EITN90 Radar and Remote Sensing Lecture 11: Fundamentals of pulse compression waveforms

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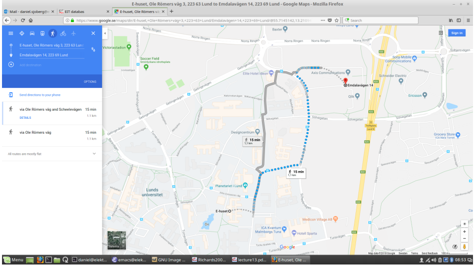
Lunch lectures at Axis Communications next week

- Wednesday March 6.
- 12:15 Lunch sandwich is served
- 13:00–15:00 Radar lectures with speakers from Axis AB and Acconeer AB, two local companies with radar activities.
- Companies interested in students who know radar.

The lunch and the lectures are held at Axis Communications AB in Lund. It is at the main Axis building at Emdalavägen 14 (15 minute walk from LTH). **Sign up on the sheet circulated at this lecture or send Daniel an email!**



Learning outcomes of this lecture

In this lecture we will

- Introduce matched filters
- See how pulse compression can improve range resolution
- Study the linear frequency modulated waveform
- See how the ambiguity function can be used to analyze waveforms

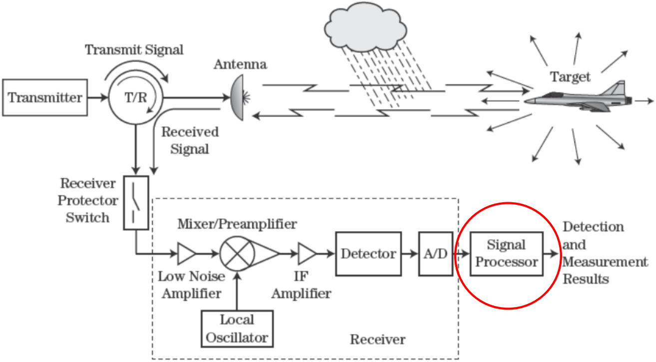


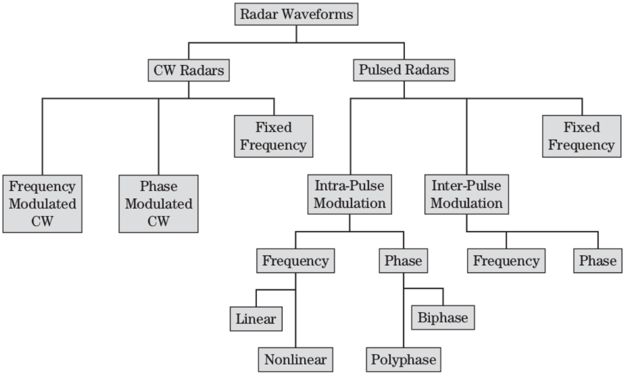
FIGURE 1-1 ■ Major elements of the radar transmission/ reception process.

Outline

Outline

Radar waveforms

FIGURE 20-1 ■ Modern radars select from and employ many waveform modulations.



Many different waveforms are used in radars, taking many system requirements and constraints into account: bandwidth, power, Doppler tolerance, sidelobes, range resolution etc.

General time-invariant filtering

After filtering the received signal $x_{\mathrm{r}}(t)$ through any linear, time-invariant filter $h(\cdot)$ the signal is

$$y(t) = \int_{-\infty}^{\infty} h(t-\alpha)x_{\mathrm{r}}(\alpha)\,\mathrm{d}\alpha$$

With a time delayed received signal $x_{\mathrm{r}}(t) = x(t-t_{\mathrm{d}})$ we have

$$y(t) = \int_{-\infty}^{\infty} h(t-\alpha)x(\alpha-t_{\mathrm{d}})\,\mathrm{d}\alpha$$

The amplitude $|y(t)|$ can be estimated using the Schwartz inequality

$$\begin{aligned}|y(t)| &\leq \left(\int_{-\infty}^{\infty} |h(t-\alpha)|^2\,\mathrm{d}\alpha\right)^{1/2} \cdot \left(\int_{-\infty}^{\infty} |x(\alpha-t_{\mathrm{d}})|^2\,\mathrm{d}\alpha\right)^{1/2} \\ &= (\text{energy of filter})^{1/2} \cdot (\text{energy of signal})^{1/2}\end{aligned}$$

where the values of t or t_{d} do not matter in the last expression.

Matched filter

With knowledge of the transmitted signal $x(t)$, we can choose the matched filter

$$h(t) = x^*(-t)$$

With this particular choice, we have the output

$$y(t) = \int_{-\infty}^{\infty} h(t-\alpha)x(\alpha - t_\mathrm{d})\,\mathrm{d}\alpha = \int_{-\infty}^{\infty} x^*(\alpha - t)x(\alpha - t_\mathrm{d})\,\mathrm{d}\alpha$$

This is maximized at $t = t_\mathrm{d}$ (demonstrating the optimality of the matched filter since the maximum is attained)

$$\max_t |y(t)| = y(t_\mathrm{d}) = \int_{-\infty}^{\infty} |x(\alpha - t_\mathrm{d})|^2\,\mathrm{d}\alpha = \int_{-\infty}^{\infty} |x(\alpha)|^2\,\mathrm{d}\alpha$$

which is proportional to the energy of the pulse waveform $x(t)$.

Matched filter as maximizing SNR

Convolution in time domain corresponds to multiplication in frequency domain, or

$$Y(\omega) = H(\omega)X_{\mathrm{r}}(\omega) = H(\omega)X(\omega)\mathrm{e}^{-\mathrm{j}\omega t_{\mathrm{d}}}$$

$$\Rightarrow \quad y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)X(\omega)\mathrm{e}^{\mathrm{j}\omega(t-t_{\mathrm{d}})}\,\mathrm{d}\omega$$

With white noise $N(\omega) = N_0$, the total received noise power is

$$\overline{n^2(t)} = \frac{N_0}{2\pi}\int_{-\infty}^{\infty} |H(\omega)|^2\,\mathrm{d}\omega$$

Hence the SNR at $t = t_{\mathrm{d}}$ is

$$\mathrm{SNR} = \frac{|y(t_{\mathrm{d}})|^2}{\overline{n^2(t)}} = \frac{|\int_{-\infty}^{\infty} H(\omega)X(\omega)\,\mathrm{d}\omega|^2}{N_0\int_{-\infty}^{\infty}|H(\omega)|^2\,\mathrm{d}\omega}$$

which is maximized for $H(\omega) = X^*(\omega)$ or $h(t) = x^*(-t)$.

▸ Discussion

Example: rectangular pulse

For the simple rectangular pulse (setting $t_\mathrm{d} = 0$)

$$x(t) = A, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

the matched filter is

$$h(t) = A, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

and the filtered response is

$$y(t) = \begin{cases} A^2(\tau + t) & -\tau \le t \le 0 \\ A^2(\tau - t) & 0 \le t \le \tau \end{cases}$$

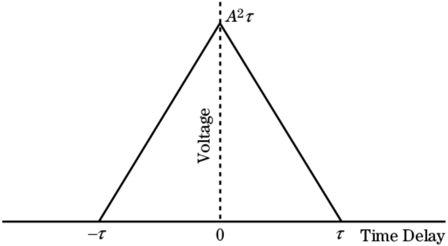
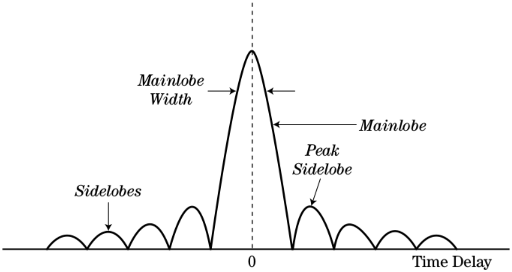


FIGURE 20-2 ■ The simple pulse of duration τ has a match filtered response of duration 2τ.

Generic response

FIGURE 20-3 ■ A generic match filtered response includes the mainlobe and sidelobes.



For general waveforms, the filtered response is typically described in terms of mainlobe and sidelobes.

Outline

Resolution

The Rayleigh resolution criterion is that the peak of one target is at the null of the second target.

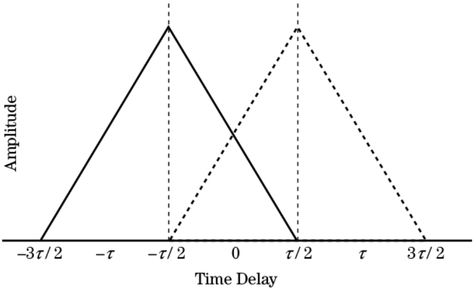


FIGURE 20-4 Individual responses from two point targets separated by the Rayleigh resolution.

The above figure corresponds to the matched filter response of rectangular pulses.

Fourier uncertainty principle

The widths of a signal of zero mean in time and frequency domain can be defined by

$$D_t = \sqrt{\frac{\int_{-\infty}^{\infty} t^2 |y(t)|^2 \, \mathrm{d}t}{\int_{-\infty}^{\infty} |y(t)|^2 \, \mathrm{d}t}}$$

$$D_\omega = \sqrt{\frac{\int_{-\infty}^{\infty} \omega^2 |Y(\omega)|^2 \, \mathrm{d}\omega}{\int_{-\infty}^{\infty} |Y(\omega)|^2 \, \mathrm{d}\omega}}$$

The product of these widths is bounded below as

$$D_t D_\omega \geq \sqrt{\frac{\pi}{2}}$$

with equality for Gaussian signals. This motivates that resolution in time (range) is inversely proportional to frequency bandwidth.

$$\delta R = \kappa \frac{c}{2B}$$

$\kappa \approx 1$, definitions of resolution and bandwidth often chosen to conform with this formula.

Phase difference between two targets

Two targets separated by the Rayleigh resolution can present radically different responses depending on phase difference.

FIGURE 20-5 ■ Combined response for two point targets with phase difference equal to 0°.

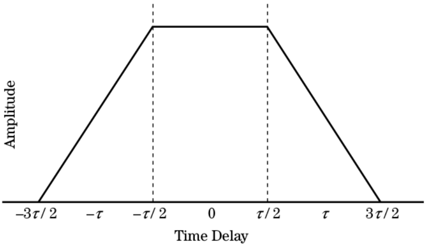
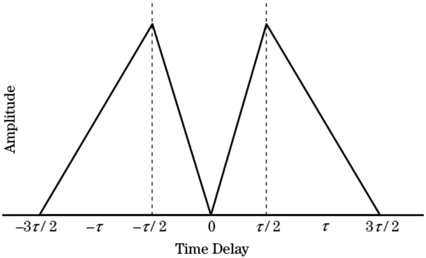


FIGURE 20-6 ■ Combined response for two point targets with phase difference equal to 180°.



Outline

LFM waveform

A baseband linear frequency modulated waveform (LFM) is

$$x(t) = A\cos\left[\pi\tau B\left(\frac{t}{\tau}\right)^2\right], \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

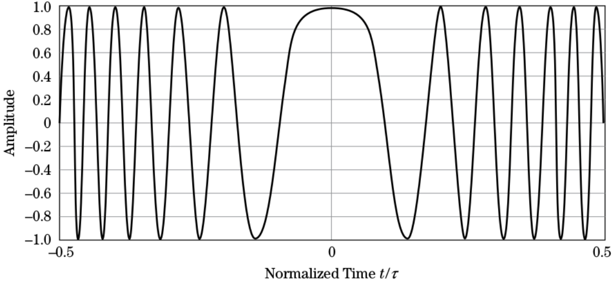


FIGURE 20-7 ■ Time-domain response, within the pulse, of a linear frequency modulated (LFM) waveform with a time-bandwidth product equal to 50.

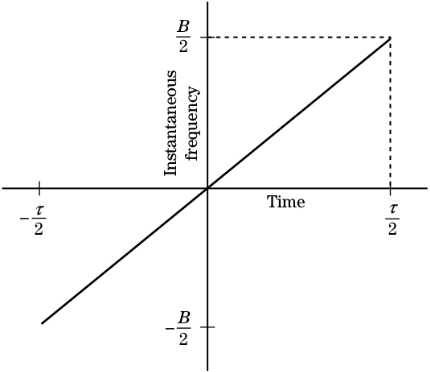
The waveform is characterized by the time-bandwidth product τB and normalized time t/τ.

Instantaneous frequency

The instantaneous phase is $\phi(t) = \pi\tau B(t/\tau)^2$, and instantantaneous frequency is

$$f_{\mathrm{i}}(t) = \frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{B}{\tau}t, \quad -\frac{\tau}{2} \le t \le \frac{\tau}{2}$$

FIGURE 20-8 Instantaneous frequency versus time for an LFM waveform.



The linear change motivates the term linear frequency modulation.

LFM spectrum

The LFM spectrum has a relatively flat spectrum across bandwidth B. Flatness and roll-off improves as time-bandwidth product τB increases.

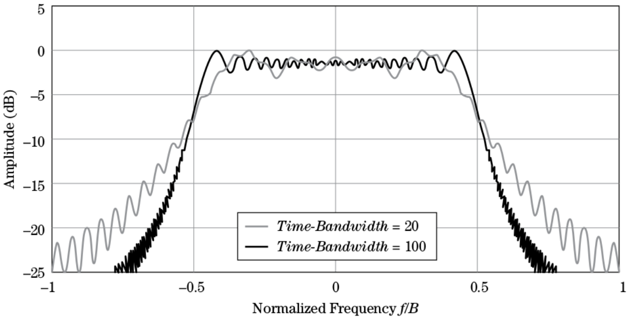


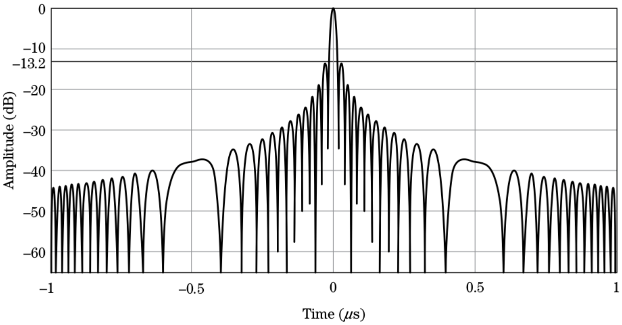
FIGURE 20-9 ■ Comparison of the spectra of LFM waveforms with time-bandwidth products of 20 (light curve) and 100 (dark curve).

Matched filter response

The matched filter response for the LFM waveform is

$$y(t) = \int_{-\infty}^{\infty} x^*(\alpha - t)x(\alpha)\,\mathrm{d}\alpha = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\left(1 - \frac{|t|}{\tau}\right)\pi\tau B \frac{t}{\tau}\right]}{\left(1 - \frac{|t|}{\tau}\right)\pi\tau B \frac{t}{\tau}}, \quad |t| \leq \tau$$

FIGURE 20-10 Match filtered response for a 50 MHz, 1 μsec LFM waveform.



The peak is much more narrow than total pulse width τ!

Discussion

Range resolution

For large values of τB, the first null occurs at $t \approx 1/B$. With range $R = ct/2$, the Rayleigh resolution in range is

$$\delta R = \frac{c}{2B}$$

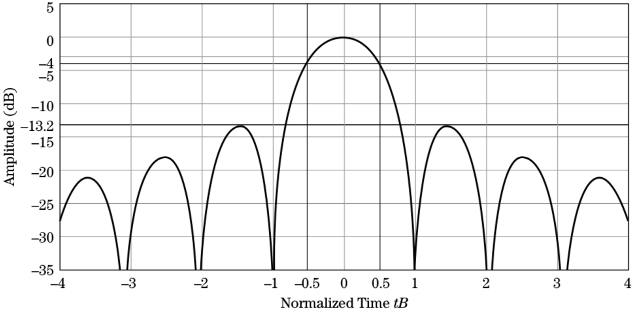


FIGURE 20-11 ■ Mainlobe and first 3 sidelobes for the LFM waveform match filtered response with a time-bandwidth product equal to 100.

The $-4\,\mathrm{dB}$ pulsewidth is $1/B$. The width of the main lobe is compressed by a factor of about $\frac{\tau}{1/B} = \tau B$.

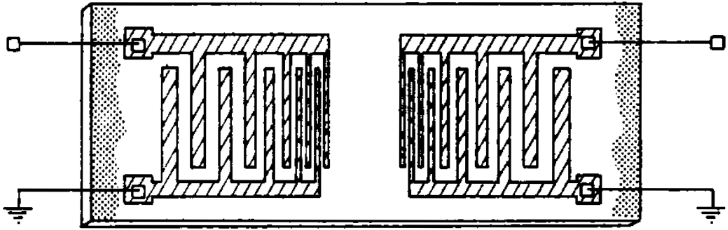
Outline

Dispersive filters

Filters having frequency dependent group delay

$$t_{\mathrm{gd}} = -\frac{\mathrm{d}\phi(\omega)}{\mathrm{d}\omega} = \frac{\tau}{2\pi B}\omega$$

can both stretch and compress waveforms. One implementation is surface acoustic wave (SAW) technology:



The device couples electromagnetic energy to acoustic waves, where the coupling is strongest when the distance between the metal fingers correspond to $\lambda/2$ for the acoustic wave. Chirping is obtained by different acoustic propagation lengths. Works up to about $3\,\mathrm{GHz}$, high insertion loss.

Digital filters

With a digitized signal, the matched signal can be implemented using the Fast Fourier Transform (FFT) of the analytic signal $x[n] = x_{\mathrm{I}}[n] + \mathrm{j}x_{\mathrm{Q}}[n]$:

$$y[n] = \mathrm{FFT}^{-1}\{H[\cdot]\,X[\cdot]\}[n], \quad X[k] = \mathrm{FFT}\{x[\cdot]\}$$

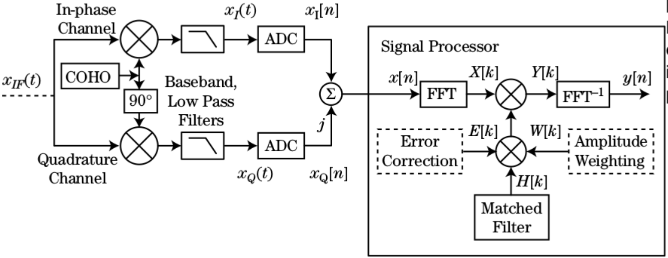


FIGURE 20-12 ■ Fast convolution is one way to implement digital pulse compression.

Error correction is obtained by transmitting a pilot pulse and recording the received (distorted) signal, taking into consideration imperfections in the transmit/receive chain.

Outline

Sidelobe reduction

Sidelobes of the compressed pulse can be reduced by weighting the filter in amplitude. The cost is an increased mainlobe width.

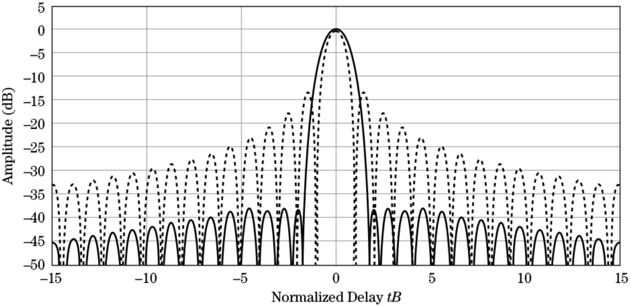


FIGURE 20-13 ■ A -40 dB, $\overline{n} = 4$, Taylor-weighted LFM waveform compressed response (solid curve) has significantly reduced sidelobes versus an unweighted LFM waveform response (dashed curve).

Reduced resolution

When increasing the sidelobe suppression, the resolution is decreased.

TABLE 20-2 ■ 4 dB Resolution Associated with a Taylor Weighting Function

	Peak Sidelobe Ratio (dB)								
	−20	−25	−30	−35	−40	−45	−50	−55	−60
$\bar{n}$	4 dB Resolution Normalized by $c/2B$								
2	1.15	1.19	1.21						
3	1.14	1.22	1.28	1.33					
4	1.12	1.22	1.29	1.36	1.42	1.46			
5	1.11	1.20	1.29	1.36	1.43	1.49	1.54		
6	1.10	1.19	1.28	1.36	1.43	1.50	1.56	1.61	
7	1.09	1.19	1.28	1.36	1.43	1.50	1.56	1.62	1.67
8	1.08	1.18	1.27	1.35	1.43	1.50	1.57	1.63	1.68

Sidelobe suppression, time-bandwidth product

The theoretical sidelobe reduction is achieved when the weighting is applied to a rectangular spectrum. A real LFM has some additional spread, which is reduced as $\tau B \to \infty$.

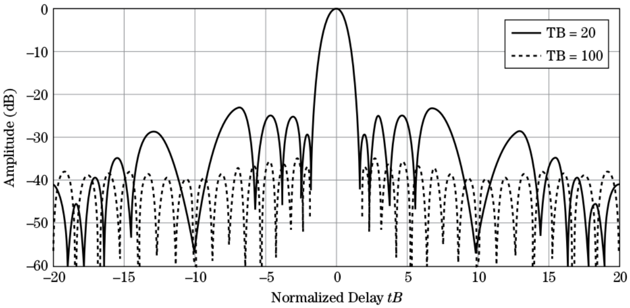
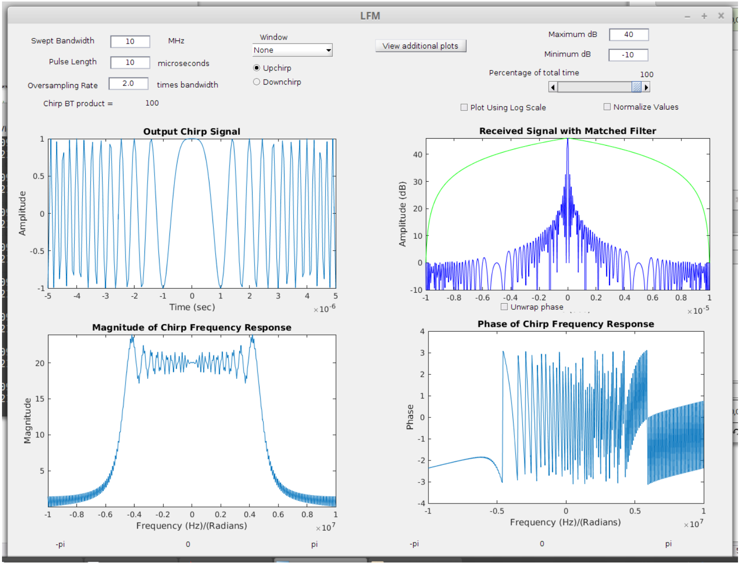


FIGURE 20-14 A comparison of time-sidelobe responses for time-bandwidth products of 20 (solid curve) and 100 (dashed curve) when applying a -40 dB Taylor weighting.

Matlab demo, http://radarsp.com

FRSP Demos/FRSP GUI Demos/FRSP LFM-GUI



Outline

Ambiguity function

Taking into account the possibility of both time delay and Doppler shift, the received signal is

$$x_{\mathrm{r}}(t) = \mathrm{e}^{\mathrm{j}2\pi f_{\mathrm{d}} t} x(t - t_{\mathrm{d}})$$

Centering the waveform over $t_{\mathrm{d}} = 0$ and applying the matched filter and normalizing x with its energy, we find the ambiguity function

$$A(t, f_{\mathrm{d}}) = \frac{\int_{-\infty}^{\infty} x(\alpha) \mathrm{e}^{\mathrm{j}2\pi f_{\mathrm{d}} \alpha} x^*(\alpha - t)\, \mathrm{d}\alpha}{\int_{-\infty}^{\infty} |x(\alpha)|^2 \, \mathrm{d}\alpha}$$

This function satisfies

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A(t, f_{\mathrm{d}})|^2 \, \mathrm{d}t \, \mathrm{d}f_{\mathrm{d}} = 1 \quad \text{and} \quad |A(t, f_{\mathrm{d}})| \leq |A(0,0)| = 1$$

Discussion

Ambiguity for a simple rectangular pulse

For an unmodulated pulse,

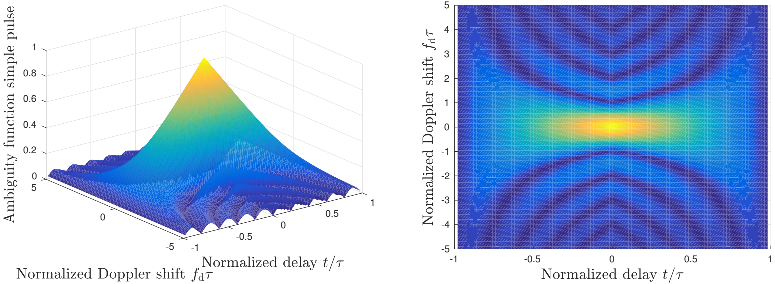
$$x(t) = \frac{1}{\sqrt{\tau}}, \quad -\frac{\tau}{2} \leq t \leq \frac{\tau}{2}$$

the ambiguity function can be calculated as

$$A(t, f_\mathrm{d}) = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\pi f_\mathrm{d} \tau \left(1 - \frac{|t|}{\tau}\right)\right]}{\pi f_\mathrm{d} \tau \left(1 - \frac{|t|}{\tau}\right)}, \quad |t| \leq \tau$$

Depends on normalized time t/τ and normalized Doppler shift $f_\mathrm{d}\tau$.

Ambiguity for a simple rectangular pulse



Similar to Fig 20-15 in the book. Using a matlab script, you can plot the figure in 3D and rotate.

Ambiguity function for LFM waveform

For a linear frequency modulated pulse

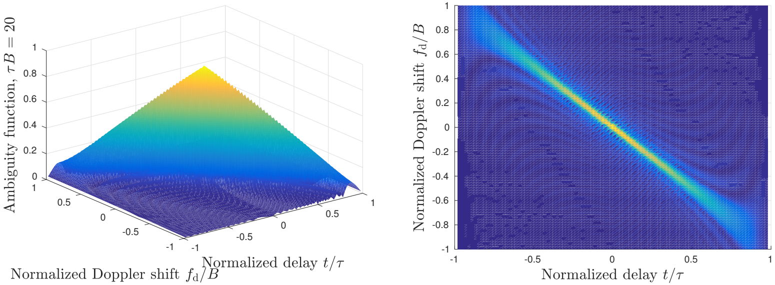
$$x(t) = \frac{1}{\sqrt{\tau}} \exp\left(\mathrm{j}\pi \frac{B}{\tau} t^2\right), \quad |t| \leq \tau$$

the ambiguity function can be calculated as

$$A(t, f_{\mathrm{d}}) = \left| \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\pi\tau B \left(1 - \frac{|t|}{\tau}\right)\left(\frac{f_{\mathrm{d}}}{B} + \frac{t}{\tau}\right)\right]}{\pi\tau B \left(1 - \frac{|t|}{\tau}\right)\left(\frac{f_{\mathrm{d}}}{B} + \frac{t}{\tau}\right)} \right|, \quad |t| \leq \tau$$

Depends on normalized time delay t/τ and normalized Doppler shift f_{d}/B, with time-bandwidth parameter τB.

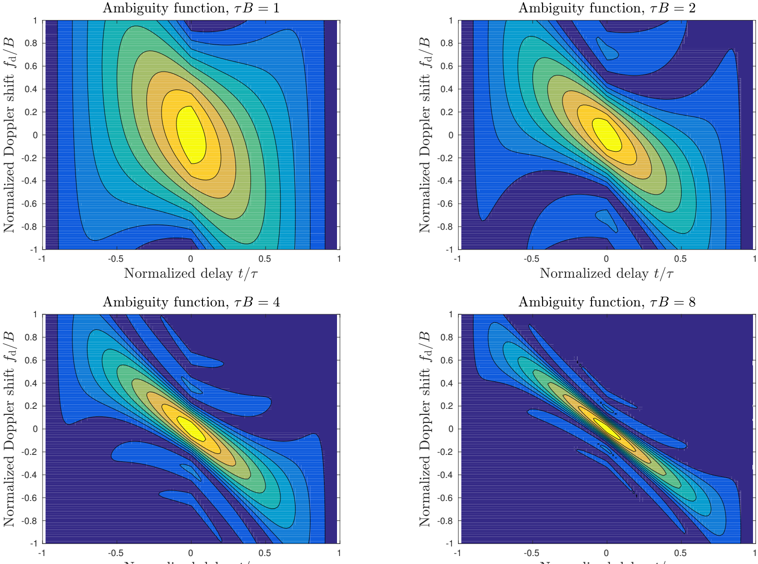
Ambiguity function for LFM waveform, $\tau B = 20$



Similar to Fig 20-16. Can also be plotted with the matlab script in 3D and rotated.

The ridge along the diagonal means delay/range can be mistaken for Doppler shift.

Ambiguity function for LFM waveform, different τB



Range-Doppler coupling

From the formula

$$A(t, f_\mathrm{d}) = \left| \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\pi\tau B \left(1 - \frac{|t|}{\tau}\right)\left(\frac{f_\mathrm{d}}{B} + \frac{t}{\tau}\right)\right]}{\pi\tau B \left(1 - \frac{|t|}{\tau}\right)\left(\frac{f_\mathrm{d}}{B} + \frac{t}{\tau}\right)} \right|, \quad |t| \le \tau$$

we see that a non-zero Doppler shift f_d can be interpreted as

- Time shift $\Delta t = -f_\mathrm{d}\tau/B$
- Amplitude reduction by $(1 - |\Delta t|/\tau) = (1 - |f_\mathrm{d}/B|)$

This leads to shifts in peak location, peak amplitude, and decreased resolution due to peak widening.

Degradations in presence of Doppler shift

FIGURE 20-17 ■ Time shift in the peak of an LFM waveform's match filtered response as a function of Doppler shift.

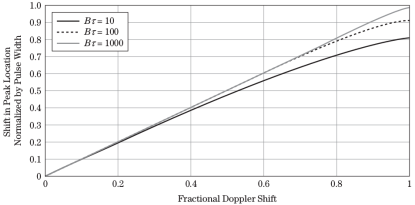
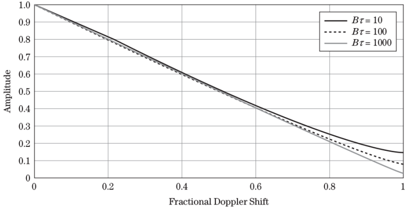


FIGURE 20-18 ■ Reduction in peak amplitude as a function of Doppler shift.



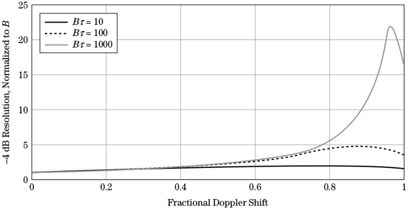


FIGURE 20-19 ■ Increase in −4 dB mainlobe width as a function of Doppler shift.

Matched filter response in presence of Doppler shift

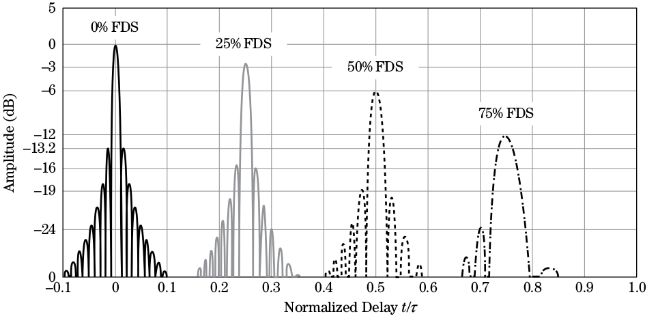


FIGURE 20-20 ■ Individual LFM match filtered responses for fractional Doppler shifts of 0%, 25%, 50%, and 75% illustrate both reduction in peak levels and broadening of the mainlobe.

Degradation of side lobes

The effect of applying an amplitude taper to control sidelobes is reduced in presence of Doppler shift.

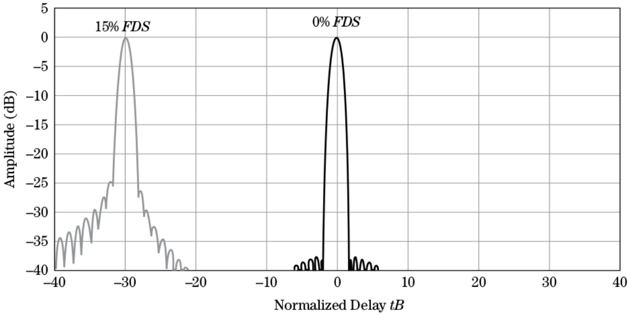


FIGURE 20-21 ■ Match filtered response for a −40 dB Taylor weighted LFM waveform with a time-bandwidth product of 200 and a fractional Doppler shift of 0% (dark curve) and 15% (light curve). Both curves have been normalized to the peak of their responses.

Outline

Phase modulation

Instead of modulating the frequency, the phase can be controlled, typically in a digital way:

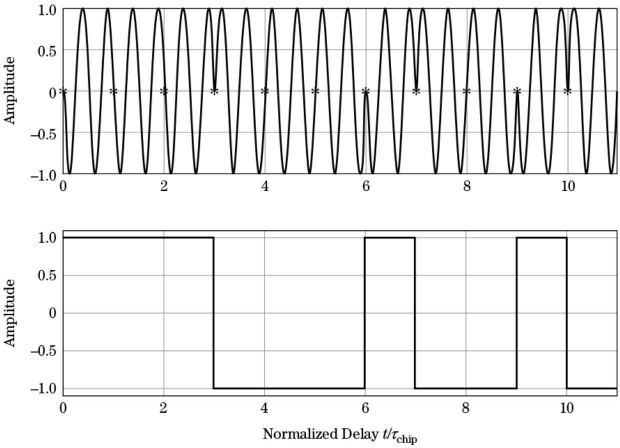


FIGURE 20-25 ■ Baseband (bottom) and RF modulated (top) phase coded waveform of length $N = 11$.

Matched filter

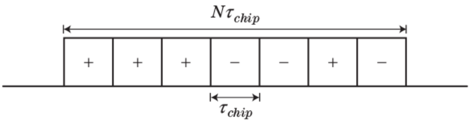


FIGURE 20-23 ■ Biphase coded waveforms consist of chips exhibiting 2 possible phase states.

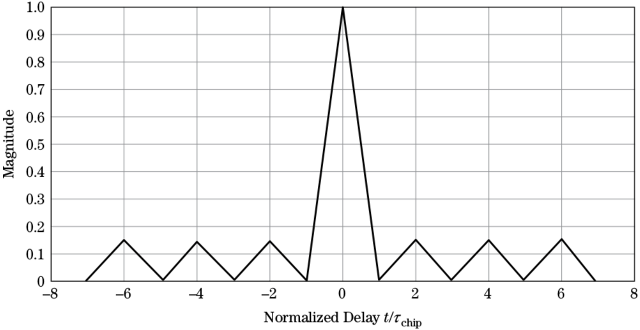


FIGURE 20-24 ■ Match filtered response for the Barker phase coded waveform (Figure 20-23) maintains equal peak sidelobes of level 1/N.

▸ Discussion

Different phase codes

- Biphase codes
  - Two phase states $(+/-)$
  - Minimum peak sidelobes (MPS)
  - Barker codes: achieve a $1 : N$ peak sidelobe to mainlobe ratio
  - Maximum length sequence (MLS): length $\ell = 2^n - 1$, peak sidelobes $\sim 1/\sqrt{\ell}$
- Polyphase codes (not treated in this lecture)
  - More than two phase states: more degrees of freedom
  - Frank, P1, P2, P3, P4

Thumb-tack ambiguity function

Careful design of the phase codes can result in a thumb-tack like ambiguity function.

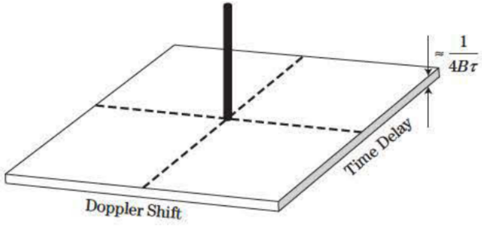


FIGURE 20-29 ■ The ambiguity surface associated with some phase coded waveforms is a thumb tack.

A number of filter banks can be used to search the Doppler space, applying the matched filter at the output of each filter. Enables simultaneous estimation of range and Doppler.

Barker codes

TABLE 20-4 ■ A List of the Known Biphase Barker Codes

Code Length	Code Sequence	Peak Sidelobe Level, dB
2	+−, ++	−6.0
3	++−	−9.5
4	++−+, +++−	−12.0
5	+++−+	−14.0
7	+++−−+−	−16.9
11	+++−−−+−−+−	−20.8
13	+++++−−++−+−+	−22.3

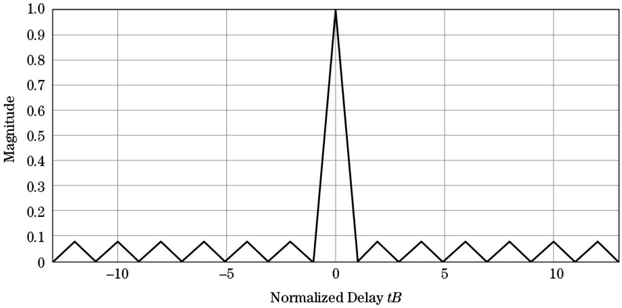


FIGURE 20-30 ■ Compressed response for the longest Barker biphase code: 13 chips.

Minimal peak sidelobe (MPS)

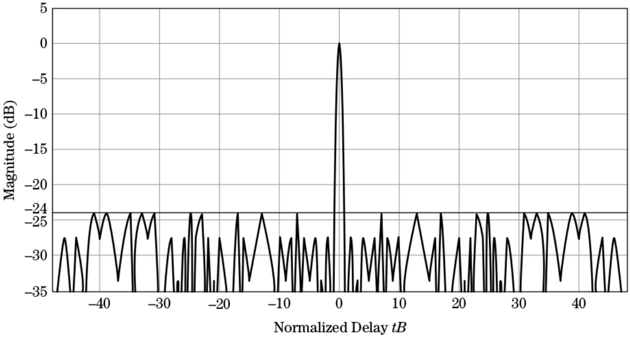


FIGURE 20-31 Compressed response for a 48-length Minimal Peak Sidelobe (MPS) code achieves a 3:48 peak sidelobe ratio.

Examples of minimal peak sidelobe (MPS)...

TABLE 20-6 ■ Example Biphase MPS Codes through Length 105

Code Length	Peak Sidelobe	Number of Codes	Example Code (Hexadecimal)
2	1	1	2
3	1	1	6
4	1	1	D
5	1	1	1D
6	2	4	0B
7	1	1	27
8	2	8	97
9	2	10	D7
10	2	5	167
11	1	1	247
12	2	16	9AF
13	1	1	159F
14	2	9	1483
15	2	13	182B
16	2	10	6877
17	2	4	774B
18	2	2	190F5
19	2	1	5BB8F
20	2	3	5181B
21	2	3	16BB83
22	3	378	E6D5F
23	3	515	38FD49
24	3	858	64AFE3
25	2	1	12540E7
26	3	242	2380AD9
27	3	388	25BBB87
28	2	2	8F1112D
29	3	283	164A80E7
30	3	86	2315240F
31	3	251	2A498C0F
32	3	422	3355A780
33	3	139	CCAA587F
34	3	51	333FE1A55
35	3	111	796AB33
36	3	161	3314A083E
37	3	52	574276F9E
38	3	17	3C34AA66
39	3	30	13350BEF3C
40	3	57	2223DC3A5A
41	3	15	38EA520364
42	3	4	4447B874B4
43	3	12	5B2ACCE1C
44	3	15	FECECB2AD7
45	3	4	2AF0CC6DBF6
46	3	1	3C0CF7B6556
47	3	1	69A7E851988
48	3	4	156B61E64FF3
49	4	Not Reported	012ABEC79E46F
50	4	Not Reported	025863ABC266F
51	3	Not Reported	71C077376ADB4
52	4	Not Reported	0945AE0F3246F
53	4	Not Reported	0132AA7F8D2C6F

TABLE 20-6 ■ *(Continued)*

Code Length	Peak Sidelobe	Number of Codes	Example Code (Hexadecimal)
54	4	Not Reported	0266A2814B3C6F
55	4	Not Reported	04C26AA1E3246F
56	4	Not Reported	099BAACB47BC6F
57	4	Not Reported	01268A8ED623C6F
58	4	Not Reported	023CE545C9ED66F
59	4	Not Reported	049D38128A1DC6F
60	4	Not Reported	0AB8DF0C973252F
61	4	Not Reported	005B44C4C79EA350
62	4	Not Reported	002D66634CB07450
63	4	Not Reported	04CF5A2471657C6F
64	4	1859	55FF84B069386665
65	4	Not Reported	002DC0B0D9BCE5450
66	4	Not Reported	0069B454739F12B42
67	4	Not Reported	007F1D164C62A5242
68	4	Not Reported	009E49E3662A8EA50
69	4	Not Reported	0231C08FDA5A0D9355
70	4	Not Reported	1A133B4E3093EDD57E
71	4	Not Reported	63383AB6B452ED93FE
72	4	Not Reported	E4CD5AF0D054433D82
73	4	Not Reported	1B66B26359C3E2BC00A
74	4	Not Reported	36DDBED681F98C70EAE
75	4	Not Reported	6399C983D03EFDB556D
76	4	Not Reported	DB69891118E2C2A1FA0
77	4	Not Reported	1961AE251DC950FDDBF4
78	4	Not Reported	328B457F0461E4ED7B73
79	4	Not Reported	76CF68F327438AC6FA80
80	4	Not Reported	CE43C8D986ED429F7D75
81	4	Not Reported	0E3C32FA1FEFD2519AB32
82	4	Not Reported	3CB25D380CE3B7765695F
83	5	Not Reported	711763AE7DBB8482D3A5A
84	5	Not Reported	CE79CCCDB6003C1E95AAA
85	5	Not Reported	19900199463E51E8B4B574
86	5	Not Reported	3603FB659181A2A52A38C7
87	5	Not Reported	7F7184F04F4E5E4D9B56AA
88	5	Not Reported	D54A9326C2C686F86F3880
89	5	Not Reported	180E09434E1BBC44ACDAC8A
90	5	Not Reported	3326D87C3A91DA8AFA84211
91	5	Not Reported	77F80E632661C3459492A55
92	5	Not Reported	CC6181859D9244A5EAA87F0
93	5	Not Reported	187B2ECB802FB4F56BCCECE5
94	5	Not Reported	319D9676CAFEADD68825F878
95	5	Not Reported	69566B2ACCC8BC3CE0DE0005
96	5	Not Reported	CF963FD09B1381657A8A098E
97	5	Not Reported	1A843DC410898B2D3AE8FC362
98	5	Not Reported	30E05C18A1525596DCCE600DF
99	5	Not Reported	72E6DB6A75E6A9E81F0846777
100	5	Not Reported	DF490FFB1F8390A54E3CD9AAE
101	5	Not Reported	1A5048216CCF18F83E910DD4C5
102	5	Not Reported	2945A4F11CE44FF664850D182A
103	5	Not Reported	77FAA82C6E065AC4BE18F274CB
104	5	Not Reported	E568ED4982F9660EBA2F611184
105	5	Not Reported	1C6387FF5DA4FA325C895958DC5

Sources: Compiled from [19–24].

Maximal length sequences

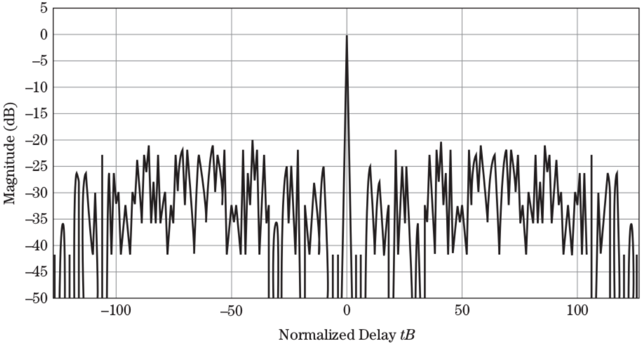


FIGURE 20-33 The compressed response for a 127-length MLS.

Comparison LFM and biphase MLS, waveform

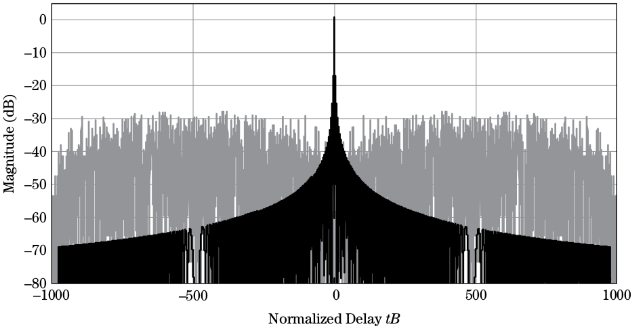
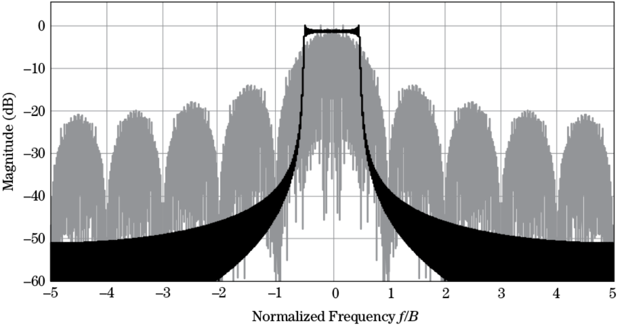


FIGURE 20-34 Comparison of a compressed LFM waveform (black curve) (TB = 1000) with a compressed biphase MLS coded waveform (gray curve) (TB = 1023).

Higher average sidelobe levels in MLS, higher peak sidelobe in LFM.

Comparison LFM and biphase MLS, spectrum

FIGURE 20-35 ■ Comparison of the spectra of an LFM waveform (black curve) with a 1023-length MLS coded waveform (gray curve).



The wide spectrum of MLS can be attributed to the abrupt changes in phase. Can create an electromagnetic interference problem.

Outline

Conclusions

- Matched filters maximize SNR for a given waveform
- The resulting pulse compression improves range resolution
- The LFM is a generic waveform, sidelobes can be improved by tapering
- Phase coding can produce very narrow ambiguity peaks, but with a wide spectrum

Discussion

Why is

$$\mathrm{SNR} = \frac{|y(t_\mathrm{d})|^2}{\overline{n^2(t)}} = \frac{|\int_{-\infty}^{\infty} H(\omega) X(\omega)\,\mathrm{d}\omega|^2}{N_0 \int_{-\infty}^{\infty} |H(\omega)|^2\,\mathrm{d}\omega}$$

maximized by $H(\omega) = X^*(\omega)$?

Go back

Discussion

Why is

$$\mathrm{SNR} = \frac{|y(t_\mathrm{d})|^2}{\overline{n^2(t)}} = \frac{|\int_{-\infty}^{\infty} H(\omega)X(\omega)\,\mathrm{d}\omega|^2}{N_0 \int_{-\infty}^{\infty} |H(\omega)|^2\,\mathrm{d}\omega}$$

maximized by $H(\omega) = X^*(\omega)$?

Answer: The Schwartz inequality

$$\left|\int_{-\infty}^{\infty} H(\omega)X(\omega)\,\mathrm{d}\omega\right|^2 \leq \left(\int_{-\infty}^{\infty} |H(\omega)|^2\,\mathrm{d}\omega\right) \cdot \left(\int_{-\infty}^{\infty} |X(\omega)|^2\,\mathrm{d}\omega\right)$$

implies

$$\mathrm{SNR} \leq \frac{1}{N_0}\int_{-\infty}^{\infty} |X(\omega)|^2\,\mathrm{d}\omega$$

with equality for the choice $H(\omega) = X^*(\omega)$.

◂ Go back

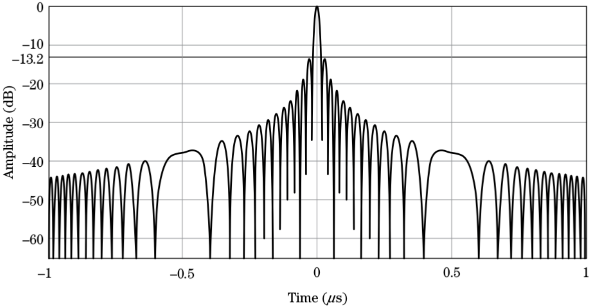
Discussion

Given the matched filter response for the LFM waveform is

$$y(t) = \int_{-\infty}^{\infty} x^*(\alpha - t)x(\alpha)\,\mathrm{d}\alpha = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\left(1 - \frac{|t|}{\tau}\right)\pi\tau B \frac{t}{\tau}\right]}{\left(1 - \frac{|t|}{\tau}\right)\pi\tau B \frac{t}{\tau}}, \quad |t| \leq \tau$$

where in the graph below can you find the value of $\pi\tau B$ in dB?

FIGURE 20-10 ■
Match filtered response for a 50 MHz, 1 μsec LFM waveform.



Go back

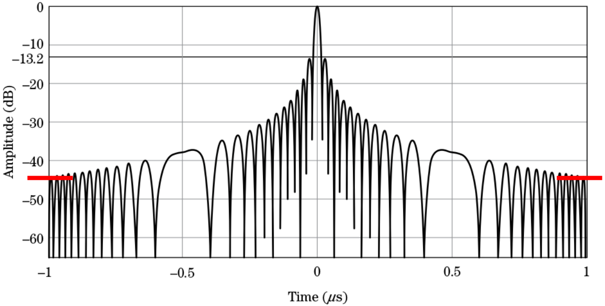
Discussion

Given the matched filter response for the LFM waveform is

$$y(t) = \int_{-\infty}^{\infty} x^*(\alpha - t)x(\alpha)\,\mathrm{d}\alpha = \left(1 - \frac{|t|}{\tau}\right) \frac{\sin\left[\left(1 - \frac{|t|}{\tau}\right)\pi\tau B\frac{t}{\tau}\right]}{\left(1 - \frac{|t|}{\tau}\right)\pi\tau B\frac{t}{\tau}}, \quad |t| \leq \tau$$

where in the graph below can you find the value of $\pi\tau B$ in dB?

FIGURE 20-10 ■ Match filtered response for a 50 MHz, 1 μsec LFM waveform.



Answer: For $|t/\tau| \approx 1$, we have $|y(t)| \approx 1/(\pi\tau B) \approx -44\,\mathrm{dB}$ in this graph.

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Discussion

The ambiguity function $A(t, f_\mathrm{d}) = \frac{\int_{-\infty}^{\infty} x(\alpha) \mathrm{e}^{\mathrm{j}2\pi f_\mathrm{d} \alpha} x^*(\alpha - t)\, \mathrm{d}\alpha}{\int_{-\infty}^{\infty} |x(\alpha)|^2 \, \mathrm{d}\alpha}$ satisfies

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |A(t, f_\mathrm{d})|^2 \, \mathrm{d}t \, \mathrm{d}f_\mathrm{d} = 1 \quad \text{and} \quad |A(t, f_\mathrm{d})| \leq |A(0,0)| = 1$$

Given the above and a pulse shape with finite support,

$$|A(t, f_\mathrm{d})|^2 = 0 \quad \text{when } |t| > \tau \text{ and } |f_\mathrm{d}| > B$$

what average value of $|A(t, f_\mathrm{d})|^2$ do you expect for $|t| < \tau$ and $|f_\mathrm{d}| < B$? How should τ be chosen to minimize the average ambiguity?

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Discussion

The ambiguity function $A(t, f_\mathrm{d}) = \frac{\int_{-\infty}^{\infty} x(\alpha) e^{j2\pi f_\mathrm{d}\alpha} x^*(\alpha - t)\,\mathrm{d}\alpha}{\int_{-\infty}^{\infty} |x(\alpha)|^2\,\mathrm{d}\alpha}$ satisfies

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |A(t, f_\mathrm{d})|^2\,\mathrm{d}t\,\mathrm{d}f_\mathrm{d} = 1 \quad \text{and} \quad |A(t, f_\mathrm{d})| \leq |A(0,0)| = 1$$

Given the above and a pulse shape with finite support,

$$|A(t, f_\mathrm{d})|^2 = 0 \quad \text{when } |t| > \tau \text{ and } |f_\mathrm{d}| > B$$

what average value of $|A(t, f_\mathrm{d})|^2$ do you expect for $|t| < \tau$ and $|f_\mathrm{d}| < B$? How should τ be chosen to minimize the average ambiguity?

Answer: Since the integral of $|A|^2$ equals 1, we should have the average value $\langle |A|^2 \rangle = 1/(4\tau B)$. To minimize this, increase the pulse length τ.

Interpretation: If you try to decrease $|A|^2$ somewhere, it needs to increase somewhere else.

Go back

Discussion

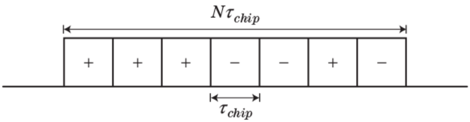

FIGURE 20-23 ■ Biphase coded waveforms consist of chips exhibiting 2 possible phase states.

With each chip consisting of a fixed frequency carrier wave $\cos(2\pi f_0 t + \phi)$, with pulse length τ_{chip}, what is the chip bandwidth B_{chip} and time-bandwidth product τB with $\tau = N\tau_{\text{chip}}$ and $B = B_{\text{chip}}$?

◂ Go back

Discussion

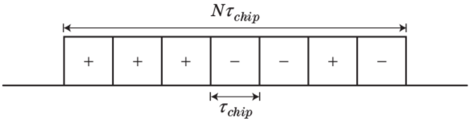

FIGURE 20-23 ■ Biphase coded waveforms consist of chips exhibiting 2 possible phase states.

With each chip consisting of a fixed frequency carrier wave $\cos(2\pi f_0 t + \phi)$, with pulse length τ_{chip}, what is the chip bandwidth B_{chip} and time-bandwidth product τB with $\tau = N\tau_{\text{chip}}$ and $B = B_{\text{chip}}$?

Answer: $B_{\text{chip}} = 1/\tau_{\text{chip}}$, $\tau B = N\tau_{\text{chip}} B_{\text{chip}} = N$. Hence, the time-bandwidth product only depends on the number of chips, N.

◂ Go back