# Home exam Electrodynamics EITN80, 2018

**The solutions has to be handed in before 3 pm, May 23.**
**For grade 3 at least 50% has to correct. For grade 4 at least 66% has to be correct and for grade 5 at least 83% has to be correct. For grade 4 and 5 one need to explain the solutions at an oral exam.**

## 1

A rectangular waveguide has cross section $6\,\mathrm{cm} \times 2.5\,\mathrm{cm}$. The boundary of the cross-section is $\Gamma$.

a) What are the cut-off frequencies for the five lowest waveguide modes?

b) What are the phase and group speeds of the fundamental mode at the frequency 4 GHz?

c) Assume that you like to use a coaxial cable to send in power to the waveguide and that you like to excite the fundamental mode. You drill a hole in the waveguide and attach the outer conductor of the coaxial cable to the surface of the hole. Suggest one position on $\Gamma$ to drill the hole if you let the inner conductor of the coaxial cable be extended along a straight line.

d) Suggest one position on $\Gamma$ to drill the hole if you let the inner conductor of the coaxial cable be extended and bent into a half circle. The tip of the inner conductor should be attached to the inner surface of the waveguide. You should also describe how the half circle is aligned.

e) Assume that the fundamental mode propagates in the positive $z-$direction in the waveguide. The frequency is 4 GHz and the amplitude of the mode is such that the largest value of the electric field in the waveguide is 100 V/m. Determine the time average of the power transported in the waveguide.

## 2

A waveguide has elliptic cross section with half axes $a = 4$ cm and $b = 2$ cm.

a) Use Comsol to determine the seven lowest cut-off frequencies for the modes.

b) Use Comsol to determine if each of the seven modes is a TE or a TM-mode.

## 3

A resonance cavity is a circular cylinder with radius $a = 10$ cm. The frequency of the $\mathrm{TM}_{010}$ mode is $f_{010}$ and the frequency of the $\mathrm{TE}_{111}$ mode is $f_{111} = 1.1 f_{010}$.

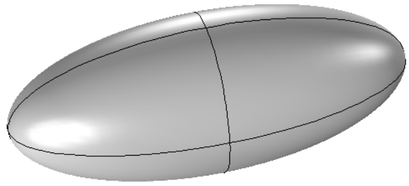
a) Determine the height $h$ of the cylinder.

b) Determine $f_{010}$

## 4

Determine the four lowest resonance frequencies of a cavity that has a cross section of a quarter of a circle. The radius of the circle is $a = 10$ cm and the height is $h = 5$ cm.

## 5



Use Comsol to determine the five lowest resonance frequencies of a resonance cavity that has the shape of a prolate spheroid, see figure. The prolate spheroid is an axially symmetric object. With the $z-$axis as symmetry axis the cross section in the $xz$-plane (and in the $yz-$plane) is an ellipse with major axis along the symmetry axis. The half axes of the ellipse are $a = 10$ cm and $b = 4$ cm.

*Help*: If you use the axially symmetric solver then the default value of the azimuthal index is $m = 0$. You need to check also $m > 0$ in order to find all of the five frequencies. Click on **Electromagnetic waves, frequency domain** and you will find the Azimuthal mode number.

## 6

A proton travels in the $xy$-plane with constant speed $v = 0.9c$, where $c$ is the speed of light. In the region where the proton travels there is a constant magnetic flux density $\boldsymbol{B} = B_0\hat{\boldsymbol{z}}$, where $B_0 = 1$ T.

a) Determine the radius $R$ of the circle that the proton travels along.

b) Introduce a coordinate system with its origin in the center of the circle and its $z-$axis along $\boldsymbol{B}$. Let $t = 0$ when the proton passes $(x, y, z) = (R, 0, 0)$. Determine the retarded time $t_{\mathrm{r}}$ when $t = 0$, for the field point $\boldsymbol{r} = (0, 0, R)$.

c) Determine the electric field $\boldsymbol{E}$ at $\boldsymbol{r} = (0, 0, R)$ as functions of $t$. You can write the solution as $\boldsymbol{E} = \alpha_1\hat{\boldsymbol{z}} + \alpha_2\boldsymbol{v}(t_{\mathrm{r}}) + \alpha_3\boldsymbol{w}(t_{\mathrm{r}})$, where $\boldsymbol{v}$ is the velocity and $\boldsymbol{w}$ the position at time $t_{\mathrm{r}}$. Give explicit expressions of $t_{\mathrm{r}}$, $\boldsymbol{w}(t_{\mathrm{r}})$, $\boldsymbol{v}(t_{\mathrm{r}})$, $\alpha_1$, $\alpha_2$ and $\alpha_3$.

d) Determine the static electric field $\boldsymbol{E}_{\mathrm{stat}}(0, 0, R)$ of a line charge $\rho_\ell = \dfrac{q}{2\pi R}$ along the circle with radius $R$. Is there any similarity between this field and the field in c)?

e) Run your Matlab code and check that you get the same electric fields as in c). Enclose the graph of the three components of $\boldsymbol{E}$ as a function of $t$ for $t$ in the interval $0 < t < 10\pi R/v$, where $v = 0.9c$ is the speed of the particle.

f) Use your Matlab code to determine $\boldsymbol{E}(R, 0, R, t)$. Enclose the graph of the three components of $\boldsymbol{E}$ as a function of $t$ for $t$ in the interval $0 < t < 10\pi R/v$, where $v = 0.9c$ is the speed of the particle.

g) Use your Matlab code to determine $\boldsymbol{E}(10R, 0, 0, t)$. Enclose the graph of the three components of $\boldsymbol{E}$ as a function of $t$ for $t$ in the interval $0 < t < 10\pi R/v$, where $v$ is the speed of the particle.

## 7

An inertial system $\bar{S}$ travels with velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ relative a system $S$. In $S$ there is a ladder with length $L = 5$ m that has one end at $(x, y, z) = (0, 0, 0)$ and the other at $(x, y, z) = (5, 5, 0)/\sqrt{2}$. What is the length $\bar{L}$ of the ladder, measured from $\bar{S}$?

## 8

An inertial system $\bar{S}$ travels with velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ relative a system $S$. In $S$ there is a coaxial waveguide with radius $a = 1$ cm of the inner conductor and radius $b = 2$ cm of the outer conductor. The symmetry axis is along the $z-$axis in $S$. In $S$ the outer conductor is grounded and the inner conductor has voltage $V = 10$ V.

a) Determine the electric field in $S$ as a function of $x$, for $a < x < b$, along the positive $x-$axis and as a function of $y$, for $a < y < b$, along the positive $y-$axis.

b) What does the cross section of the coaxial waveguide look like in $\bar{S}$? Draw a figure and describe its geometry.

c) Determine the electric and magnetic fields in $\bar{S}$ along the lines that in $S$ are the positive $x-$ and $y-$axis.

d) In $S$ the electric field is perpendicular to the inner and outer conductor. Does that also hold for all parts of the conductors in $\bar{S}$? If not try to explain why.

## 9

A long straight conductor carries a current of 30 A. The symmetry axis of the conductor is the $z-$axis. One centimeter from the symmetry axis there is a sharp needle. At time $t = 0$ an electron is emitted from the tip of the needle, and hence its position is $\boldsymbol{r}(0) = (1\,\text{cm}, 0, 0)$. The velocity of the electron is then $\boldsymbol{v}(0) = (\beta c, 0, 0)$, where $c$ is the speed of light.

You should use your Matlab program to determine the trajectory $(x(t), y(t), z(t))$ of the electron for $t > 0$. It turns out that the electron will travel outwards and then return if $\beta$ is small enough. The needle does not affect the trajectory.

a) First plot the trajectory during the time interval $0 \leq t \leq 500$ ns for $\beta = 0.0001$, $0.001$ and $0.01$.

b) Increase the time interval and $\beta$ and check how large $\beta$ you can reach so that the electron still turns and come back towards the conductor.

Plot the trajectories in two-dimensional plots with $(x(t), 0, z(t))$. You may check that the accuracy is ok by also plotting the speed $v$ of the particle as a function of time. It should not differ more than 5% from the initial value, in order for the trajectory curve to be accurate enough.

*Comment:* Don't spend too much time on finding the maximum $\beta$. An approximate value is enough.

## 10

Two beams of protons are parallell with the $z-$axis and travel with velocity $\boldsymbol{v} = \beta c\hat{\boldsymbol{z}}$ relative a system $S$. The distance between the symmetry axes of the beams is $a$, their charge per unit length, seen from $S$, is $\rho_\ell$, and the radius of the beams is much smaller than $a$. Let one of the beams be at $(x, y) = (0, 0)$ and the other at $(x, y) = (a, 0)$

a) Determine the force per unit length, measured in $S$, on the beam at $(x, y) = (a, 0)$ as a function of $\beta$. What happens when $\beta \to 1$?

b) A system $\bar{S}$ is also traveling with velocity $\boldsymbol{v} = \beta c\hat{\boldsymbol{z}}$. What is the line charge density $\bar{\rho}_\ell$ in $\bar{S}$?

c) Determine the force per unit length on the beam at $(x, y) = (a, 0)$ as a function of $\beta$ in $\bar{S}$.