EITN90 Radar and Remote Sensing
Lecture 6: Target fluctuation models

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1 Radar cross section of simple targets

2 Radar cross section of complex targets

3 Statistical characteristics of the RCS of complex targets

4 Target fluctuation models

5 Doppler spectrum of fluctuating targets

6 Conclusions
Learning outcomes of this lecture

In this lecture we will

- Learn the origin of fluctuations in target RCS
- Describe the statistics of the fluctuations
- Characterize RCS decorrelation lengths
- Understand the basic assumptions of the Swerling fluctuation models
Outline

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Sphere scattering

The scattering from a sphere can be computed analytically. This gives very precise results, and spheres are often used as calibrating targets at measurement sites.

**FIGURE 7-1**
Examples of RCS calibration spheres. (Courtesy of Professor Nadav Levanon, Tel-Aviv University.)
Sphere scattering

A sphere has the same backscattering in all directions due to symmetry.

\[ \sigma / m^2 \]

\[ k \alpha = \frac{2 \pi \alpha}{\lambda} \]

Sphere Circumference in Wavelengths

FIGURE 6-12 Sphere scattering from Rayleigh, resonance, and optics regions, [1].
Corner reflectors

A corner reflector has a strong backscattering in a large angular interval. Dihedral = two walls, trihedral = three walls.

FIGURE 7-3
Corner reflectors.
(a) Dihedrals.
(b) Trihedrals.
(Courtesy of Professor Nadav Levanon, Tel-Aviv University.)
When several scatterers are subjected to an incident wave $E_0 e^{-j k \cdot R}$, the backscattering is (complex addition)

$$\sigma_{tot} = \left| \sum_{i=1}^{N} \sqrt{\sigma_i} e^{-j 2 k \cdot R_i} \right|^2$$
Two scatterers

\[ \sigma_{\text{tot}} = \left| \sum_{i=1}^{N} \sqrt{\sigma_i} e^{-j2k \cdot R_i} \right|^2 = \left| \sqrt{\sigma_0} e^{-j2k \frac{D}{2} \sin \theta} + \sqrt{\sigma_0} e^{-j2k (-\frac{D}{2} \sin \theta)} \right|^2 \]

\[ = 4\sigma_0 \cos^2 \left( \frac{2\pi D \sin \theta}{\lambda} \right) \]
Two scatterers, RCS as function of angle

FIGURE 7-5 —
Relative radar cross section of the dumbbell target of Figure 7-4 when $D = 5\lambda$ and $R = 10,000 \, D$.

FIGURE 7-6 —
Polar plot of the data of Figure 7-5.
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Randomly distributed targets

Put out a number of randomly distributed targets.

With \( R_i \) a random variable, the RCS

\[
\sigma = \left| \sum_{i=1}^{N} \sqrt{\sigma_i} e^{-j2k \cdot R_i} \right|^2
\]

also becomes a random variable, with some probability density function as angle or frequency varies. However, once the scatterers are fixed the model (including \( \sigma \)) is completely deterministic.
Randomly distributed targets

The RCS as function of angle appears as a random variable, with some probability density.
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Equal scatterers

A large number of approximately equal scatterers with uniformly distributed phase have exponentially distributed RCS:

\[
p(\sigma) = \begin{cases} 
\frac{1}{\bar{\sigma}} \exp \left[ -\frac{\sigma}{\bar{\sigma}} \right] & \sigma \geq 0 \\
0 & \sigma < 0 
\end{cases}
\]

where \( \bar{\sigma} \) is the mean. The amplitude voltage \( (\varsigma = \sqrt{\sigma}) \) is Rayleigh distributed:

\[
p(\varsigma) = \begin{cases} 
\frac{2\varsigma}{\bar{\sigma}} \exp \left[ -\frac{\varsigma^2}{\bar{\sigma}} \right] & \varsigma \geq 0 \\
0 & \varsigma < 0 
\end{cases}
\]

For any RCS distribution \( p_\sigma(\sigma) \), the corresponding amplitude distribution \( p_\varsigma(\varsigma) \) is given by \( p_\varsigma(\varsigma) = 2\varsigma p_\sigma(\varsigma^2) \). This is seen from

\[
1 = \int p_\sigma(\sigma) \, d\sigma \quad \overset{\sigma = \varsigma^2}{\Rightarrow} \quad \int p_\sigma(\varsigma^2) 2\varsigma \, d\varsigma
\]

\[= p_\varsigma(\varsigma) \]
One dominating scatterer

When the situation has one dominating and many small scatterers, the RCS can be modeled by a fourth-degree chi-square distribution (easier to handle analytically than the true Rician distribution):

\[
p(\sigma) = \begin{cases} 
\frac{4\sigma}{\sigma^2} \exp \left[ -\frac{2\sigma}{\sigma} \right] & \sigma \geq 0 \\
0 & \sigma < 0 
\end{cases}
\]

**FIGURE 7-10** Comparison of a fourth-degree chi-square PDF and the histogram of linear-scale RCS data for one dominant scatterer with many small scatterers. See text for details.
<table>
<thead>
<tr>
<th>Model Name</th>
<th>PDF for RCS $\sigma$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nonfluctuating, Marcum,</td>
<td>$p(\sigma) = \delta_D (\sigma - \bar{\sigma})$</td>
<td>Constant echo power, e.g. calibration sphere or perfectly stationary reflector with no radar or target motion.</td>
</tr>
<tr>
<td>Swerling 0, or Swerling 5</td>
<td>$\text{var}(\sigma) = 0$</td>
<td></td>
</tr>
<tr>
<td>Exponential (chi-square of</td>
<td>$p(\sigma) = \frac{1}{\bar{\sigma}} \exp \left[ -\frac{\sigma}{\bar{\sigma}} \right]$</td>
<td>Many scatterers, randomly distributed, none dominant. Used in Swerling case 1 and 2 models.</td>
</tr>
<tr>
<td>degree 2)</td>
<td>$\text{var}(\sigma) = \bar{\sigma}^2$</td>
<td></td>
</tr>
<tr>
<td>Chi-square of degree 4</td>
<td>$p(\sigma) = \frac{4\sigma}{\bar{\sigma}^2} \exp \left[ -2\frac{\sigma}{\bar{\sigma}} \right]$</td>
<td>Approximation to case of many small scatterers + one dominant, with RCS of dominant equal to $1 + \sqrt{2}$ times the sum of RCS of others. Used in Swerling case 3 and 4 models.</td>
</tr>
<tr>
<td></td>
<td>$\text{var}(\sigma) = \frac{\bar{\sigma}^2}{2}$</td>
<td></td>
</tr>
<tr>
<td>Chi-square of</td>
<td>$p(\sigma) = \frac{m}{\Gamma(m)\bar{\sigma}} \left[ \frac{m\sigma}{\bar{\sigma}} \right]^{m-1} \exp \left[ -\frac{m\sigma}{\bar{\sigma}} \right]$</td>
<td>Generalization of the two preceding cases. Weinstock cases correspond to $0.6 \leq 2m \leq 4$. Higher degrees correspond to presence of a more dominant single scatterer.</td>
</tr>
<tr>
<td>degree 2m, Weinstock</td>
<td>$\text{var}(\sigma) = \frac{\bar{\sigma}^2}{m}$</td>
<td></td>
</tr>
<tr>
<td>Weibull</td>
<td>$p(\sigma) = CB\sigma^{C-1} \exp \left[ -B\sigma^C \right]$</td>
<td>Empirical fit to many measured target and clutter distributions. Can have longer “tail” than previous cases.</td>
</tr>
<tr>
<td></td>
<td>$\bar{\sigma} = \Gamma \left(1 + 1/C\right) B^{-1/C}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{var}(\sigma) = B^{-2/C} \left[ \Gamma \left(1 + 2/C\right) - \Gamma^2 \left(1 + 1/C\right) \right]$</td>
<td></td>
</tr>
<tr>
<td>Log-normal</td>
<td>$p(\sigma) = \frac{1}{\sqrt{2\pi} s\sigma} \exp \left[ -\ln^2 \left(\sigma/\sigma_m\right)/2s^2 \right]$</td>
<td>Empirical fit to many measured target and clutter distributions. “Tail” is longest of previous cases. $\sigma_m$ is the median value of $\sigma$.</td>
</tr>
<tr>
<td></td>
<td>$\bar{\sigma} = \sigma_m \exp \left(s^2/2\right)$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\text{var}(\sigma) = \sigma_m^2 \exp \left(s^2\right) \left[ \exp \left(s^2\right) - 1 \right]$</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 7-12
Comparison of five models for the probability density function of radar cross section with the same mean (except for the exponential) and variance. See text for additional details. (a) Linear scale. (b) Log scale, showing detail of the PDF “tails.”
Matlab demo (see http://radarsp.com)
RCS decorrelation

Decorrelation occurs when the observation of RCS is significantly changed by an alteration of time, frequency, or angle. To estimate when this occurs, consider the situation below.

\[ \sigma = \left| \sum_{i=-M}^{M} \sqrt{\sigma_i} e^{-j2k \cdot R_i} \right|^2 = \sigma_0 \left| \sum_{i=-M}^{M} e^{-j2ki \Delta x \sin \theta} \right|^2 = \sigma_0 \frac{\sin^2[(2M + 1)k\Delta x \sin \theta]}{\sin^2(k\Delta x \sin \theta)} \]
RCS decorrelation

Introduce the total length $L = (2M + 1)\Delta x$ of the collection of scatterers:

$$\sigma = \sigma_0 \frac{\sin^2(kL \sin \theta)}{\sin^2(k\Delta x \sin \theta)}$$

The first zero corresponds to $kL \sin \theta = \pi$, implying the correlation lengths (using $k = 2\pi/\lambda = 2\pi f/c$ and $\sin \theta \approx \theta$ for small $\theta$)

$$\Delta f = \frac{c}{2L \sin \theta} \quad \text{(frequency decorrelation)}$$

$$\Delta \theta = \frac{\lambda}{2L} \quad \text{(angle decorrelation)}$$

$L \sin \theta$ is the projected length of the target in boresight.

The book derives these values differently (from an autocorrelation function), but the results are the same. However, different definitions of decorrelation may apply in different applications.
RCS correlation properties, width of target

Autocorrelation in angle for a rectangular target $5\text{ m} \times 10\text{ m}$. Note that a wider target has shorter correlation length.

**FIGURE 7-14**
Decorrelation in angle of RCS of target from Figure 7-8. See text for details.
RCS correlation properties, frequency agility

Variation in RCS due to frequency agility for $5 \text{ m} \times 10 \text{ m}$ target.

Frequency agility can be used to decorrelate contributions from clutter and atmospheric effects.
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Detection framework

A radar with azimuth beamwidth $\theta_3$ and pulse repetition frequency PRF is rotating at angular speed $\Omega \, \text{rad/s}$. This provides

$$N = \frac{\theta_3}{\Omega} \text{PRF}$$

main beam samples per $360^\circ$ sweep
Swerling models

Extreme cases for decorrelation. The non-fluctuating case is sometimes listed as the Swerling 0 or Swerling 5 case.

**TABLE 7-3 ▶ Swerling Models**

<table>
<thead>
<tr>
<th>Probability Density Function of RCS</th>
<th>Decorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Scan-to-Scan</td>
</tr>
<tr>
<td>Exponential</td>
<td>Case 1</td>
</tr>
<tr>
<td>Chi-square, degree 4</td>
<td>Case 3</td>
</tr>
</tbody>
</table>

- The exponential distribution models many equal scatterers.
- Chi-square degree 4 models one dominant scatterer surrounded by many small.
- Scan-to-scan decorrelation may occur for long scan times.
- Pulse-to-pulse decorrelation may occur for frequency agile systems.
Different decorrelations, scan or pulse

**FIGURE 7-17**
Notional sequences of Swerling target samples. Results from three scans with 10 pulses per scan are shown.
(a) Swerling case 1.
(b) Swerling case 4.
Example: choice of model

Consider the following scenario:

- **Target:** 10 m long, complex aircraft.
- **Frequency:** 10 GHz.
- **Range:** 30 km.
- **Cross-range velocity:** 200 m/s.

Choice of model:

- The complex nature of the aircraft with no clear dominating scatterer suggests an exponential PDF.
- Decorrelation angle: \( \Delta \theta = \frac{\lambda}{2L} = 1.5 \text{ mrad} = 0.86^\circ \). This corresponds to a flight time \( \Delta t = R\Delta \theta/v = 225 \text{ ms} \).
- If dwell time \( T_d < 225 \text{ ms} \) we expect coherence during pulse-to-pulse and choose Swerling 1.
- If dwell time \( T_d > 225 \text{ ms} \) the signal is decorrelated within the PRI, and we choose Swerling 2.
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Doppler shift

A target traveling at radial velocity $v \cos \psi$ towards the radar causes a frequency shift $f \rightarrow f + f_d$. After demodulation (removal of carrier frequency $f$), the baseband signal will ideally be

$$y[m] = Ae^{j2\pi f_dmT}$$

However, amplitude fluctuations may change this.

**FIGURE 8-1**

Doppler shift is determined by the radial component of relative velocity between the target and radar.
Doppler spectrum of fluctuating targets

**FIGURE 7-18**
Effect of amplitude fluctuations on target Doppler spectrum.
(a) 20-pulse Rayleigh fluctuating amplitude sequence.
(b) Spectrum of the fluctuating and nonfluctuating data.

More on Doppler processing in next lecture!
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Conclusions

- Interference causes fluctuations in RCS with viewing angle and frequency, which may be modeled as random.
- Typical probability densities have been reviewed.
- An interactive matlab program is available for investigating effects of number of scatterers, frequency, range etc.
- Decorrelations in angle and frequency have been quantified.
- The bounding cases of Swerling models have been presented.