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Wireless Communications Channels
Lecture 3: Fading

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Fading – Statistical description of the wireless channel

- ❑ **Why statistical description**
- ❑ **Large scale fading**
- ❑ **Small scale fading:**
  - ❑ without dominant component
  - ❑ with dominant component
- ❑ **Statistical models**
- ❑ Measurement example

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Why "statistical" description?

- Complex, unknown environment
- Can not describe everything in detail
  - Maxwell's equations far too complex in real scenarios
- Large variations depending on the TX, RX and interacting object locations
- **Need a statistical measure since we can not describe every point everywhere**

"There is a x% probability that the amplitude/power will be above the level y"

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The WSSUS model: Assumptions

A very common channel model is the WSSUS-model: Statistical properties remain the same over the considered time (or area)

Recall: the channel is composed of a number of different contributions (incoming waves), the following is assumed:

- The channel is **Wide-Sense Stationary (WSS),** meaning that the **correlation** of the channel is **invariant** over time.
- The channel is built up by **Uncorrelated Scatterers (US), meaning** that the **frequency** correlation of the channels is **invariant** over frequency. (Contributions with different delays are uncorrelated.)

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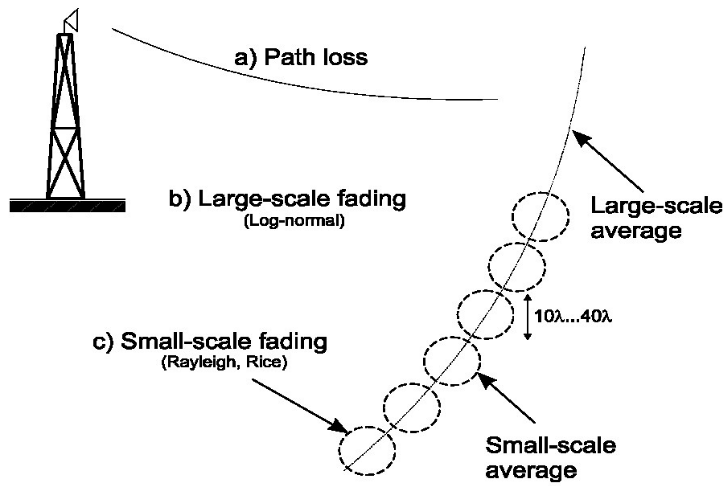
What is large scale and small scale?



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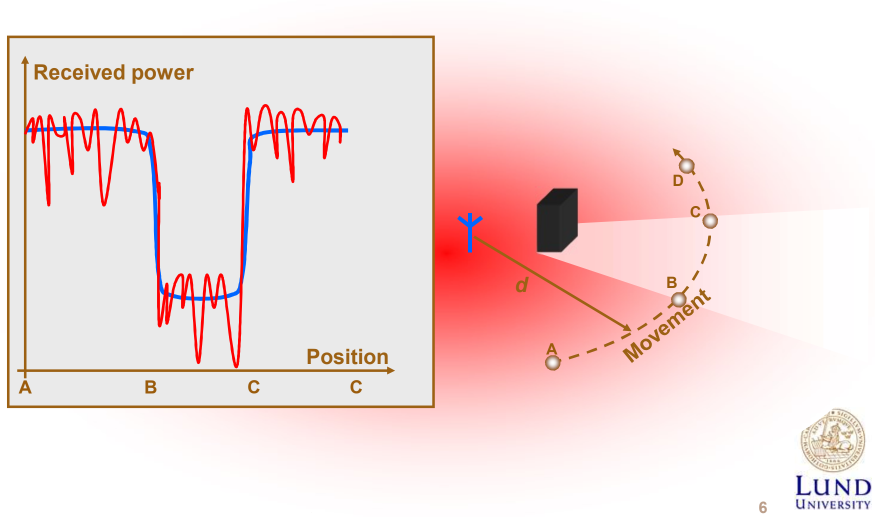
Large-scale fading: Basic principle



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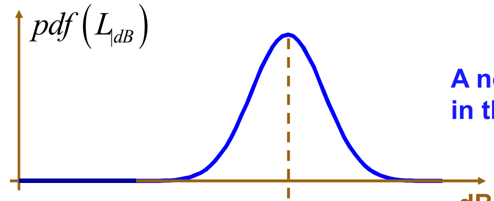
Large-scale fading: Log normal distribution

Confirmed by propagation channel measurements over the past 50 years.



A normal distribution in the dB domain.

$$pdf\left(L_{|dB}\right)=\frac{1}{\sqrt{2\pi}\sigma_{F|dB}}\exp\left(-\frac{\left(L_{|dB}-L_{0|dB}\right)^2}{2\sigma_{F|dB}^{\;2}}\right)$$

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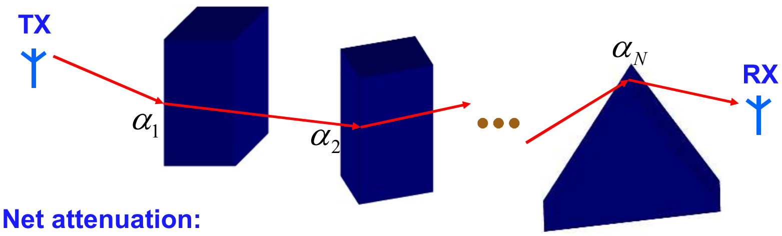
Large-scale fading: Why log-normal?

Many diffraction points adding extra attenuation to the pathloss.
This is, however, only one of several possible explanations.



Net attenuation:

$$L_{tot}=L(d)\times\alpha_1\times\alpha_2\times\ldots\times\alpha_N$$
$$L_{tot|dB}=L(d)_{|dB}+\alpha_{1|dB}+\alpha_{2|dB}+\ldots+\alpha_{N|dB}$$

If these are considered random and independent, we should get a normal distribution in the dB domain.

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Example: Shadowing from people

Two persons communicating with each other using cell phones, signal sometimes blocked by randomly moving humans

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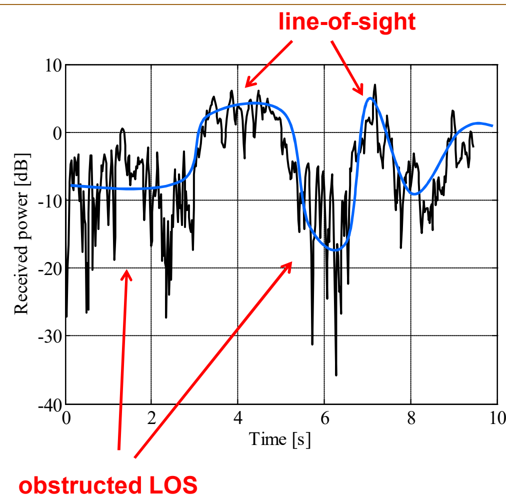
Example: Shadowing from humans



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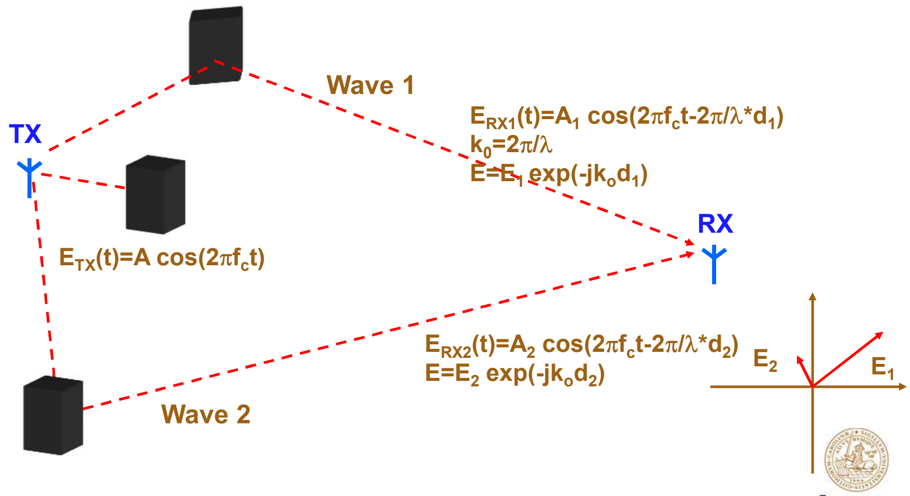
Small-scale fading: Two waves



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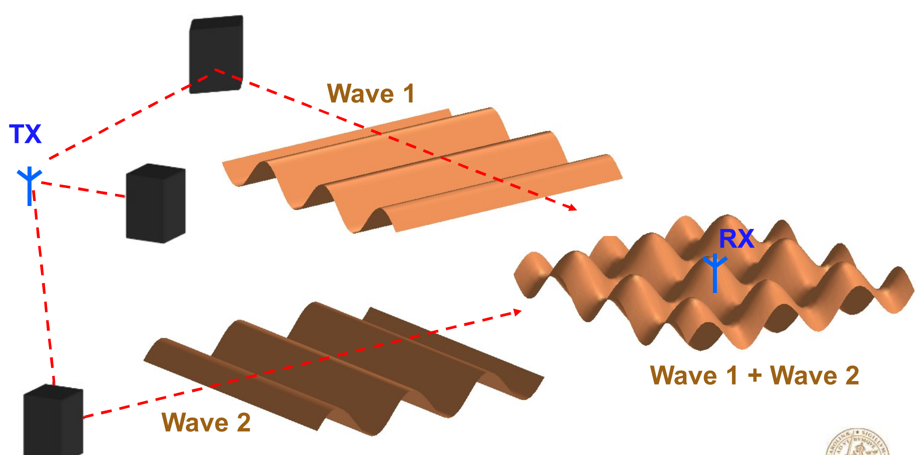
Small-scale fading: Two waves



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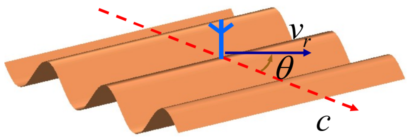
Small-scale fading: Doppler shifts



Receiving antenna moves with speed v_r at an angle θ relative to the propagation direction of the incoming wave, which has frequency f_0.

Frequency of received signal:

$$f = f_0 + \nu$$

where the doppler shift is

$$\nu = -f_0 \frac{v_r}{c} \cos(\theta)$$

The maximal Doppler shift is

$$\nu_{\max} = f_0 \frac{v}{c}$$

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Doppler shift

How large is the maximum Doppler frequency at pedestrian speeds for 5.2 GHz WLAN and at highway speeds using GSM 900?

$$\nu_{\max} = f_0 \frac{v}{c}$$

- f_0=5.2 10^9 Hz, v=5 km/h, (1.4 m/s) ➡ 24 Hz
- f_0=900 10^6 Hz, v=110 km/h, (30.6 m/s) ➡ 92 Hz

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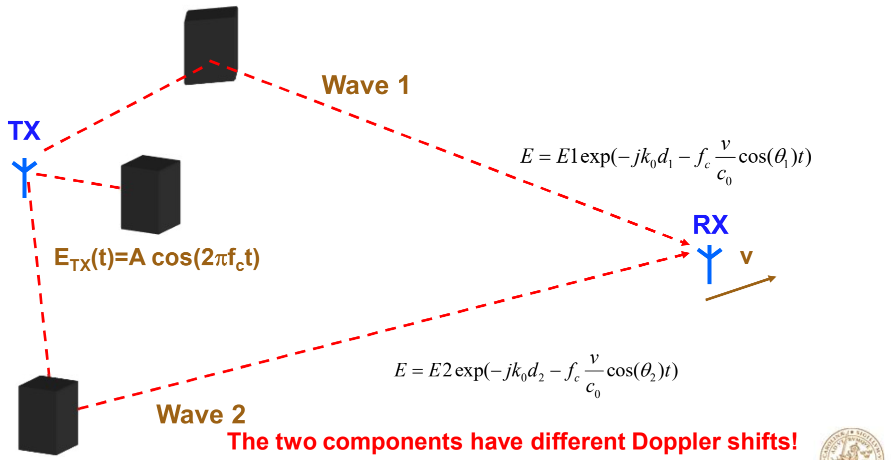
Small-scale fading
Two waves with Doppler



The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation

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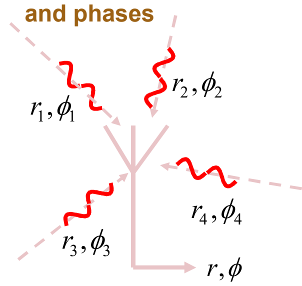
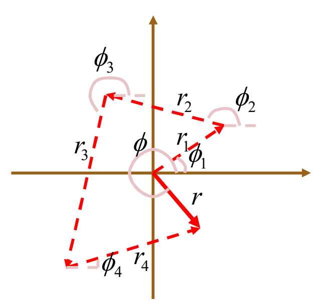
Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases



Add them up as phasors



$$r\exp(j\phi) = r_1\exp(j\phi_1) + r_2\exp(j\phi_2) + r_3\exp(j\phi_3) + r_4\exp(j\phi_4)$$

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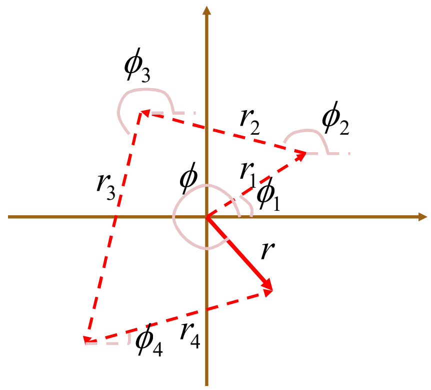
Small-scale fading
Many incoming waves

Re and Im components are sums of many independent equally distributed components

$$\mathrm{Re}(r) \in N(0,\sigma^2)$$

Re(r) **and** ***Im(r)*** **are independent**



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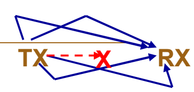
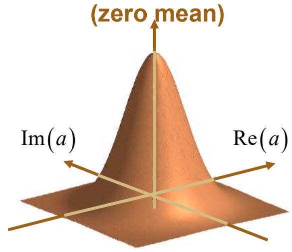
Small-scale fading
Rayleigh fading



No dominant component (non line-of-sight)

Tap distribution
2D Gaussian
(zero mean)

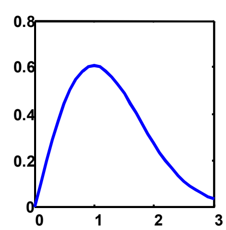


$r = |a|$

No line-of-sight component

Amplitude distribution

Rayleigh



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

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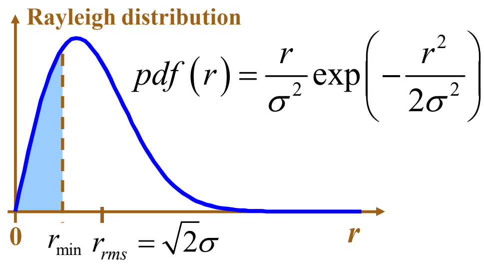
Small-scale fading
Rayleigh fading



Probability that the amplitude is below some threshold $r_{\min}$:

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right)$$

Small-scale fading
Rayleigh fading – outage probability

- What is the probability that we will receive an amplitude 20 dB below the r_{rms}?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1 - \exp(-0.01) \approx 0.01$$

- What is the probability that we will receive an amplitude below r_{rms}?

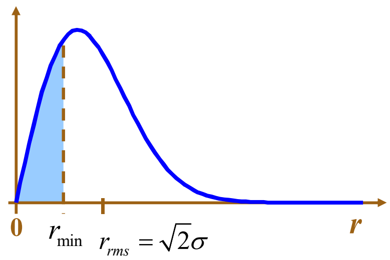
$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1 - \exp(-1) \approx 0.63$$

Small-scale fading
Rayleigh fading – fading margin

To ensure that in most cases we **receive** enough power, we transmit **extra** power by including the so-called "fading margin"

$$M = \frac{r_{rms}^2}{r_{\min}^2}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^2}{r_{\min}^2}\right)$$



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Small-scale fading
Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln\left(0.99\right) = 0.01 \quad \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

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Small-scale fading
Rayleigh fading – signal and interference

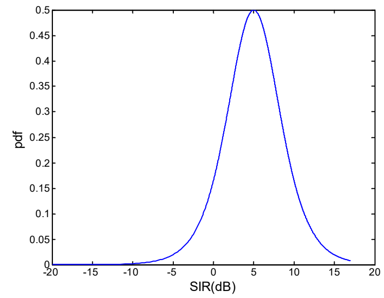
Both the desired signal and the interference undergo fading

For a single user inteferer and Rayleigh fading:

$$pdf_{SIR}(r) = \frac{2\bar{\sigma}^2 r}{(\bar{\sigma}^2 + r^2)^2}$$

$$cdf_{SIR}(r) = 1 - \frac{\bar{\sigma}^2 r}{(\bar{\sigma}^2 + r^2)}$$

where $\bar{\sigma}^2 = \frac{\sigma_2^2}{\sigma_1^2}$ is the mean signal to interference ratio



pdf for 10 dB mean signal to interference ratio

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Small-scale fading
Rayleigh fading – signal and interference

What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 10 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\bar{\sigma}^2 r_{\min}}{(\bar{\sigma}^2 + r_{\min}^2)} = 1 - \frac{10}{(10+1)} \approx 0.09$$

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Small-scale fading: One dominating component

In case of Line-of-Sight (LOS) one component dominates.

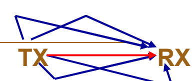
- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

- The recieved amplitude has now a Ricean distribution instead of a Rayleigh
  - The fluctuations are smaller
  - The phase is dominated by the LOS component
  - In real cases the mean propagation loss is often smaller due to the LOS
- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

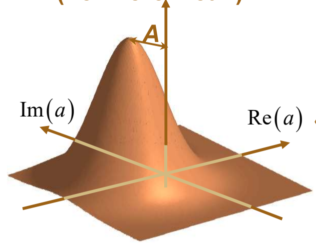
$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

Small-scale fading Ricean fading



A dominant component (line of sight)

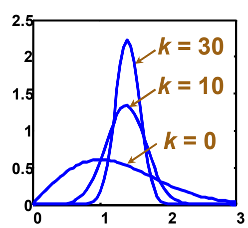
Tap distribution 2D Gaussian (non-zero mean)



$r = |\alpha|$

Amplitude distribution

Rice

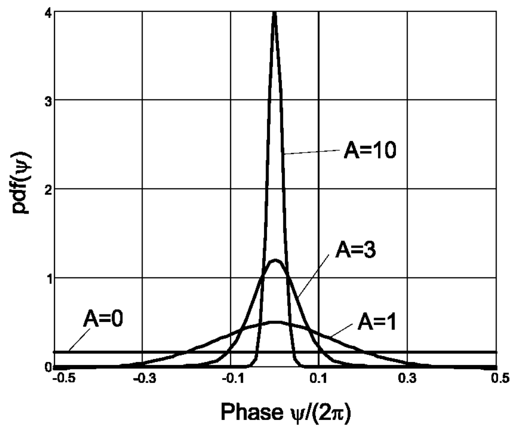


$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

Line-of-sight (LOS) component with amplitude *A*.

Small-scale fading
Ricean fading, phase distribution

The distribution of the phase is dependent on the K-factor



Small-scale fading: Nakagami distribution

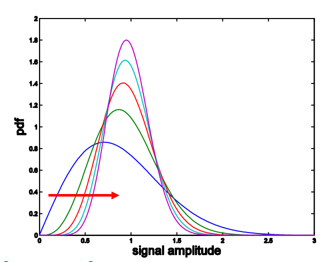
In many cases the received signal can not be described as a pure LOS + diffuse components

The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{1}{\Gamma(m)}\left(\frac{m}{\sigma^2}\right)^m r^{2m-1} \exp\left(-\frac{m}{2\sigma^2} r^2\right)$$

where $\Gamma(m)$ is the Gamma function

$$m = \left(\frac{2\sigma^2}{r^2 - 2\sigma^2}\right)^2$$



increasing m
{1 2 3 4 5}

with *m* it is possible to adjust the dominating power

Some special cases

Rayleigh fading

$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Rice fading, *K*=0

$$pdf(r) = \frac{r}{\sigma^2} \exp(-\frac{r^2+A^2}{2\sigma^2}) I_0(-\frac{rA}{\sigma^2}) = \frac{r}{\sigma^2} \exp(-\frac{r^2}{2\sigma^2})$$

Rice fading with K=0 becomes Rayleigh

Nakagami, *m*=1

$$pdf(r) = \frac{1}{\Gamma(m)} (\frac{m}{\sigma^2})^m r^{2m-1} \exp(-\frac{m}{2\sigma^2} r^2) = \frac{r}{\sigma^2} \exp(-\frac{r^2}{2\sigma^2})$$

Nakagami with *m*=1 becomes Rayleigh