Wireless Communications Channels
Lecture 3: Fading

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Why “statistical” description?

- Complex, unknown environment
- Can not describe everything in detail
  - Maxwell’s equations far too complex in real scenarios
- Large variations depending on the TX, RX and interacting object locations
- **Need a statistical measure since we can not describe every point everywhere**

> **“There is a x% probability that the amplitude/power will be above the level y”**

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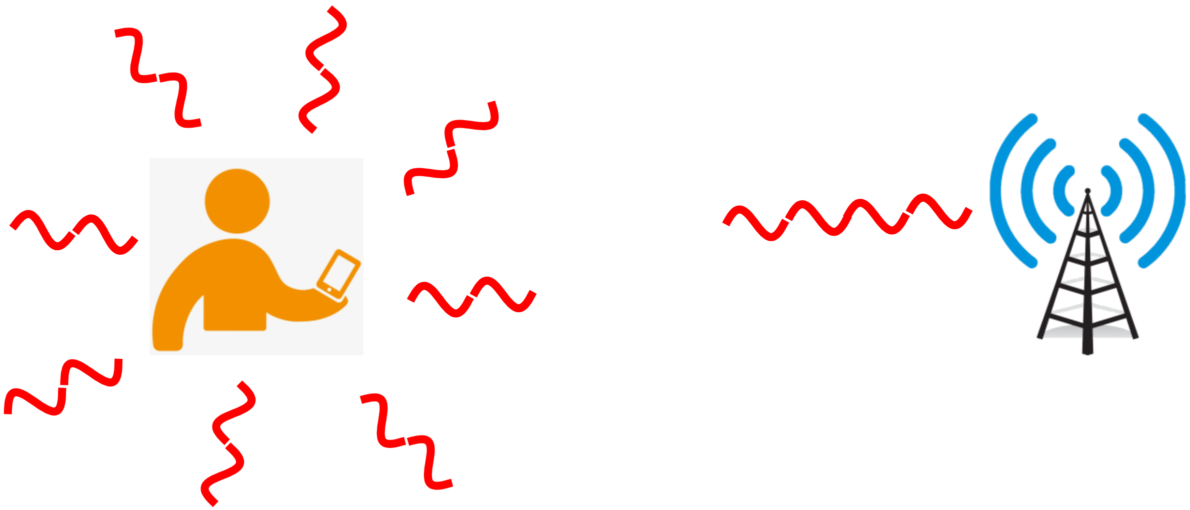
The WSSUS model: Assumptions

Recall: the channel is composed of a number of different contributions (incoming waves), the following is assumed:



A very common channel model is the WSSUS-model: Statistical properties remain the same over the considered time (or area)

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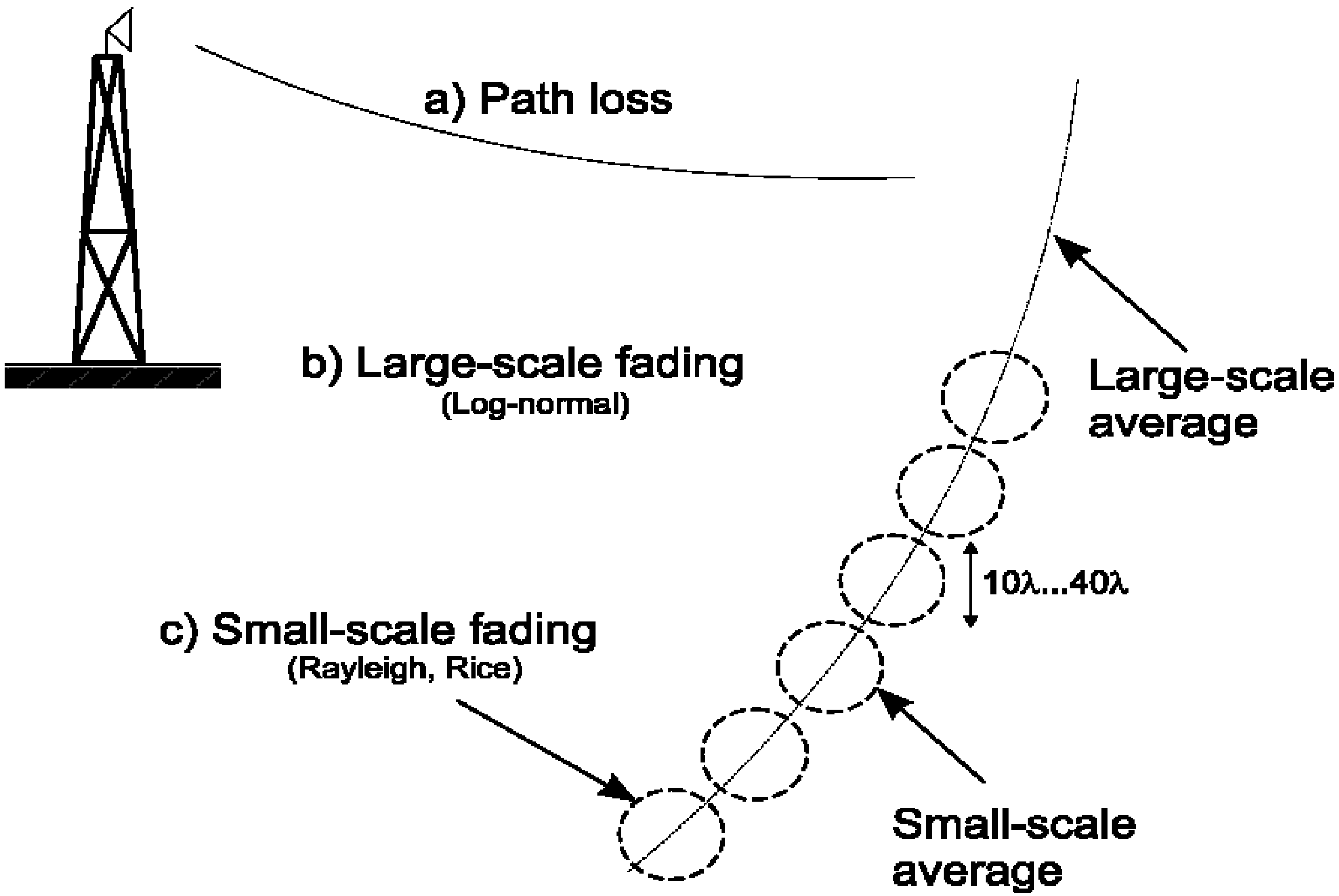
WSSUS model

- The channel is **Wide-Sense Stationary (WSS),** meaning
  a. $\mathrm{E}(\mathrm{h}(\mathrm{t})) =$ constant, for all t, the expectation of the channel is constant over time
  b. $R_h(t_1, t_2) = R_h(t_1 - t_2)$, the **correlation** of the channel is **invariant** over time.

- The channel is built up by **Uncorrelated Scatterers (US), meaning** that the **frequency** correlation of the channels is **invariant** over frequency. (Contributions with different delays are uncorrelated.)
  a. $R_h(t_1, t_2; \tau_1, \tau_2) = R_h(t_1 - t_2; \tau_1)\delta(\tau_1 - \tau_2)$

What is large scale and small scale?



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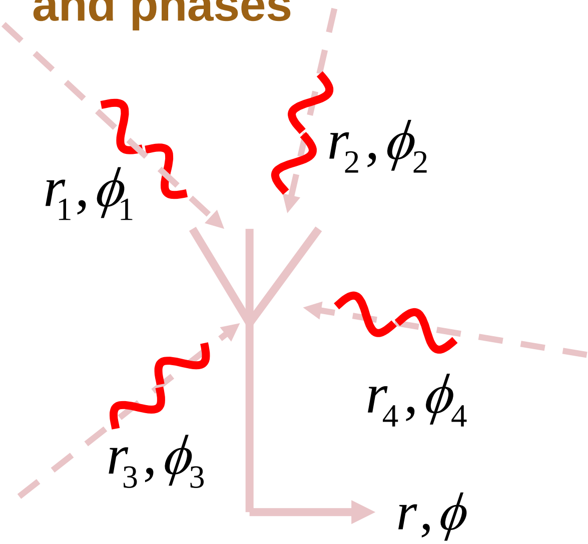
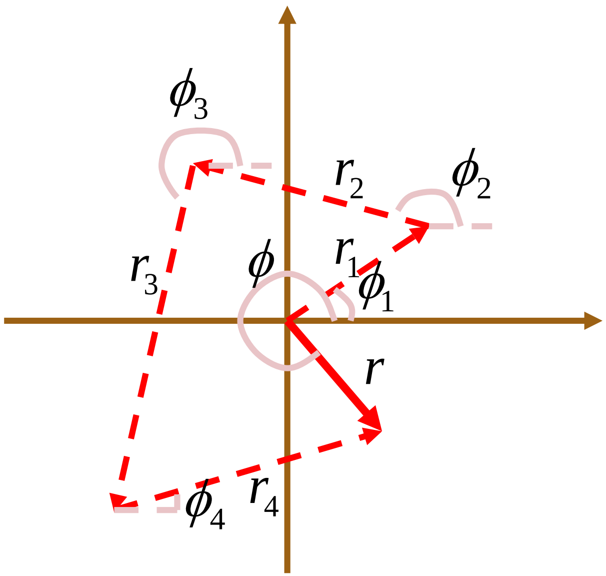
Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases



Add them up as phasors



$$r \exp\left(j\phi\right) = r_1 \exp\left(j\phi_1\right) + r_2 \exp\left(j\phi_2\right) + r_3 \exp\left(j\phi_3\right) + r_4 \exp\left(j\phi_4\right)$$

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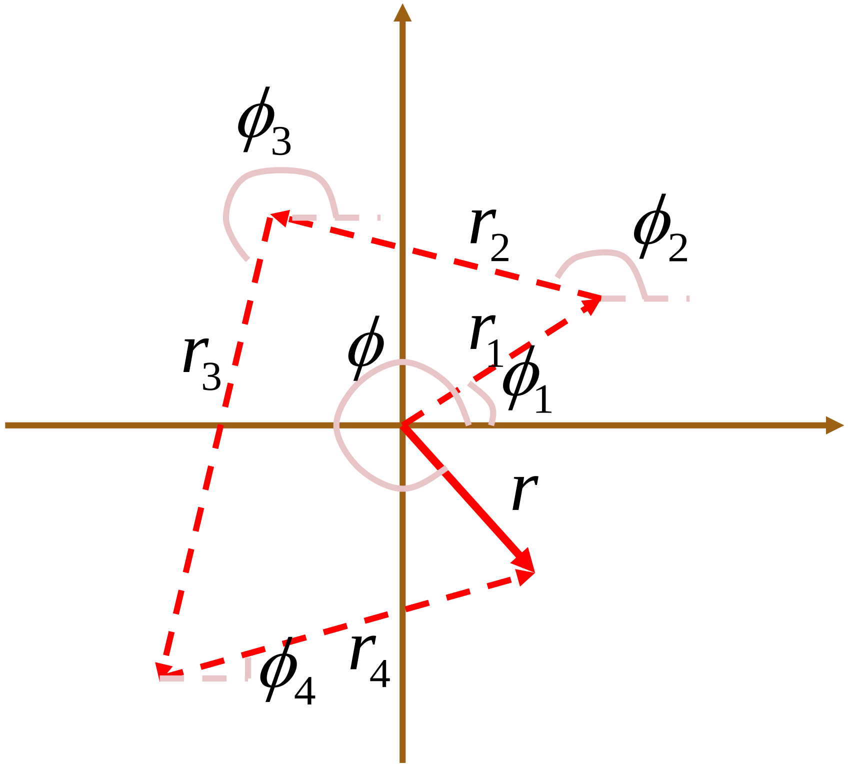
Small-scale fading
Many incoming waves

Re and Im components are sums of many independent equally distributed components

$$\mathrm{Re}(r) \in N(0, \sigma^2)$$

Re(r) **and** ***Im(r)*** **are independent**



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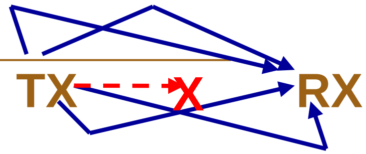
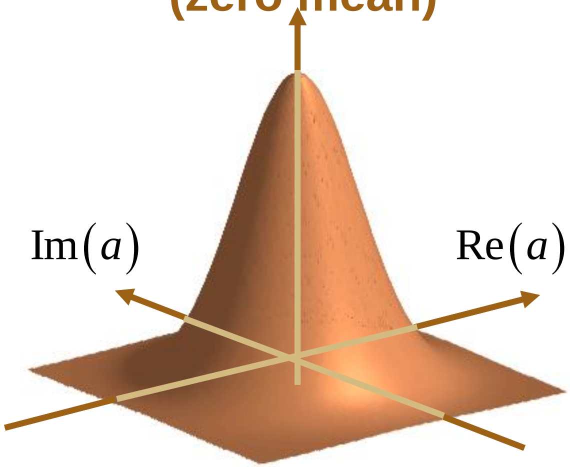
Small-scale fading
Rayleigh fading



No dominant component (non line-of-sight)

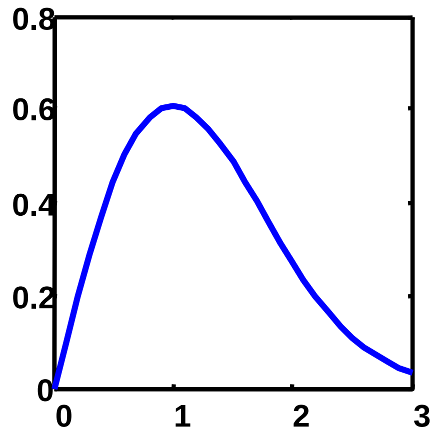
Tap distribution
2D Gaussian
(zero mean)



$$r = |a|$$

Amplitude distribution

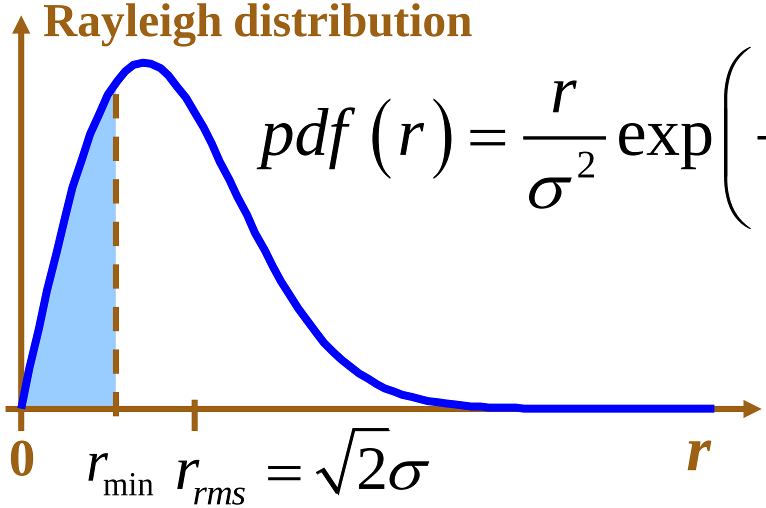
Rayleigh



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2} \right)$$

No line-of-sight component

Small-scale fading
Rayleigh fading



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2} \right)$$

Probability that the amplitude is below some threshold $r_{\min}$:

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2} \right)$$

Small-scale fading
Rayleigh fading – outage probability

- What is the probability that we will receive an amplitude 20 dB below the r_{rms}?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^{2}}{r_{rms}^{2}}\right) = 1 - \exp(-0.01) \approx 0.01$$

- What is the probability that we will receive an amplitude below r_{rms}?

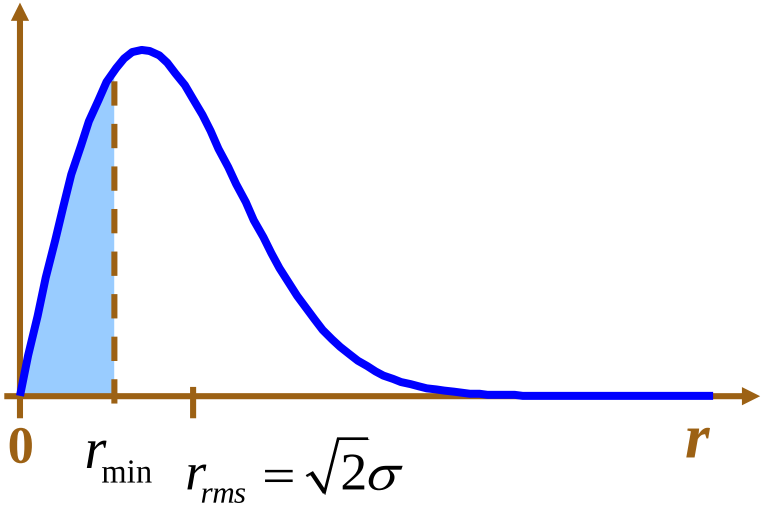
$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^{2}}{r_{rms}^{2}}\right) = 1 - \exp(-1) \approx 0.63$$

Small-scale fading
Rayleigh fading – fading margin

To ensure that in most cases we **receive** enough power, we transmit **extra** power by including the so-called "fading margin"

$$M = \frac{r_{rms}^2}{r_{\min}^2}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^2}{r_{\min}^2}\right)$$



Small-scale fading
Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln(0.99) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln(0.99) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

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Small-scale fading
Rayleigh fading – signal and interference

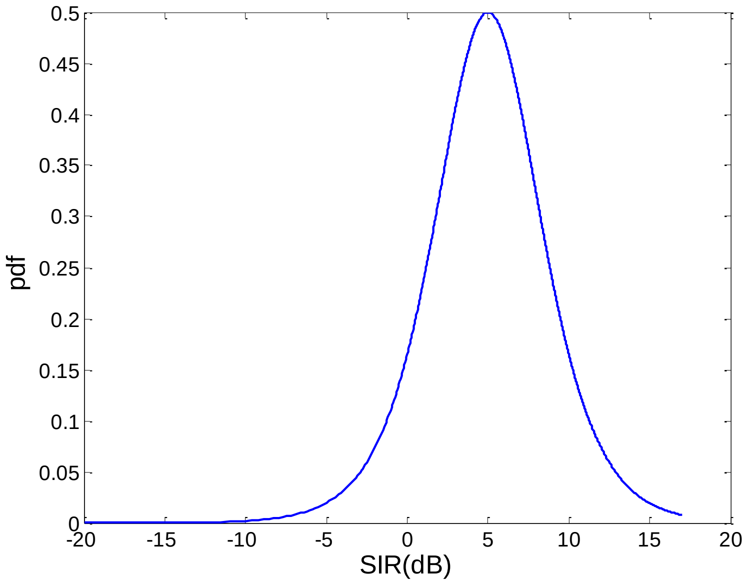
Both the desired signal and the interference undergo fading

For a single user inteferer and Rayleigh fading:

$$pdf_{SIR}(r) = \frac{2\bar{\sigma}^2 r}{(\bar{\sigma}^2 + r^2)^2}$$

$$cdf_{SIR}(r) = 1 - \frac{\bar{\sigma}^2 r}{(\bar{\sigma}^2 + r^2)}$$

where $\bar{\sigma}^2 = \frac{\sigma_2^2}{\sigma_1^2}$ is the mean signal to interference ratio



pdf for 10 dB mean signal to interference ratio

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Small-scale fading
Rayleigh fading – signal and interference

What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 10 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\overline{\sigma}^2 r_{\min}}{(\overline{\sigma}^2 + r_{\min}^2)} = 1 - \frac{10}{(10+1)} \approx 0.09$$

Small-scale fading: One dominating component

In case of Line-of-Sight (LOS) one component dominates.

- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

- The received amplitude has now a Ricean distribution instead of a Rayleigh
  - The fluctuations are smaller
  - The phase is dominated by the LOS component
  - In real cases the mean propagation loss is often smaller due to the LOS
- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

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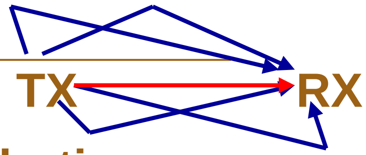
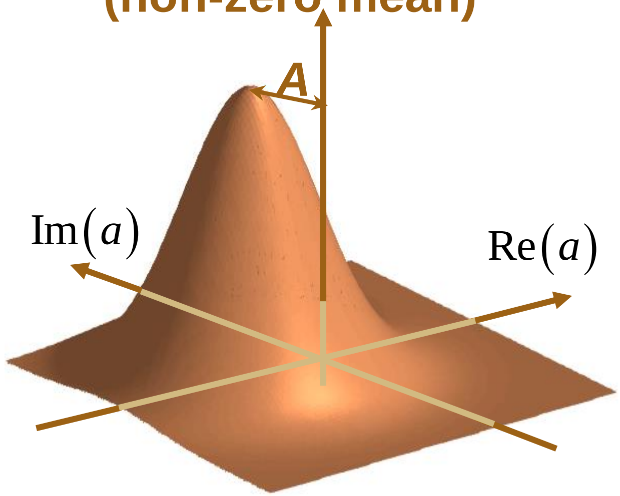
Small-scale fading
Ricean fading

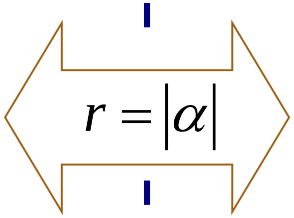


A dominant component (line of sight)

Tap distribution

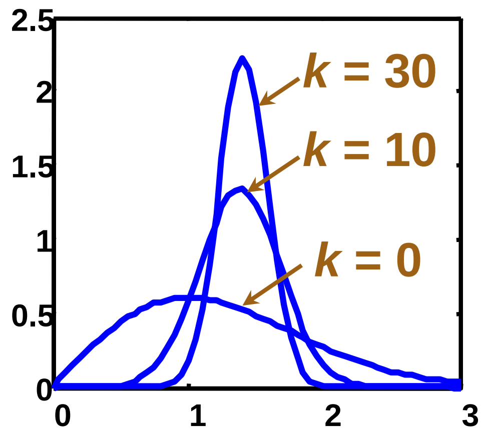
2D Gaussian (non-zero mean)





Amplitude distribution

Rice



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

Line-of-sight (LOS) component with amplitude *A*.

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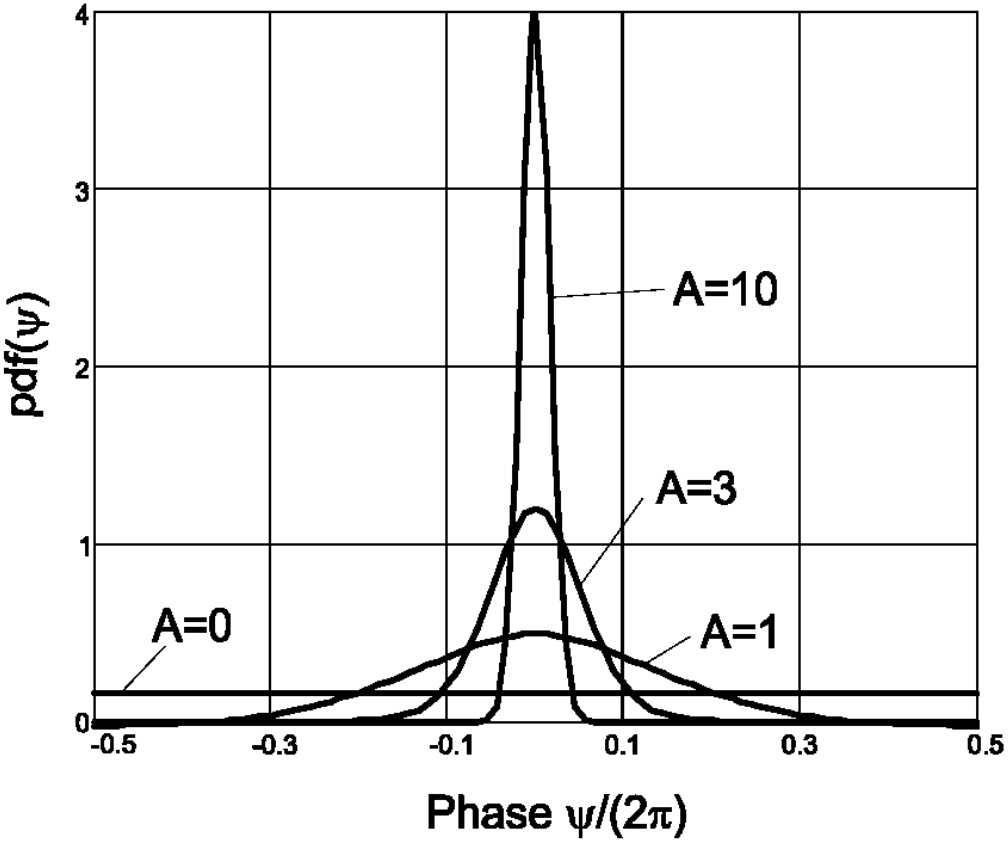
Small-scale fading
Ricean fading, phase distribution

The distribution of the phase is dependent on the K-factor



Small-scale fading: Nakagami distribution

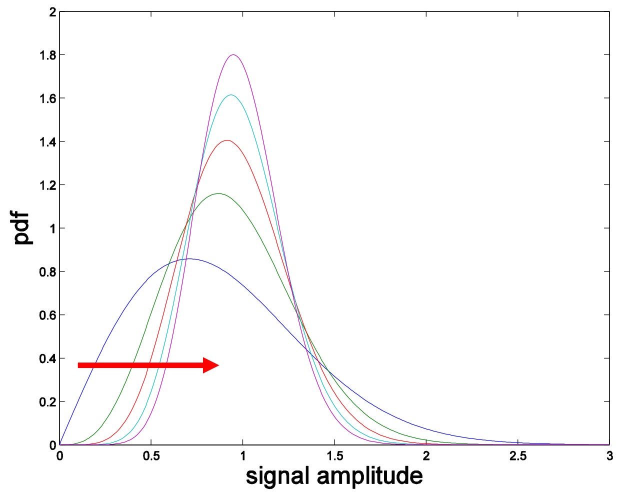
In many cases the received signal can not be described as a pure LOS + diffuse components

The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{1}{\Gamma(m)}\left(\frac{m}{\sigma^2}\right)^m r^{2m-1} \exp\left(-\frac{m}{2\sigma^2}r^2\right)$$

where $\Gamma(m)$ is the Gamma function

$$m = \left(\frac{2\sigma^2}{r^2 - 2\sigma^2}\right)^2$$



increasing m
{1 2 3 4 5}

with *m* it is possible to adjust the dominating power

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