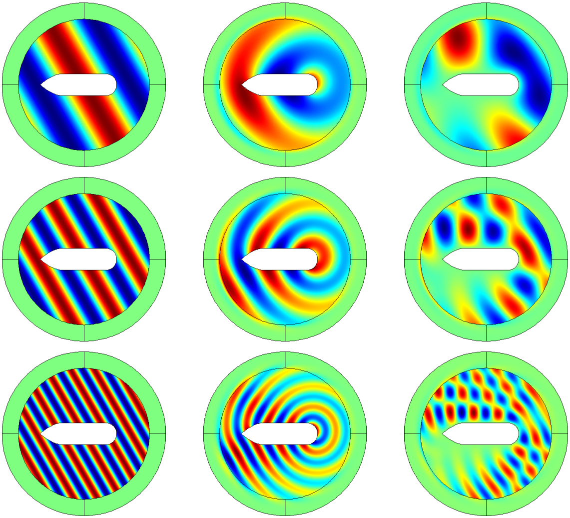
Incident field   Scattered field   Total field

Intermediate frequency: resonant scattering

$f = f_0$

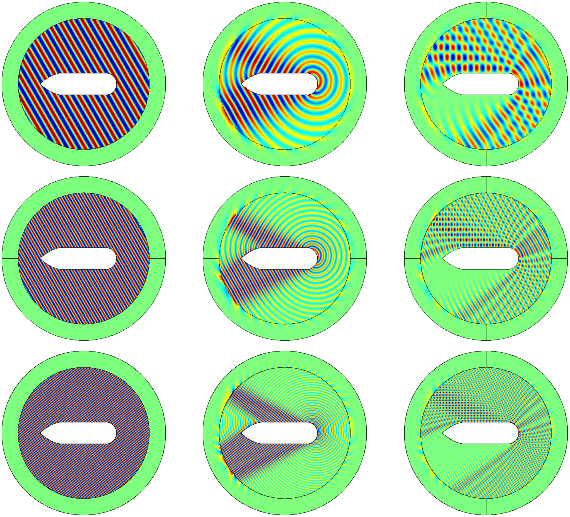
$f = 2f_0$

$f = 4f_0$



Incident field Scattered field Total field

High frequency: optical scattering



$f = 8f_0$

$f = 16f_0$

$f = 32f_0$

Incident field   Scattered field   Total field

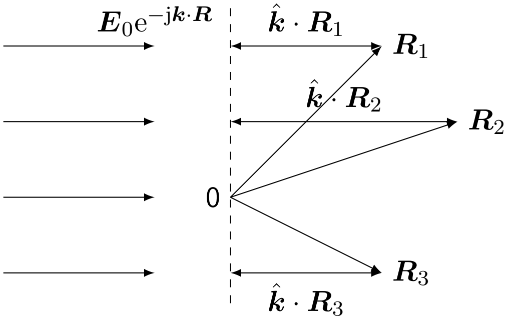
Scattering mechanisms

TABLE 6-1 ■ Scattering Mechanism and Relevant Scattering Regime

Scattering Mechanisms	Scattering Regime	Comments
Dipole	Dipole	Small scattering varies as the fourth power of frequency and the sixth power of size.
Surface waves	Resonance	Traveling, edge, and creeping waves; grazing angle phenomena; depends on polarization
Specular	Optics, resonance	Angle of reflection = angle of incidence for planar, single-, and double-curved surfaces
Multiple bounce	Optics, resonance	Few bounces (e.g., corner); many bounces (e.g., cavities)
End region	Optics, resonance	Sidelobes of a plate or cylinder from the ends of the surface
Edge diffraction	Optics, resonance	Diffraction in the specular direction; depends on polarization
Discontinuities, gaps, cracks	Optics	Surface imperfections important at higher frequencies

Outline

Superposition from several scatterers



When several scatterers are subjected to an incident wave $\boldsymbol{E}_0 \mathrm{e}^{-\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{R}}$, the backscattering is (complex addition)

$$\sigma_{\mathrm{tot}} = \left| \sum_{i=1}^{N} \sqrt{\sigma_i}\, \mathrm{e}^{-\mathrm{j}2\boldsymbol{k}\cdot\boldsymbol{R}_i} \right|^2$$

Phasor addition

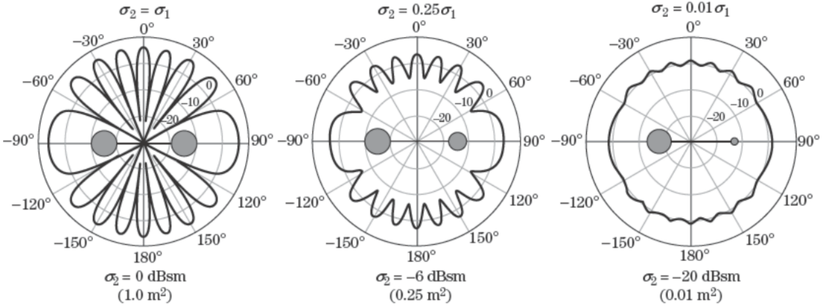


FIGURE 6-14 ■ Phase sum of two point scattering centers of different magnitudes. Scale: dBsm, 10 dB/div.

Two scatterers spaced by 2λ. Strong interference when scatterers have equal amplitude, dominated by the strong scatterer when they are very different.

Specular scattering

When the surface normal $\hat{\boldsymbol{n}}$ of a relatively flat surface points toward the radar, there is little variation of $\hat{\boldsymbol{k}} \cdot \boldsymbol{R}$ over the surface. Hence, the phase does not change much, and we have coherent addition:

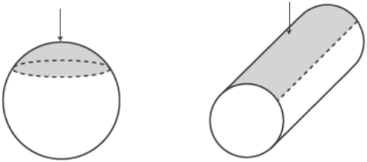


FIGURE 6-17 ■ Specular point constant phase current region for backscatter when viewed from the top.

$$\sigma_{\text{specular}} = 4\pi \frac{A_{\text{cp}}^2}{\lambda^2}, \quad A_{\text{cp}} = \text{area of constant phase}$$

$$L_{\text{cp}} \sim \sqrt{\frac{R_{\text{c}}\lambda}{2}} \quad \text{length of constant phase, } R_{\text{c}} = \text{radius of curvature}$$

$$\sigma = \pi R_{\text{c}1} R_{\text{c}2} \quad \text{double curved surfaces}$$

$$\sigma_{\text{cyl}} = \frac{2\pi}{\lambda} R_{\text{c}} L^2 \quad \text{cylindrical surface}$$

Discussion

End-region scattering

Scattering from a metal plate shows significant side-lobes. At off-specular directions, only the edges are scattering in the back-direction.

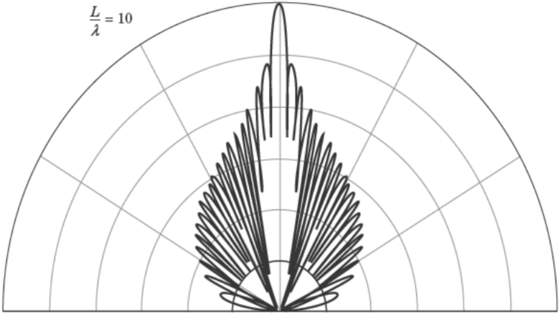
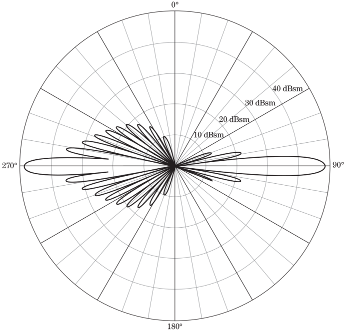


FIGURE 6-18 ■ Backscatter from a flat plate when viewed perpendicular to an edge. The side lobes are due to the truncated end region currents phase adding/subtracting. The first side lobe is 13 dB down from specular. Scale is 10 dB/div.

The effect is due to truncation of currents on the flat plate.

Metal plate at different orientations

FIGURE 6-19 ■ Plate backscatter showing end region side lobe scattering: Left) Viewed perpendicular to edges; Right) Viewed along diagonal.



Scattering from edges is stronger than from corners.

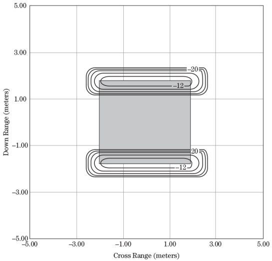
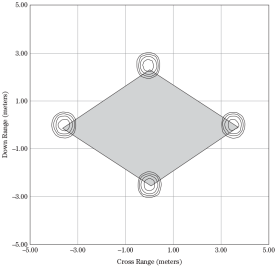


FIGURE 6-20 ■ Flat-plate physical optics end-region return for oblique 45 degree backscatter looking perpendicular to edge of plate. These end regions produce the side lobes as they phase add or subtract. Image analytically computed using physical optics currents; that is, edge diffraction is not included, which at this angle is not yet significant. Plate size is 5λ.

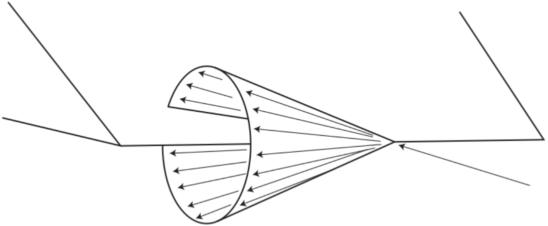
FIGURE 6-21 ■ Flat-plate physical optics end-region return for oblique 45 degree incidence looking along diagonal of plate. These much smaller end regions produce lower side lobe envelopes as they phase add/subtract compared to looking perpendicular to an edge. Image analytically computed using physical optics current; that is, edge diffraction is not included, which at this angle is not yet significant. Plate size is 5λ.



Edge diffraction

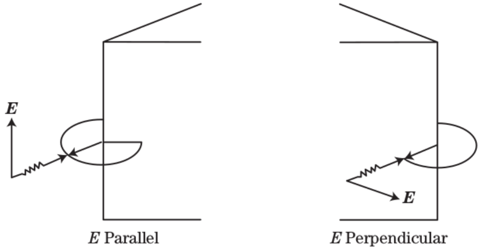
When a wave is incident on an edge, a line source current is induced. At oblique incidence it radiates in a cone.

FIGURE 6-22 ■ Keller cone of edge specular reflected rays. Cone is due to symmetry of wire like local edge currents. Cone is the specular direction(s) of the incident ray, (from Knott [1]).



Monostatic return only at normal incidence, $\sigma \approx L^2/\pi$.

FIGURE 6-23 ■ Edge diffraction depends on polarization: $\boldsymbol{E}$ parallel or perpendicular to edge.



Multiple bounces

When two specular reflections combine at 90° angle, strong backscattering occurs, so called corner reflectors.

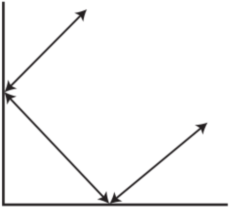


FIGURE 6-24 ■ Multiple bounce is two or more specular scatters which reflect back to a radar.

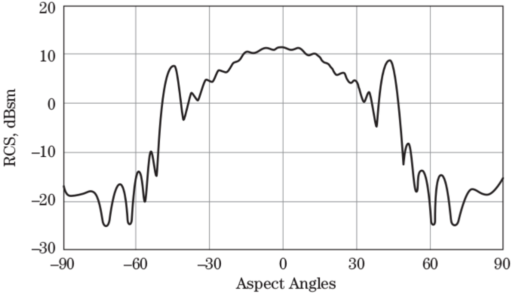
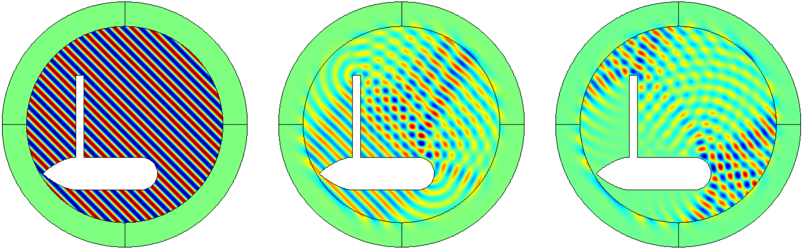


FIGURE 6-25 ■ Multiple bounce dihedral backscatter showing a large central region of scattering [Knott, 1].

Corner reflections



Incident field Scattered field Total field

(Corner reflector, cat eye)

Multiple bounces

The specular reflections do not need to be at flat surfaces. Many different combinations can occur.

FIGURE 6-26 Multiple bounces can exist for a variety of geometric arrangements.

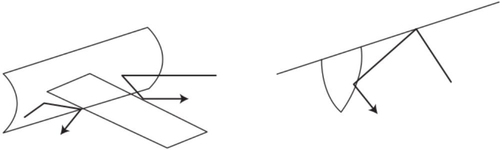
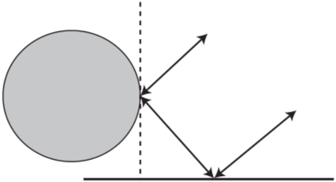


FIGURE 6-27 Doubly curved surfaces often form corner reflectors with other surfaces.



When designing stealthy objects, it is important to find shapes with as little corner reflections as possible.

Outline

Metal plate, different methods

Diffraction important at low levels of scattering.

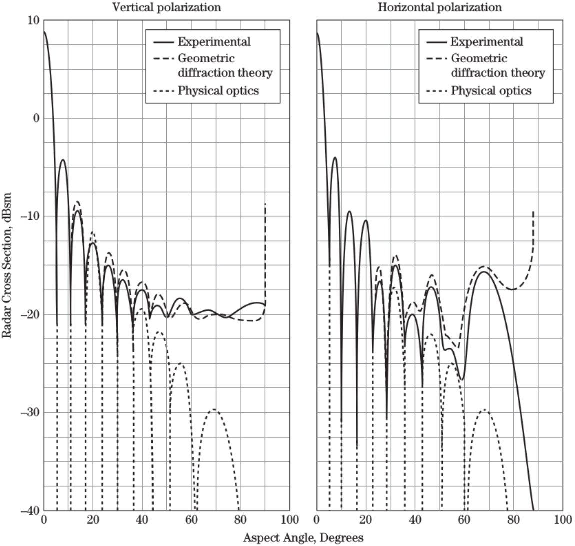
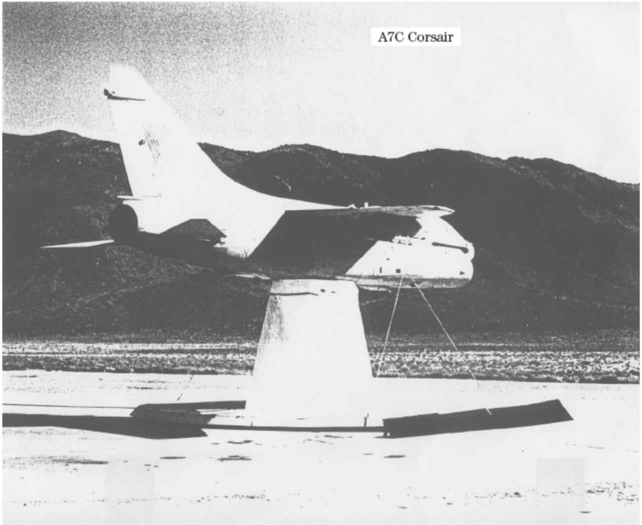


FIGURE 6-28 ■ RCS patterns of a 6.5 in. square plate at a wavelength of 1.28 inches viewed perpendicular to its edges [from 7].

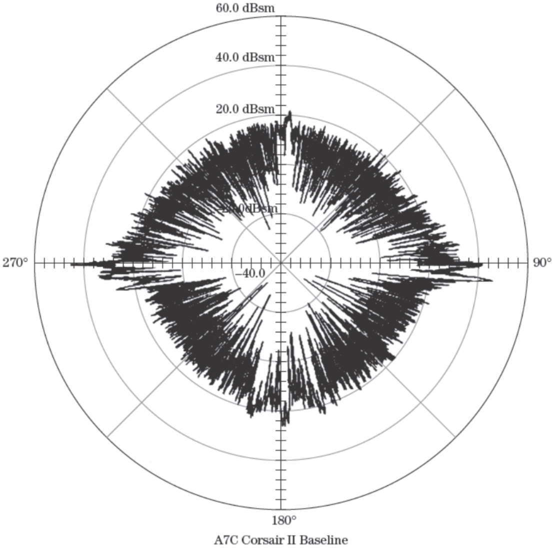
A7 aircraft

FIGURE 6-29 ■
A7C Corsair RCS measurement set up at the Navy Junction Ranch Range.



A7 aircraft

FIGURE 6-30 ■ A7 C measured backscatter for horizontal polarization at 9.5 GHz, 20 dB/div.



Discussion

A7 aircraft

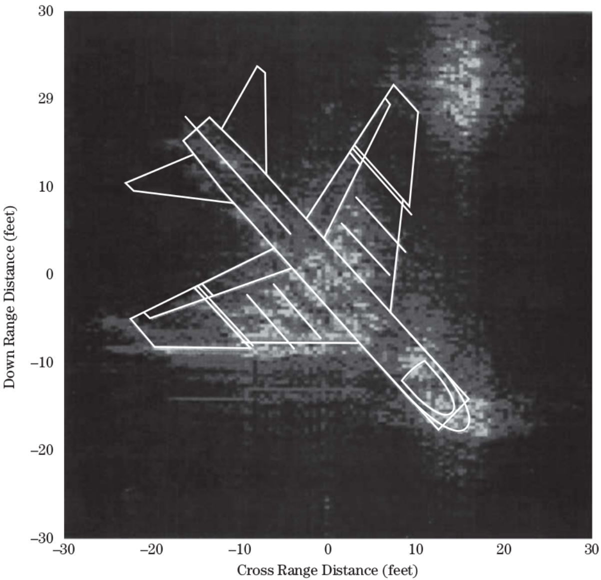
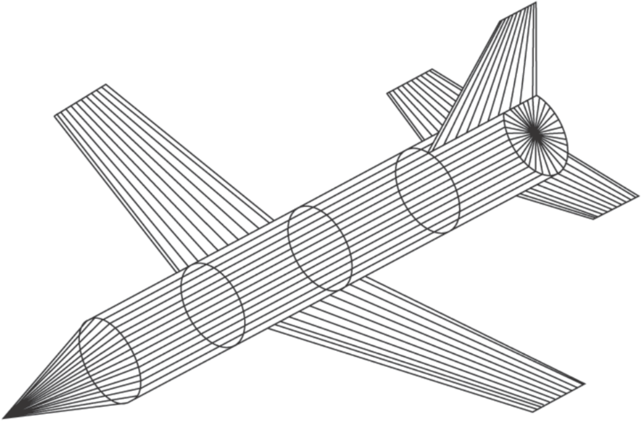


FIGURE 6-31 ■ A7 down/cross range image measurements when perpendicular to wing leading edge for horizontal polarization at X-band [Navy Junction Range Range].

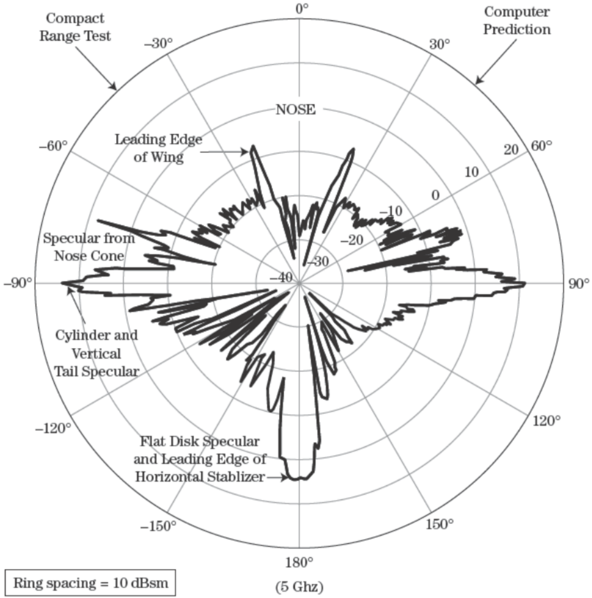
Stovepipe aircraft

FIGURE 6-32 ■ "Stovepipe" RCS backscatter model.



Stovepipe aircraft

FIGURE 6-33 ■ Measured and Physical Optics predicted RCS for "Stovepipe" geometry at C band.



Stovepipe aircraft

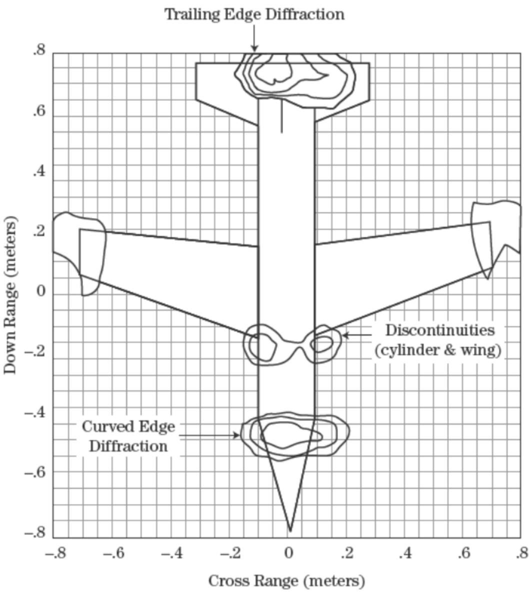
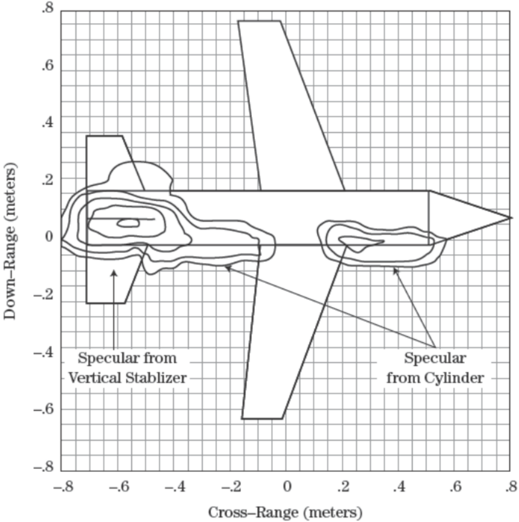


FIGURE 6-34 ■ "Stovepipe" nose view scattering centers.

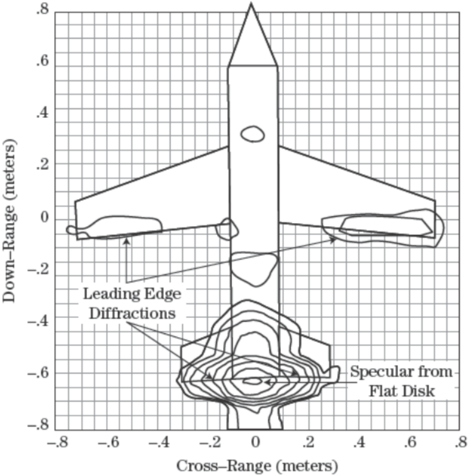
Stovepipe aircraft

FIGURE 6-35 ■ "Stovepipe" broadside view scattering centers.



Stovepipe aircraft

FIGURE 6-36 ■ "Stovepipe" tail view scattering centers.



Stovepipe aircraft

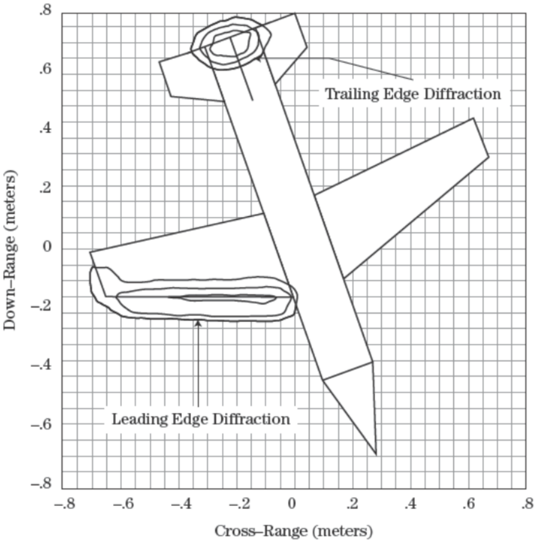


FIGURE 6-37 ■ "Stovepipe" wing leading edge view scattering centers.

Stovepipe aircraft

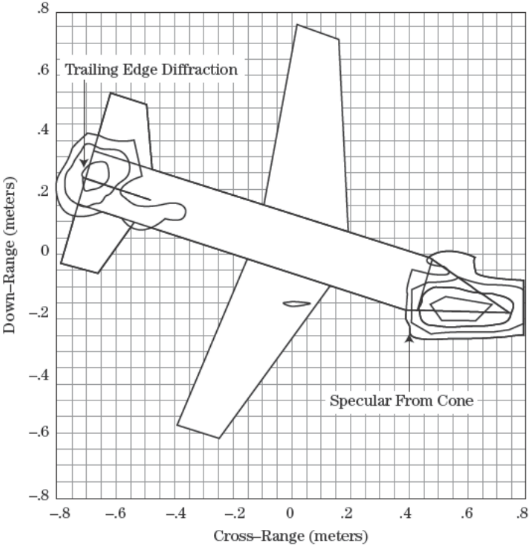


FIGURE 6-38 ■ "Stovepipe" scattering centers when viewed normal to front nose cone.

Outline

Conclusions

- We have reviewed basic scattering theory and how it relates to RCS.
- Three different scattering regimes: Rayleigh, resonance, optical.
- Interaction between multiple targets.
- Scattering mechanisms: dipole, surface waves, specular, multiple bounces, end regions, edge diffraction, discontinuities.

Discussion

Pair up the different materials with the corresponding relative permittivity!

1. Vacuum
2. Teflon
3. Rubber
4. Barium titanate

A. 2.1
B. 5000
C. 7
D. 1

Go back

Discussion

Pair up the different materials with the corresponding relative permittivity!

1. Vacuum	A. 2.1
2. Teflon	B. 5000
3. Rubber	C. 7
4. Barium titanate	D. 1

Answer:

1. Vacuum	D. 1
2. Teflon	A. 2.1
3. Rubber	C. 7
4. Barium titanate	B. 5000

A rule of thumb is that permittivity is higher the denser the material. $BaTiO_3$ is a ferro-electric material, which has exceptionally high permittivity and is used in capacitors.

Go back

Discussion (tough one!)

The scattering in the forward direction is significantly larger than in other directions. Why?

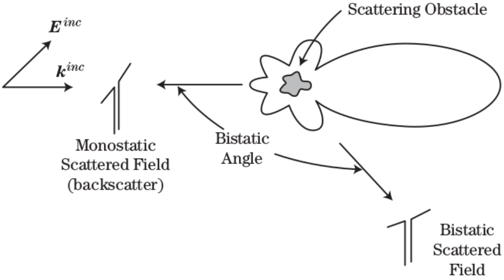


FIGURE 6-10 Monostatic or bistatic target cross section cases.

Go back

Discussion (tough one!)

The scattering in the forward direction is significantly larger than in other directions. Why?

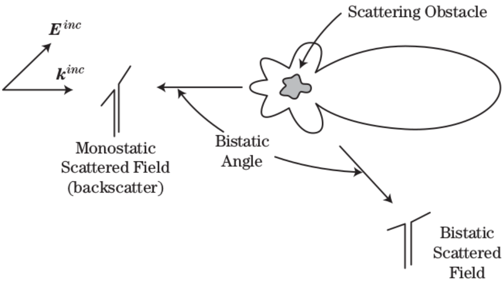


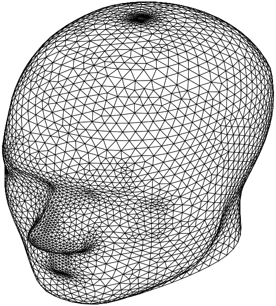
FIGURE 6-10 Monostatic or bistatic target cross section cases.

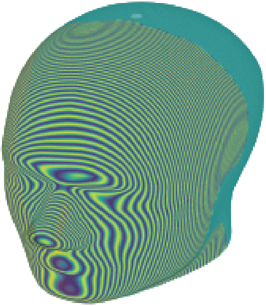
Answer: For a large object, we expect a shadow. The scattering in the forward direction needs to cancel the incident field in order to create a shadow, $\boldsymbol{E}^{\text{tot}} = \boldsymbol{E}^{\text{inc}} + \boldsymbol{E}^{\text{scat}} \approx \boldsymbol{0}$. Since $\boldsymbol{E}^{\text{inc}} = \boldsymbol{E}_0 \mathrm{e}^{-\mathrm{j}\boldsymbol{k}\cdot\boldsymbol{R}}$ does not decrease with distance but $\boldsymbol{E}^{\text{scat}}$ does, it needs to be sharply focused to maintain the shadow at large distance.

Go back

Discussion

The face below is illuminated from the front and below by a $60\,\mathrm{GHz}$ radar ($\lambda = 5\,\mathrm{mm}$). Numerical mesh on the left, phase distribution on the right. Identify some of the strongest specular reflections!

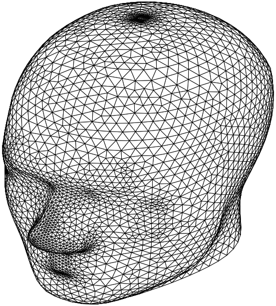


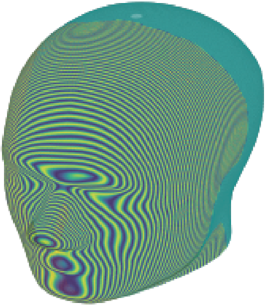


◂ Go back

Discussion

The face below is illuminated from the front and below by a $60\,\mathrm{GHz}$ radar ($\lambda = 5\,\mathrm{mm}$). Numerical mesh on the left, phase distribution on the right. Identify some of the strongest specular reflections!



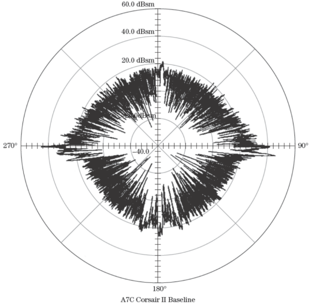


Answer: Between the eye brows, on the nose tip, on the upper lip, and on the chin.

◂ Go back

Why is the RCS so sensitive to angle?

FIGURE 6-30 ■ A7 C measured backscatter for horizontal polarization at 9.5 GHz, 20 dB/div.

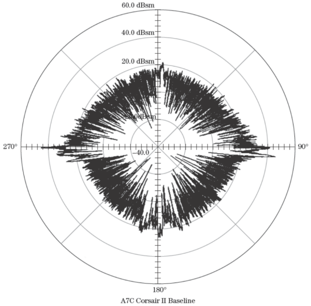


Go back

Discussion

Why is the RCS so sensitive to angle?

FIGURE 6-30 ■ A7 C measured backscatter for horizontal polarization at 9.5 GHz, 20 dB/div.



Answer: Many different specular reflections and multiple bounces are interfering to form the total RCS

$$\sigma_{\text{tot}} = \left| \sum_{i=1}^{N} \sqrt{\sigma_i}\, \mathrm{e}^{-\mathrm{j}2\boldsymbol{k}\cdot\boldsymbol{R}_i} \right|^2$$

Go back