

EITN90 Radar and Remote Sensing Lecture 11: Fundamentals of pulse compression waveforms

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Lunch lectures at Axis Communications next week

- Wednesday March 6.
- 12:15 Lunch sandwich is served
- ► 13:00–15:00 Radar lectures with speakers from Axis AB and Acconeer AB, two local companies with radar activities.
- Companies interested in students who know radar.

The lunch and the lectures are held at Axis Communications AB in Lund. It is at the main Axis building at Emdalavägen 14 (15 minute walk from LTH). Sign up on the sheet circulated at this lecture or send Daniel an email!



Learning outcomes of this lecture

In this lecture we will

- Introduce matched filters
- See how pulse compression can improve range resolution
- Study the linear frequency modulated waveform
- See how the ambiguity function can be used to analyze waveforms



FIGURE 1-1 = Major elements of the radar transmission/ reception process.

Outline

- Matched filters
- 2 Range resolution
- 6 Linear frequency modulated waveforms
- **4** Matched filter implementations
- 5 Sidelobe reduction in an LFM waveform
- **6** Ambiguity functions
- Phase-coded waveforms
- **8** Conclusions

Outline

Matched filters

- 2 Range resolution
- Einear frequency modulated waveforms
- **4** Matched filter implementations
- **5** Sidelobe reduction in an LFM waveform
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- **8** Conclusions

Radar waveforms



Many different waveforms are used in radars, taking many system requirements and constraints into account: bandwidth, power, Doppler tolerance, sidelobes, range resolution etc.

General time-invariant filtering

After filtering the received signal $x_{\rm r}(t)$ through any linear, time-invariant filter $h(\cdot)$ the signal is

$$y(t) = \int_{-\infty}^{\infty} h(t - \alpha) x_{\mathbf{r}}(\alpha) \,\mathrm{d}\alpha$$

With a time delayed received signal $x_{
m r}(t)=x(t-t_{
m d})$ we have

$$y(t) = \int_{-\infty}^{\infty} h(t - \alpha) x(\alpha - t_{\rm d}) \,\mathrm{d}\alpha$$

The amplitude $\left|y(t)\right|$ can be estimated using the Schwartz inequality

$$\begin{split} |y(t)| &\leq \left(\int_{-\infty}^{\infty} |h(t-\alpha)|^2 \,\mathrm{d}\alpha\right)^{1/2} \cdot \left(\int_{-\infty}^{\infty} |x(\alpha-t_{\mathrm{d}})|^2 \,\mathrm{d}\alpha\right)^{1/2} \\ &= (\mathrm{energy \ of \ filter})^{1/2} \cdot (\mathrm{energy \ of \ signal})^{1/2} \end{split}$$

where the values of t or t_d do not matter in the last expression.

Matched filter

With knowledge of the transmitted signal $\boldsymbol{x}(t),$ we can choose the matched filter

 $h(t) = x^*(-t)$

With this particular choice, we have the output

$$y(t) = \int_{-\infty}^{\infty} h(t-\alpha)x(\alpha-t_{\rm d})\,\mathrm{d}\alpha = \int_{-\infty}^{\infty} x^*(\alpha-t)x(\alpha-t_{\rm d})\,\mathrm{d}\alpha$$

This is maximized at $t = t_d$ (demonstrating the optimality of the matched filter since the maximum is attained)

$$\max_{t} |y(t)| = y(t_{\mathrm{d}}) = \int_{-\infty}^{\infty} |x(\alpha - t_{\mathrm{d}})|^2 \,\mathrm{d}\alpha = \int_{-\infty}^{\infty} |x(\alpha)|^2 \,\mathrm{d}\alpha$$

which is proportional to the energy of the pulse waveform x(t).

Matched filter as maximizing SNR

Convolution in time domain corresponds to multiplication in frequency domain, or

$$\begin{split} Y(\omega) &= H(\omega) X_{\rm r}(\omega) = H(\omega) X(\omega) {\rm e}^{-{\rm j}\omega t_{\rm d}} \\ \Rightarrow \quad y(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) X(\omega) {\rm e}^{{\rm j}\omega(t-t_{\rm d})} \, {\rm d}\omega \end{split}$$

With white noise $N(\omega)=N_0,$ the total received noise power is

$$\overline{n^2(t)} = \frac{N_0}{2\pi} \int_{-\infty}^{\infty} |H(\omega)|^2 \,\mathrm{d}\omega$$

Hence the SNR at $t = t_{\rm d}$ is

$$SNR = \frac{|y(t_d)|^2}{\overline{n^2(t)}} = \frac{\left|\int_{-\infty}^{\infty} H(\omega)X(\omega) \,\mathrm{d}\omega\right|^2}{N_0 \int_{-\infty}^{\infty} |H(\omega)|^2 \,\mathrm{d}\omega}$$

which is maximized for $H(\omega) = X^*(\omega)$ or $h(t) = x^*(-t)$.

Example: rectangular pulse

For the simple rectangular pulse (setting $t_{\rm d}=0$)

$$x(t) = A, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

the matched filter is

$$h(t) = A, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

and the filtered response is



FIGURE 20-2 The simple pulse of duration τ has a match filtered response of duration 2τ .

Generic response



For general waveforms, the filtered response is typically described in terms of mainlobe and sidelobes.

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Matched filters

2 Range resolution

- **③** Linear frequency modulated waveforms
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Resolution

The Rayleigh resolution criterion is that the peak of one target is at the null of the second target.



FIGURE 20-4 ■ Individual responses from two point targets separated by the Rayleigh resolution.

The above figure corresponds to the matched filter response of rectangular pulses.

Fourier uncertainty principle

The widths of a signal of zero mean in time and frequency domain can be defined by

$$D_t = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 |y(t)|^2 \, \mathrm{d}t}{\int_{-\infty}^{\infty} |y(t)|^2 \, \mathrm{d}t}}$$
$$D_\omega = \sqrt{\frac{\int_{-\infty}^{\infty} \omega^2 |Y(\omega)|^2 \, \mathrm{d}\omega}{\int_{-\infty}^{\infty} |Y(\omega)|^2 \, \mathrm{d}\omega}}$$

The product of these widths is bounded below as

$$D_t D_\omega \ge \sqrt{\frac{\pi}{2}}$$

with equality for Gaussian signals. This motivates that resolution in time (range) is inversely proportional to frequency bandwidth.

$$\delta R = \kappa \frac{c}{2B}$$

 $\kappa\approx 1,$ definitions of resolution and bandwidth often chosen to conform with this formula.

Phase difference between two targets

Two targets separated by the Rayleigh resolution can present radically different responses depending on phase difference.

FIGURE 20-5 ■ Combined response for two point targets with phase difference equal to 0°.



FIGURE 20-6 ■ Combined response for two point targets with phase difference equal to 180°.

Outline

1 Matched filters

2 Range resolution

6 Linear frequency modulated waveforms

- **4** Matched filter implementations
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LFM waveform

A baseband linear frequency modulated waveform (LFM) is

$$x(t) = A \cos\left[\pi \tau B\left(\frac{t}{\tau}\right)^2\right], \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$



FIGURE 20-7 ■ Time-domain response, within the pulse, of a linear frequency modulated (LFM) waveform with a time-bandwidth product equal to 50.

The waveform is characterized by the time-bandwidth product τB and normalized time $t/\tau.$

Instantaneous frequency

The instantaneous phase is $\phi(t)=\pi\tau B(t/\tau)^2$, and instantaneous frequency is



The linear change motivates the term linear frequency modulation.

The LFM spectrum has a relatively flat spectrum across bandwidth B. Flatness and roll-off improves as time-bandwidth product τB increases.



FIGURE 20-9 Comparison of the spectra of LFM waveforms with time-bandwidth products of 20 (light curve) and 100 (dark curve).

Matched filter response

The matched filter response for the LFM waveform is

$$y(t) = \int_{-\infty}^{\infty} x^*(\alpha - t) x(\alpha) \, \mathrm{d}\alpha = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\left(1 - \frac{|t|}{\tau}\right) \pi \tau B \frac{t}{\tau}\right]}{\left(1 - \frac{|t|}{\tau}\right) \pi \tau B \frac{t}{\tau}}, \quad |t| \le \tau$$

FIGURE 20-10 ■ Match filtered response for a 50 MHz, 1 µsec LFM waveform.



The peak is much more narrow than total pulse width τ !



Range resolution

 $\delta R = \frac{c}{2B}$

For large values of τB , the first null occurs at $t \approx 1/B$. With range R = ct/2, the Rayleigh resolution in range is



FIGURE 20-11 Mainlobe and first 3 sidelobes for the LFM waveform match filtered response with a time-bandwidth product equal to 100.

The -4 dB pulsewidth is 1/B. The width of the main lobe is compressed by a factor of about $\frac{\tau}{1/B} = \tau B$.

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- 2 Range resolution
- Iinear frequency modulated waveforms

4 Matched filter implementations

- **5** Sidelobe reduction in an LFM waveform
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Dispersive filters

Filters having frequency dependent group delay

$$t_{\rm gd} = -\frac{\mathrm{d}\phi(\omega)}{\mathrm{d}\omega} = \frac{\tau}{2\pi B}\omega$$

can both stretch and compress waveforms. One implementation is surface acoustic wave (SAW) technology:



The device couples electromagnetic energy to acoustic waves, where the coupling is strongest when the distance between the metal fingers correspond to $\lambda/2$ for the acoustic wave. Chirping is obtained by different acoustic propagation lengths. Works up to about 3 GHz, high insertion loss.

Digital filters

With a digitized signal, the matched signal can be implemented using the Fast Fourier Transform (FFT) of the analytic signal $x[n] = x_{\rm I}[n] + jx_{\rm Q}[n]$:

 $y[n] = \operatorname{FFT}^{-1}\{H[\cdot] X[\cdot]\}[n], \quad X[k] = \operatorname{FFT}\{x[\cdot]\}$



Error correction is obtained by transmitting a pilot pulse and recording the received (distorted) signal, taking into consideration imperfections in the transmit/receive chain.

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Sidelobe reduction

Sidelobes of the compressed pulse can be reduced by weighting the filter in amplitude. The cost is an increased mainlobe width.



FIGURE 20-13 ■ A -40 dB, $\bar{n} = 4$, Taylor-weighted LFM waveform compressed response (solid curve) has significantly reduced sidelobes versus an unweighted LFM waveform response (dashed curve). When increasing the sidelobe suppression, the resolution is decreased.

TABLE 20-2 4 dB Resolution Associated with a Taylor Weighting Function

	Peak Sidelobe Ratio (dB)								
	-20	-25	-30	-35	-40	-45	-50	-55	-60
\overline{n}	4 dB Resolution Normalized by c/2B								
2	1.15	1.19	1.21						
3	1.14	1.22	1.28	1.33					
4	1.12	1.22	1.29	1.36	1.42	1.46			
5	1.11	1.20	1.29	1.36	1.43	1.49	1.54		
6	1.10	1.19	1.28	1.36	1.43	1.50	1.56	1.61	
7	1.09	1.19	1.28	1.36	1.43	1.50	1.56	1.62	1.67
8	1.08	1.18	1.27	1.35	1.43	1.50	1.57	1.63	1.68

The theoretical sidelobe reduction is achieved when the weighting is applied to a rectangular spectrum. A real LFM has some additional spread, which is reduced as $\tau B \rightarrow \infty$.



FIGURE 20-14 ■ A comparison of time-sidelobe responses for time-bandwidth products of 20 (solid curve) and 100 (dashed curve) when applying a -40 dB Taylor weighting.

Matlab demo, http://radarsp.com

FRSP Demos/FRSP GUI Demos/FRSP LFM-GUI



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Ambiguity function

Taking into account the possibility of both time delay and Doppler shift, the received signal is

$$x_{\rm r}(t) = \mathrm{e}^{\mathrm{j}2\pi f_{\rm d}t} x(t - t_{\rm d})$$

Centering the waveform over $t_{\rm d}=0$ and applying the matched filter and normalizing x with its energy, we find the ambiguity function

$$A(t, f_{\rm d}) = \frac{\int_{-\infty}^{\infty} x(\alpha) \mathrm{e}^{\mathrm{j}2\pi f_{\rm d}\alpha} x^*(\alpha - t) \,\mathrm{d}\alpha}{\int_{-\infty}^{\infty} |x(\alpha)|^2 \,\mathrm{d}\alpha}$$

This function satisfies

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A(t, f_{\rm d})|^2 \, \mathrm{d}t \, \mathrm{d}f_{\rm d} = 1 \quad \text{and} \quad |A(t, f_{\rm d})| \le |A(0, 0)| = 1$$

Ambiguity for a simple rectangular pulse

For an unmodulated pulse,

$$x(t) = \frac{1}{\sqrt{\tau}}, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

the ambiguity function can be calculated as

$$A(t, f_{\rm d}) = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\pi f_{\rm d}\tau \left(1 - \frac{|t|}{\tau}\right)\right]}{\pi f_{\rm d}\tau \left(1 - \frac{|t|}{\tau}\right)}, \quad |t| \le \tau$$

Depends on normalized time t/τ and normalized Doppler shift $f_{\rm d}\tau$.



Similar to Fig 20-15 in the book. Using a matlab script, you can plot the figure in 3D and rotate.

Ambiguity function for LFM waveform

For a linear frequency modulated pulse

$$x(t) = \frac{1}{\sqrt{\tau}} \exp\left(\mathrm{j}\pi \frac{B}{\tau} t^2\right), \quad |t| \leq \tau$$

the ambiguity function can be calculated as

$$A(t, f_{\rm d}) = \left| \left(1 - \frac{|t|}{\tau} \right) \frac{\sin \left[\pi \tau B \left(1 - \frac{|t|}{\tau} \right) \left(\frac{f_{\rm d}}{B} + \frac{t}{\tau} \right) \right]}{\pi \tau B \left(1 - \frac{|t|}{\tau} \right) \left(\frac{f_{\rm d}}{B} + \frac{t}{\tau} \right)} \right|, \quad |t| \le \tau$$

Depends on normalized time delay t/τ and normalized Doppler shift f_d/B , with time-bandwidth parameter τB .

Ambiguity function for LFM waveform, $\tau B = 20$



Similar to Fig 20-16. Can also be plotted with the matlab script in 3D and rotated.

The ridge along the diagonal means delay/range can be mistaken for Doppler shift.

Ambiguity function for LFM waveform, different τB



From the formula

$$A(t, f_{\rm d}) = \left| \left(1 - \frac{|t|}{\tau} \right) \frac{\sin \left[\pi \tau B \left(1 - \frac{|t|}{\tau} \right) \left(\frac{f_{\rm d}}{B} + \frac{t}{\tau} \right) \right]}{\pi \tau B \left(1 - \frac{|t|}{\tau} \right) \left(\frac{f_{\rm d}}{B} + \frac{t}{\tau} \right)} \right|, \quad |t| \le \tau$$

we see that a non-zero Doppler shift $f_{\rm d}$ can be interpreted as

- Time shift $\Delta t = -f_{\rm d}\tau/B$
- Amplitude reduction by $(1 |\Delta t|/\tau) = (1 |f_d/B|)$

This leads to shifts in peak location, peak amplitude, and decreased resolution due to peak widening.

Degradations in presence of Doppler shift



Matched filter response in presence of Doppler shift



FIGURE 20-20 = Individual LFM match filtered responses for fractional Doppler shifts of 0%, 25%, 50%, and 75% illustrate both reduction in peak levels and broadening of the mainlobe. The effect of applying an amplitude taper to control sidelobes is reduced in presence of Doppler shift.

FIGURE 20-21 = Match filtered response for a -40 dB Taylor weighted LFM waveform with a time-bandwidth product of 200 and a fractional Doppler shift of 0% (dark curve) and 15% (light curve). Both curves have been normalized to the peak of their responses.



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Phase modulation

Instead of modulating the frequency, the phase can be controlled, typically in a digital way:



Matched filter



FIGURE 20-23 Biphase coded waveforms consist of chips exhibiting 2 possible phase states.



FIGURE 20-24 Match filtered response for the Barker phase coded waveform (Figure 20-23) maintains equal peak sidelobes of level 1/N.



Different phase codes

Biphase codes

- ▶ Two phase states (+/-)
- Minimum peak sidelobes (MPS)
- Barker codes: achieve a 1: N peak sidelobe to mainlobe ratio
- ► Maximum length sequence (MLS): length l = 2ⁿ 1, peak sidelobes ~ 1/√l
- Polyphase codes (not treated in this lecture)
 - More than two phase states: more degrees of freedom
 - Frank, P1, P2, P3, P4

Careful design of the phase codes can result in a thumb-tack like ambiguity function.



FIGURE 20-29 The ambiguity surface associated with some phase coded waveforms is a thumb tack.

A number of filter banks can be used to search the Doppler space, applying the matched filter at the output of each filter. Enables simultaneous estimation of range and Doppler.

Barker codes

TABLE 20-4 A List of the Known Biphase Barker Codes

Code Length	Code Sequence	Peak Sidelobe Level, dB
2	+-,++	-6.0
3	+ + -	-9.5
4	+ + - +, + + + -	-12.0
5	+ + + - +	-14.0
7	+ + + + -	-16.9
11	++++-+-	-20.8
13	+++++++-+-+	-22.3



Normalized Delay tB

Minimal peak sidelobe (MPS)



FIGURE 20-31 Compressed response for a 48-length Minimal Peak Sidelobe (MPS) code achieves a 3:48 peak sidelobe ratio.

Examples of minimal peak sidelobe (MPS)...

TABLE 20-6 = Example Biphase MPS Codes through Length 105

Cool. Largely Paule Statubbe Number Ofcodes Example Code Headscients Code Largely Paule 2 1 1 2 4 5 3 1 1 2 4 5 4 1 1 0 55 5 1 1 0 7 3 6 2 4 00 35 7 7 2 1 77 64 3 10 2 16 77 64 3 11 1 197 64 3 4 12 2 16 77 64 4 13 1 197 64 4 4 14 2 3 143 67 4 15 2 13 143 67 4 16 2 3 143 77 4 17 2 4 773						
1 1 2 3 1 1 6 5 1 1 0 35 6 2 1 10 37 7 2 4 00 35 7 2 1 0 35 7 2 1 0 35 8 7 0 0 0 9 2 1 0 10 0 10 2 1 10 198 0 0 11 1 1 198 0 0 0 12 2 13 183 0 0 0 13 1 1 1 198 0 0 0 14 1 1 198 0 0 0 0 14 1 1 198 0 0 0 0 0 0 0	Code Length	Peak Sidelobe	Number of Codes	Example Code (Hexadecimal)	Code Length	Pea
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1 2 1 10883 75 22 3 377 16055 75 23 3 157 16057 75 24 3 159 34442 76 25 2 1 15462 77 26 3 324 2800.02 79 25 2 3 324 2800.02 79 26 3 323 2800.02 79 70 27 3 323 164.402 82 70 28 2 323.3 164.4027 82 70 30 3 323 164.4027 83 86 31 3 323 154.4027 85 86 31 3 323 75.45.466 90 86 31 3 327 75.45.466 90 91 35 3 37 75.45.466 90 92	20	2	1	5181B	72	
12 3 778 1260,69 74 23 3 135 30799 75 24 3 145 50799 75 25 3 431 644781 77 26 3 42 200,007 79 27 3 388 200,007 79 28 2 333 1611,007 81 30 3 36 21,5346 84 31 3 221 200,007 84 31 3 212 200,007 84 31 3 212 200,007 84 31 3 212 200,007 84 34 3 311 399 200,007 86 34 3 311 399 200,007 86 35 311 397 304 307 306 36 3 317 394,007 96	21	2	3	160093	73	
13 3 113 3PFD# 75 24 3 4458 644ED 77 25 3 1 2300.05 76 26 3 1 230.05 77 27 3 383 2888.7 79 28 3 288.8 2888.7 79 29 3 383 2888.7 84 30 3 312 2.000.07 84 313 3 213.5467 86 3 313 3 312 2.000.07 84 313 3 311 33.9FLAS 87 313 3 311 33.9FLAS 87 314 3 313 71.71494 89 315 73.7121947 89 91 91 314 3 312 73.7121947 92 41 3 315 74.7121947 92 42 3	22	3	378	F6D5F	74	
1 3 158 64AFE 76 25 2 1 154EF 77 26 2 1 154EF 77 26 3 202 2000.02 79 27 3 202 2000.02 79 28 2 28 110.0 60 29 3 203 14AAEC 42 20 3 203 14AAEC 42 21 3 423 3444E 42 31 3 12 255.470 45 32 3 422 355.470 45 31 3 12 754.483 46 31 3 17 754.446 49 32 3 17 754.446 49 42 3 12 1256.454 49 43 3 12 1256.454 49 44 447.477.444 49	22	1	515	2810.00	75	
1 1 1.55482 77 26 3 32 230009 797 70 3 381 230009 797 72 3 381 230009 797 73 3 36 231007 80 80 3 164 81 81 31 3 169 2.15367 81 31 3 121 2.060000 84 81 31 3 129 CCAASTP 86 86 34 3 110 794433 87 76 35 3 110 794433 88 96 36 3 110 794433 88 96 37 3 32 7372497 90 97 38 3 17 7534446 91 91 41 3 15 38825054 96 96 42 3 <t< td=""><td>24</td><td>3</td><td>858</td><td>64AEE3</td><td>76</td><td></td></t<>	24	3	858	64AEE3	76	
16 3 342 2 MADDy 75 27 3 328 2 BIRDy 9 BIR 28 3 2 BIRDy 9 BIR 9 BIR 29 3 3 2 BIR 8 BIR 9 BIR 30 3 3 BIR 2 BIR 8 BIR 9 BIR 31 3 3 DIR 2 BIR 8 BIR 9 BIR 31 3 3 DIR 2 BIR 8 BIR 9 BIR 33 3 3 DIR 3 BIR 9 BIR 9 BIR 34 3 3 DIR 3 BIR 9 BIR 9 BIR 35 3 3 DIR 3 BIR 9 BIR 9 BIR 36 3 3 DIR 3 BIR 9 BIR 9 BIR 41 3 3 DIR 9 BIR 9 BIR 9 BIR 42 3 15 PRCERDADY 9 BIR 9 BIR 43 3 15 PRCERDADY 9 BIR 9 BIR	25	2	1	1254067	77	
1 5 588 2.98887 79 28 2 2 8111.0 611 3 3 30 16.44827 82 31 3 201 16.44827 82 31 3 201 2.548477 85 32 3 422 3.555.470 45 32 3 422 3.355.470 45 34 3 129 3.355.470 45 34 3 111 756.433 87 35 3 111 756.433 87 36 3 17 752.44.46 99 37 3 13.13 13.95 36 36 3 17 752.44.46 99 42 3 4 43.14.41.41 95 43 3 12 39.47.0716 96 44 4.97.4716.49 98 98 96 45 3 <td>26</td> <td>1</td> <td>242</td> <td>23804.09</td> <td>78</td> <td></td>	26	1	242	23804.09	78	
2 2 3 911120 80 29 3 233 164AWET 41 30 3 66 21/3540 43 31 3 66 21/3540 43 32 3 22 2355.700 48 33 3 19 CCAAST 66 33 3 19 CCAAST 86 33 3 11 31944635 87 34 3 11 3144635 88 37 3 22 57.21997 69 37 3 31 3144635 89 37 3 32 77.21997 69 40 3 15 382.52354 44 41 3 15 382.52354 69 42 3 15 382.52354 69 43 3 15 382.52354 69 44 3 15	27	1	199	15000007	79	
12 3 13 144AUET 11 10 3 36 2153497 63 31 3 212 2.048020 84 32 3 212 2.048020 84 33 3 212 2.048020 84 33 3 212 2.048020 84 33 3 11 3.0591245 87 34 3 11 3.0591245 87 35 3 12 7.712896 99 36 3 7.7 2.2205244 43 40 3 7.7 2.2205244 43 41 3 12 MBACREC 46 42 3 15 PRCCEADDOP 77 43 3 12 MBACREC 49 44 3 15 PRCCEADDOP 77 47 3 4 400278656 99 47 3	20	2	2	8E1112D	80	
10 3	20	2	192	1644.8007	81	
1 3 251 7.448CUP 81 22 3 4422 3355700 485 313 3 139 CCAASTS 66 313 3 139 CCAASTS 66 315 3 111 SUPERIT 86 315 3 111 SUPERIT 87 315 3 111 SUPERIT 89 315 3 161 3144080 89 317 3 52 57,27494 90 318 3 17 SUPART 91 319 3 15 SUPART 91 41 3 15 SUPART 91 42 3 1 SUPART 91 44 1580164478	20	3	285	221 62405	82	
12 3 -22 3353,270 84 33 3 1397 CCAASTT 64 34 3 11 JSPELAS 97 34 3 111 JSPELAS 97 35 3 121 JSPELAS 97 36 3 131 JSPELAS 97 37 3 52 37,11949 99 3 30 JSJSMEPIC 92 94 40 3 17 2220CAAA 93 41 3 12 PSEACEELC 95 42 3 12 PSEACEELC 96 43 3 12 PSEACEELC 97 44 3 15 PRECEDAD/F 97 44 3 15 PRECEDAD/F 98 47 3 4 4072198 109 48 3 15 PRECEDAD/F 102 49 4	30	3	251	2313240F	83	
13 3 1.09 CCAASTP 85 34 3 51 3387.55 87 35 3 111 79AA33 87 35 3 111 79AA33 88 37 3 52 2747497 89 38 3 17 X5AAAST 90 38 3 17 X5AAAS 91 41 3 13 11348874 92 42 3 13 7147474 95 41 3 15 738AA5 96 42 3 13 7307 96 43 3 13 7307 96 44 3 14 447878744 95 45 3 4 240705566 96 46 3 4 250474576 101 47 3 4 250474566 102 48 4 505164787	22		133	2366 4 700	84	
14 3 51 339ELAS 86 35 3 111 766.033 88 36 3 181 331.44.032 89 36 3 181 331.44.032 89 37 3 23 57.17496 99 3 3 10 133.988EFA 91 40 3 3 10 133.988EFA 92 41 3 15 382.5316.4 94 94 42 3 15 382.5216.4 94 94 43 3 15 382.5216.4 94 94 44 3 15 PECCE3.207 97 96 44 3 15 PECCE3.207 97 96 45 3 4 2.447C.2019.6 98 96 46 3 1 2.967.5219.8 101 101 47 3 1 2.967.5219.8 102	32	3	422	33334780	85	
15 3 111 7956303 97 36 3 1614 314,40031 97 37 3 32 37,111967 99 37 3 32 37,111967 99 39 3 32 37,111967 92 40 3 37 222305,454 93 41 3 12 398,0421 93 42 3 12 398,0421 96 43 3 12 398,0421 96 44 41619144 46 97 97 44 3 15 198,04216 98 44 3 14 407,0139 99 44 3 14 407,0139 99 45 3 1 407,0139 100 46 3 1 407,0139 100 47 3 4 407,0139 100 48 3	33	3	139	2221014.64	86	
35 3 11.6 31.444020 88 36 3 12 31.444020 88 37 3 52 37.27499E 69 38 3 17 75.344.46 91 30 3 37 32.2257.45A 69 41 3 15 31.82.5258.4 64 42 3 15 31.82.5258.4 64 43 3 15 31.82.5258.4 69 44 3 15 31.82.5258.4 69 45 3 4 2.3470.007 69 46 3 1 2.3490.007 69 47 3 4 2.3490.007.966.6 99 48 3 4 30.101.101.101.101.101.101.101.101.101.1	34	3	51	353PEDA55	87	
3 13 31/176492 99 38 3 17 XXAA46 91 39 3 30 13/3808F5 92 40 3 57 22210CA4A 93 41 3 17 22210CA4A 94 42 3 12 22210CA4A 95 43 3 12 2230CA4A 96 44 3 15 PECCBAA7 97 45 3 4 244708704 96 44 3 15 PECCBAD7 97 45 3 4 244708704 99 46 3 4 24470CB148 99 47 3 1 99/3785198 100 48 3 4 1580B475 101 49 4 Net Reprot 0200707 102 51 4 Net Reprot 0200707 103 53 Net Reprot	35	3	111	790/4833	88	
33 3 27 2-54,47,69 00 39 3 30 133,98476 01 40 3 37 22,2162,45,4 03 40 3 37 22,2162,45,4 03 41 43 3 12 93,24,216 04 42 3 4 444787444 03 04 43 3 12 93,24,216 05 04 44 3 12 93,24,216 06 06 06 07 07 07 07 44 3 12 193,24,216 06 07 07 08 06 07 07 08 07 07 08 07 07 08 07 07 07 08 07	30	3	181	3314A083E	89	
59 3 10 13.30825C 49 40 3 57 23235204A 49 41 3 15 38262394 49 41 3 15 38262394 49 42 3 12 44778344 69 43 3 15 98262394 69 44 3 15 9826209 69 45 3 4 244702096 69 45 3 4 244702096 69 46 3 1 X002796356 69 47 3 4 1506104475 101 48 3 4 1506104475 101 49 4 Net Reported 012804206476 113 50 4 Net Reported 0128043020678 103 51 4 Net Reported 0128043020578 1015 52 4 Net Reported 01280430205788 105	37	3	32	3/42/08/98	90	
3 3 3 12 223 BE AA 92 41 3 15 382620364 94 42 3 4 442BE 784 95 43 3 12 882.002 96 44 3 15 BECKER 2A/D 97 44 3 1 AVATBS 1984 100 46 3 4 XACTBS 1984 100 47 3 1 AVATBS 1984 100 48 3 4 1950 EBHTP 101 49 4 Not Reported 012A/ABC79648 102 51 3 Not Reported 012A/ABC79648 103 52 4 Not Reported 012A/ABC79648 103 53 4 Not Reported 012A	38	3	17	30.544,400	91	
41 3 13 308A82084 64 42 3 4 447878444 65 43 3 12 493A7216 65 43 3 12 493A7216 66 45 3 4 2447646 66 46 3 4 24476466 69 47 3 4 2447646666 99 47 3 4 64476865 99 47 3 4 644768656 10 10 49 4 Net Reprod 012MAEC76467 102 50 4 Net Reprod 013MAEC76467 103 51 4 Net Reprod 013MAEC76467 103 52 4 Net Reprod 013MAEC76467 103 53 4 Net Reprod 013MAEC76567 103	39	3	30	13350BEF3C	92	
14 3 5 Architectulat 44 13 3 12 Statutat 44 43 3 12 Statutat 66 44 3 15 Statutat 66 44 3 15 Statutat 69 45 3 4 2447CC0016 69 47 3 1 Control 100 69 48 3 4 15601E0487 101 49 4 Net Reprotid 0230ABC706467 102 51 4 Net Reprotid 0230ABC706467 103 52 4 Net Reprotid 013AABC706468 105 53 4 Net Reprotid 013AABC706478 105	40	3	31	222300.37634	93	
41 3 12 SBLACETIC 65 44 3 15 FREERADOT 97 45 3 4 2APCCOMP6 98 46 3 1 ACCTSS65 99 47 3 1 AVATES198 100 48 5 1 AVATES198 100 48 6 1 AVATES198 100 49 6 1 1000000000000000000000000000000000000	41	3	15	38EA520364	94	
44 3 15 TRECED AU7 67 45 3 4 2APCCEMP6 69 46 3 1 2COPTRES 99 47 3 1 2COPTRES 99 46 3 4 SMERTER 101 47 3 4 SMERTER 101 48 3 4 SMERTER 101 49 4 No. Reported DISS/AUGCE/FM47 102 50 4 No. Reported DISS/AUGCE/FM47 103 52 4 No. Reported OUXA/RED/SME7 103 53 4 No. Reported OUXA/RED/SME7 105	42	3	4	4447D874D4	95	
44 3 15 11 10 46 3 4 200 10 47 3 1 200 10 10 48 3 4 1560 10 10 49 4 Net Reported 02,040 102 102 40 4 Net Reported 02,040 102 102 51 3 Net Reported 02,040 104 104 52 4 Net Reported 043,ABE72449 103 104 53 4 Net Reported 043,ABE72449 105 105	43	3	12	SBZACCEIC	96	
46 3 4 249/04/X0008 98 47 3 1 06/X1918 99 47 3 1 06/X1918 99 48 3 4 556016/4171 101 49 4 No Reported 0120/REF0946F 102 50 4 No Reported 0258/X007264F 103 51 3 No Reported 0258/X007264F 104 52 4 Nor Reported 0153/X07304F 104 53 4 Nor Reported 0153/X07304F 105	44	3	15	FECECB2AD/	97	
46 3 1 0.64 (780) 084 9 (97) 48 3 4 15560 (1546) 175 101 49 4 Net Reported 0.23, MEC 706-967 102 50 4 Net Reported 0.23, MEC 706-967 102 51 4 Net Reported 0.13, MALEC 706-967 103 52 4 Net Reported 0.93, MET 705-976 105 53 4 Net Reported 0.13, AND 705 Securic cancel from (19.34)	45	3	4	2AP0CC6DBP6	98	
44 3 4 WALRSDAM 100 49 4 No Reported 0.234ET/SPR4FF 101 50 4 No Reported 0.234ET/SPR4FF 101 51 3 No Reported 0.254ET/SPR4FF 103 52 4 No Reported 0.954ABT/SPR4FF 104 53 4 No Reported 0.95AABT/SPR4FF 105	46	3	1	3C0CF7B6556	99	
48 3 4 1560 [15/1473 10] 49 4 Net Reported 012ABEC/95467 102 50 4 Net Reported 015805ABC/25667 103 51 3 Net Reported 015805ABC/2567 104 52 4 Net Reported 015AA/BC/2567 105 53 4 Net Reported 013AA/BC/2567 Terrors/mails/monitors/milling/moniling/monitors/milling/moniling/monitors/milling/monitors/	47	3	1	69A /E851988	100	
40 4 Not Reported 012.BEC/PHeaP 102 51 3 Not Reported 012.VT75ADTB4 104 52 4 Not Reported 015AJE073AAF 104 53 4 Not Reported 015AAJE073AAF 105	48	3	4	156B61E64FF3	101	
50 4 Not Reported 025853ABC266F 103 51 3 Not Reported 71007376ADB4 104 52 4 Not Reported 0953ABC266F 105 53 4 Not Reported 0153AA7BD2C6F 3000000000000000000000000000000000000	49	4	Not Reported	012ABEC79E46F	102	
51 5 Not Reported 71C07376ADB4 104 52 4 Not Reported 0445AE007346F 105 53 4 Not Reported 0132AA7F8D2C6F Sweep: Correlia from (19-24).	50	4	Not Reported	025863ABC266F	103	
52 4 Not Reported 0945AE0F3246F 105 53 4 Not Reported 0132AA7F8D2C6F Sovrees: Centriled from (19-24).	51	3	Not Reported	71C077376ADB4	104	
53 4 Not Reported 0132AA7F8D2C6F Source: Comeiled from (19-24).	52	4	Not Reported	0945AE0F3246F	105	
	55	4	Not Reported	0132AA7F8D2C6F	Sources: Compiled from	[19-24].

TABLE 20-6 = (Continued)

de Length	Peak Sidelobe	Number of Codes	Example Code (Hexadecimal)
54	4	Not Reported	0266 A2814B3C6F
55	4	Not Reported	04C26AA1E3246F
56	4	Not Reported	099BAACB47BC6F
57	4	Not Reported	01268A8ED623C6F
58	4	Not Reported	023CE545C9ED66F
59	4	Not Reported	049D38128A1DC6F
60	4	Not Reported	0AB8DF0C973252F
61	4	Not Reported	005B44C4C79EA350
62	4	Not Reported	002D66634CB07450
63	4	Not Reported	04CF5A2471657C6F
64	4	1859	55FF84B069386665
65	4	Not Reported	002DC0B0D9BCE5450
66	4	Not Reported	0069B454739F12B42
67	4	Not Reported	007F1D164C62A5242
68	4	Not Reported	009E49E3662A8EA50
69	4	Not Reported	0231C08FDA5A0D9355
70	4	Not Reported	1A133B4E3093EDD57E
71	4	Not Reported	63383AB6B452ED93EE
72	4	Not Reported	E4CD5AF0D054433D82
73	4	Not Reported	1B66B26359C3E2BC00A
74	4	Not Reported	36DDBED681F98C70EAE
75	4	Not Reported	6399C983D03EFDB556D
76	4	Not Reported	DB69891118E2C2A1EA0
77	4	Not Reported	1961AE251DC950EDDRE4
78	4	Not Reported	328B457F0461E4ED7B73
79	4	Not Reported	76CF68F327438AC6FA80
80	4	Not Reported	CE43C8D986ED429F7D75
81	4	Not Reported	0E3C32FA1FEFD2519AB32
82	4	Not Reported	3CB25D380CE3B7765695F
83	ŝ	Not Reported	711763AE7DBB8482D3A5A
84	5	Not Reported	CE79CCCDB6003C1E95AAA
85	5	Not Reported	19900199463E51E8B4B574
86	ŝ	Not Reported	3603FB659181A2A52A3807
87	5	Not Reported	7F7184F04F4E5E4D9B56AA
88	5	Not Reported	D54A9326C2C686F86F3880
89	š	Not Reported	180F09434F1BBC44ACDAC8A
90	5	Not Reported	3326D87C3A91DA8AFA84211
91	5	Not Reported	77F80F632661C3459492A55
92	5	Not Reported	CC6181859D9244A5EAA87F0
93	5	Not Reported	187B2ECB802FB4F56BCCECE5
94	5	Not Reported	319D9676CAFFADD68825F878
95	5	Not Reported	69566B2ACCC8BC3CE0DE0005
96	5	Not Reported	CE963ED09B1381657A8A098E
97	š	Not Reported	1A843DC410898B2D3AE8EC362
98	5	Not Reported	30E05C18A1525596DCCE600DE
99	5	Not Reported	72E6DB6A75E6A9E81E0846777
100	5	Not Reported	DE490FEB1E8390A54E3CD9AAE
101	5	Not Reported	1A5048216CCF18F83E910DD4C5
102	5	Not Reported	2945A4F11CF44FF664850D182A
103	5	Not Reported	77FAA82C6F065AC4BE18F274CB
104	5	Not Reported	E568ED4982E9560EBA2E611184
105	5	Not Reported	1C6387FF5Da4Fa325C895958DC5
Constant Constant	-		

Maximal length sequences



FIGURE 20-33 The compressed response for a 127-length MLS.

Comparison LFM and biphase MLS, waveform



FIGURE 20-34 ■ Comparison of a compressed LFM waveform (black curve) (TB = 1000) with a compressed biphase MLS coded waveform (gray curve) (TB = 1023).

Higher average sidelobe levels in MLS, higher peak sidelobe in LFM.

Comparison LFM and biphase MLS, spectrum

FIGURE 20-35 ■ Comparison of the spectra of an LFM waveform (black curve) with a 1023-length MLS coded waveform (gray curve).



The wide spectrum of MLS can be attributed to the abrupt changes in phase. Can create an electromagnetic interference problem.

Outline

- 1 Matched filters
- **2** Range resolution
- Einear frequency modulated waveforms
- **4** Matched filter implementations
- **5** Sidelobe reduction in an LFM waveform
- **6** Ambiguity functions
- Phase-coded waveforms

(B) Conclusions

Conclusions

- Matched filters maximize SNR for a given waveform
- ► The resulting pulse compression improves range resolution
- The LFM is a generic waveform, sidelobes can be improved by tapering
- Phase coding can produce very narrow ambiguity peaks, but with a wide spectrum

Why is

$$SNR = \frac{|y(t_d)|^2}{\overline{n^2(t)}} = \frac{|\int_{-\infty}^{\infty} H(\omega)X(\omega) \, d\omega|^2}{N_0 \int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega}$$

maximized by $H(\omega) = X^*(\omega)$?



Why is

$$SNR = \frac{|y(t_d)|^2}{\overline{n^2(t)}} = \frac{|\int_{-\infty}^{\infty} H(\omega)X(\omega) \, d\omega|^2}{N_0 \int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega}$$

maximized by $H(\omega)=X^*(\omega)?$

Answer: The Schwartz inequality

$$\left|\int_{-\infty}^{\infty} H(\omega)X(\omega) \,\mathrm{d}\omega\right|^2 \leq \left(\int_{-\infty}^{\infty} |H(\omega)|^2 \,\mathrm{d}\omega\right) \cdot \left(\int_{-\infty}^{\infty} |X(\omega)|^2 \,\mathrm{d}\omega\right)$$

implies

$$\operatorname{SNR} \le \frac{1}{N_0} \int_{-\infty}^{\infty} |X(\omega)|^2 \,\mathrm{d}\omega$$

with equality for the choice $H(\omega) = X^*(\omega)$.

◀ Go back

Given the matched filter response for the LFM waveform is

$$y(t) = \int_{-\infty}^{\infty} x^*(\alpha - t) x(\alpha) \, \mathrm{d}\alpha = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\left(1 - \frac{|t|}{\tau}\right) \pi \tau B \frac{t}{\tau}\right]}{\left(1 - \frac{|t|}{\tau}\right) \pi \tau B \frac{t}{\tau}}, \quad |t| \le \tau$$

where in the graph below can you find the value of $\pi\tau B$ in dB?

FIGURE 20-10 ■ Match filtered response for a 50 MHz, 1 µsec LFM waveform.



Given the matched filter response for the LFM waveform is

$$y(t) = \int_{-\infty}^{\infty} x^*(\alpha - t) x(\alpha) \, \mathrm{d}\alpha = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\left(1 - \frac{|t|}{\tau}\right) \pi \tau B \frac{t}{\tau}\right]}{\left(1 - \frac{|t|}{\tau}\right) \pi \tau B \frac{t}{\tau}}, \quad |t| \le \tau$$

where in the graph below can you find the value of $\pi \tau B$ in dB?



Answer: For $|t/\tau| \approx 1$, we have $|y(t)| \approx 1/(\pi \tau B) \approx -44 \,\mathrm{dB}$ in this graph.

The ambiguity function $A(t, f_d) = \frac{\int_{-\infty}^{\infty} x(\alpha) e^{j2\pi f_d \alpha} x^*(\alpha - t) d\alpha}{\int_{-\infty}^{\infty} |x(\alpha)|^2 d\alpha}$ satisfies

 $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A(t, f_{\rm d})|^2 \, \mathrm{d}t \, \mathrm{d}f_{\rm d} = 1 \quad \text{and} \quad |A(t, f_{\rm d})| \le |A(0, 0)| = 1$

Given the above and a pulse shape with finite support,

$$|A(t,f_{\mathrm{d}})|^2=0$$
 when $|t|> au$ and $|f_{\mathrm{d}}|>B$

what average value of $|A(t, f_d)|^2$ do you expect for $|t| < \tau$ and $|f_d| < B$? How should τ be chosen to minimize the average ambiguity?



The ambiguity function $A(t, f_d) = \frac{\int_{-\infty}^{\infty} x(\alpha) e^{j2\pi f_d \alpha} x^*(\alpha - t) d\alpha}{\int_{-\infty}^{\infty} |x(\alpha)|^2 d\alpha}$ satisfies

 $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A(t, f_{\rm d})|^2 \, \mathrm{d}t \, \mathrm{d}f_{\rm d} = 1 \quad \text{and} \quad |A(t, f_{\rm d})| \le |A(0, 0)| = 1$

Given the above and a pulse shape with finite support,

$$|A(t, f_{\mathrm{d}})|^2 = 0$$
 when $|t| > \tau$ and $|f_{\mathrm{d}}| > B$

what average value of $|A(t, f_d)|^2$ do you expect for $|t| < \tau$ and $|f_d| < B$? How should τ be chosen to minimize the average ambiguity?

Answer: Since the integral of $|A|^2$ equals 1, we should have the average value $\left<|A|^2\right>=1/(4\tau B)$. To minimize this, increase the pulse length τ .

Interpretation: If you try to decrease $|{\cal A}|^2$ somewhere, it needs to increase somewhere else.





FIGURE 20-23 Biphase coded waveforms consist of chips exhibiting 2 possible phase states.

With each chip consisting of a fixed frequency carrier wave $\cos(2\pi f_0 t + \phi)$, with pulse length $\tau_{\rm chip}$, what is the chip bandwidth $B_{\rm chip}$ and time-bandwidth product τB with $\tau = N \tau_{\rm chip}$ and $B = B_{\rm chip}$?





FIGURE 20-23 Biphase coded waveforms consist of chips exhibiting 2 possible phase states.

With each chip consisting of a fixed frequency carrier wave $\cos(2\pi f_0 t + \phi)$, with pulse length $\tau_{\rm chip}$, what is the chip bandwidth $B_{\rm chip}$ and time-bandwidth product τB with $\tau = N \tau_{\rm chip}$ and $B = B_{\rm chip}$?

Answer: $B_{\rm chip} = 1/\tau_{\rm chip}$, $\tau B = N\tau_{\rm chip}B_{\rm chip} = N$. Hence, the time-bandwidth product only depends on the number of chips, N.

◀ Go back