Wireless Communications Channels
Lecture 3: Fading

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1

Fading – Statistical description of the wireless channel

- ❑ **Why statistical description**
- ❑ **Large scale fading**
- ❑ **Small scale fading:**
  - ❑ without dominant component
  - ❑ with dominant component
- ❑ **Statistical models**

2

Why "statistical" description?

- ❑ Complex, unknown environment
- ❑ Can not describe everything in detail
  - ❑ Maxwell's equations far too complex in real scenarios
- ❑ Large variations depending on the TX, RX and interacting object locations
- ❑ **Need a statistical measure since we can not describe every point everywhere**

"There is a x% probability that the amplitude/power will be above the level y"

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The WSSUS model: Assumptions

A very common channel model is the WSSUS-model: Statistical properties remain the same over the considered time (or area)

Recall: the channel is composed of a number of different contributions (incoming waves), the following is assumed:

- ❑ The channel is **Wide-Sense Stationary (WSS),** meaning that the **correlation** of the channel is **invariant** over time.
- ❑ The channel is built up by **Uncorrelated Scatterers (US), meaning** that the **frequency** correlation of the channels is **invariant** over frequency. (Contributions with different delays are uncorrelated.)

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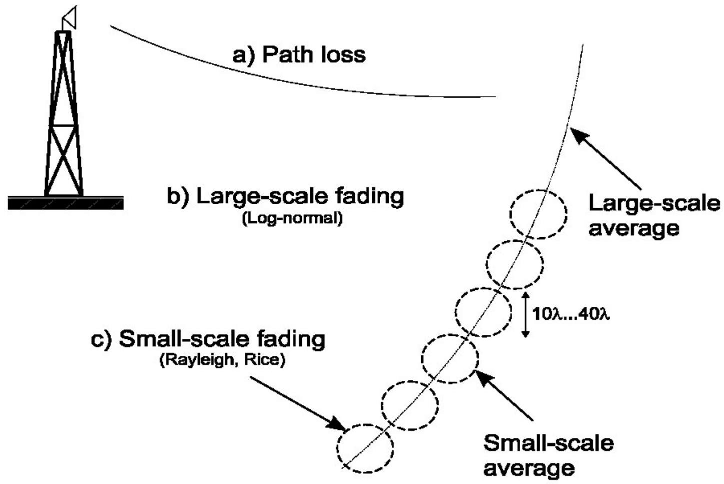
What is large scale and small scale?



Wireless Communication Channels 5

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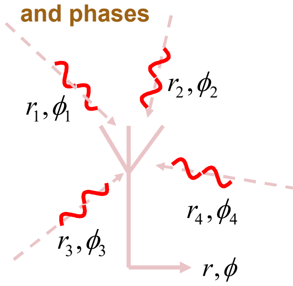
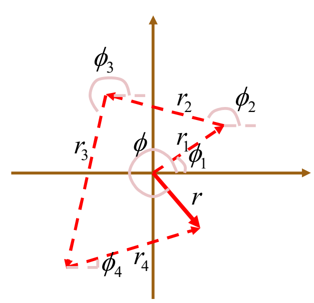
5

Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases

Add them up as phasors





$$r\exp(j\phi) = r_1\exp(j\phi_1) + r_2\exp(j\phi_2) + r_3\exp(j\phi_3) + r_4\exp(j\phi_4)$$

Wireless Communication Channels 6

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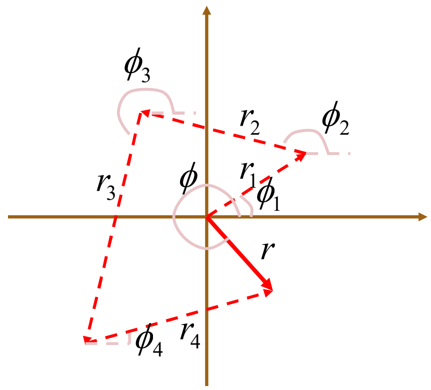
6

Small-scale fading
Many incoming waves

Re and Im components are sums of many independent equally distributed components

$$\mathrm{Re}(r) \in N(0, \sigma^2)$$

***Re(r)* and *Im(r)* are independent**



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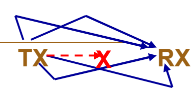
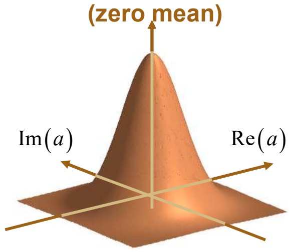
7

Small-scale fading
Rayleigh fading



No dominant component (non line-of-sight)

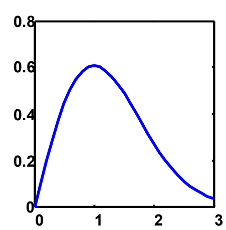
Tap distribution
2D Gaussian
(zero mean)



$r = |a|$

Amplitude distribution

Rayleigh



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

No line-of-sight component

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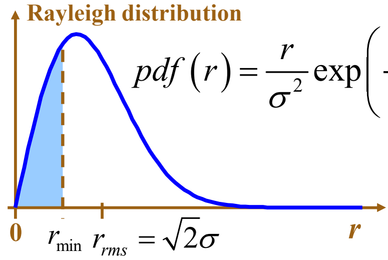
8

Small-scale fading
Rayleigh fading



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Probability that the amplitude is below some threshold $r_{\min}$:

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right)$$

Small-scale fading
Rayleigh fading – outage probability

- What is the probability that we will receive an amplitude 20 dB below the r_{rms}?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1 - \exp(-0.01) \approx 0.01$$

- What is the probability that we will receive an amplitude below r_{rms}?

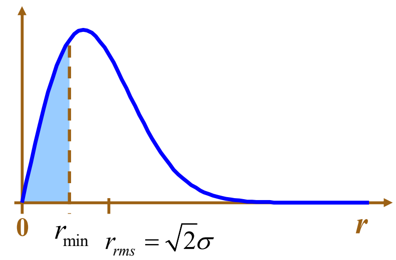
$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1 - \exp(-1) \approx 0.63$$

Small-scale fading
Rayleigh fading – fading margin

To ensure that in most cases we **receive** enough power, we transmit **extra** power by including the so-called "fading margin"

$$M = \frac{r_{rms}^2}{r_{\min}^2}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^2}{r_{\min}^2}\right)$$



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11

Small-scale fading
Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln\left(0.99\right) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

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12

Small-scale fading
Rayleigh fading – signal and interference

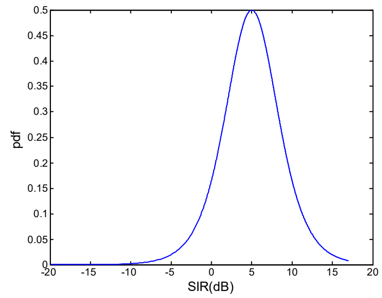
Both the desired signal and the interference undergo fading

For a single user inteferer and Rayleigh fading:

$$pdf_{SIR}(r) = \frac{2\overline{\sigma}^2 r}{(\overline{\sigma}^2 + r^2)^2}$$

$$cdf_{SIR}(r) = 1 - \frac{\overline{\sigma}^2 r}{(\overline{\sigma}^2 + r^2)}$$

where $\overline{\sigma}^2 = \frac{\sigma_2^2}{\sigma_1^2}$ is the mean signal to interference ratio



pdf for 10 dB mean signal to interference ratio

Small-scale fading
Rayleigh fading – signal and interference

What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 10 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\overline{\sigma}^2 r_{\min}}{(\overline{\sigma}^2 + r_{\min}^2)} = 1 - \frac{10}{(10+1)} \approx 0.09$$

Small-scale fading: One dominating component

In case of Line-of-Sight (LOS) one component dominates.

- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A,\sigma^2) \quad \mathrm{Im}(r) \in N(0,\sigma^2)$$

- The recieved amplitude has now a Ricean distribution instead of a Rayleigh
  - The fluctuations are smaller
  - The phase is dominated by the LOS component
  - In real cases the mean propagation loss is often smaller due to the LOS
- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

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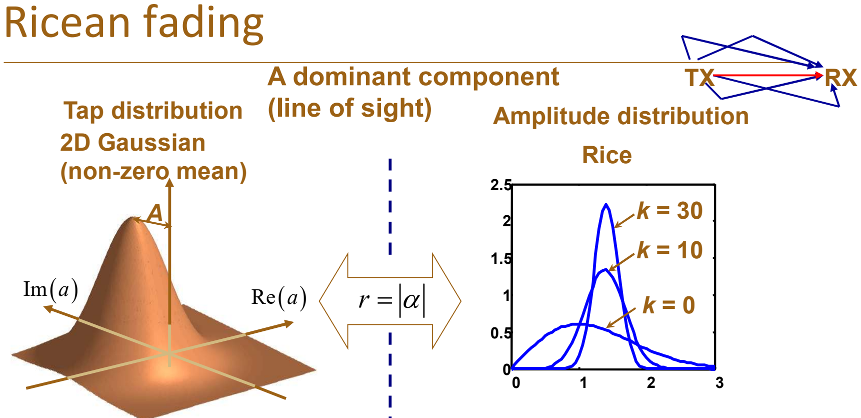
Small-scale fading Ricean fading



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

Line-of-sight (LOS) component with amplitude *A*.

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17