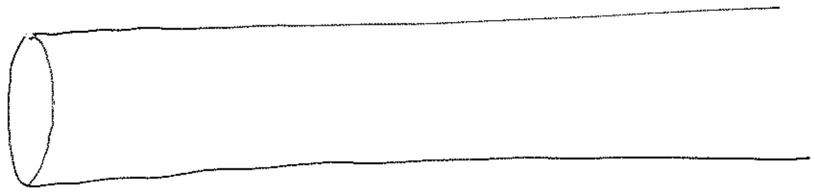
Traveling wave cavity



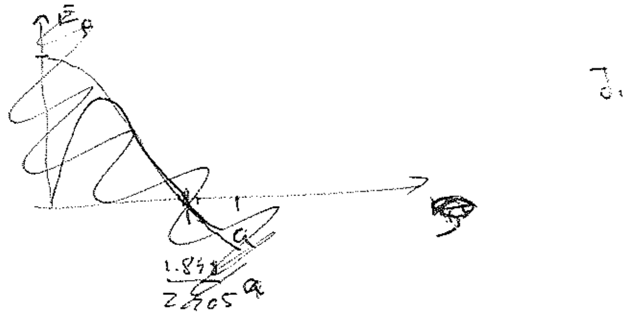
TM$_{01}$-mode $\Rightarrow$ $E_z = A\, J_0(k_t \rho)\, e^{i k_z z}$

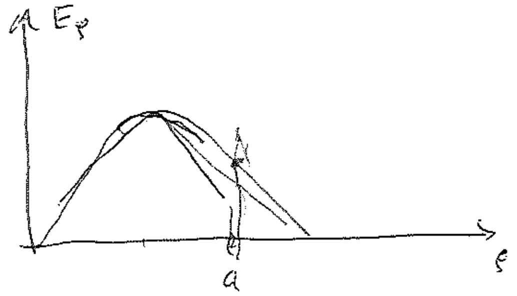
$$(15) \Rightarrow \quad \bar{E}_T = \frac{i}{k_t^2} k_z \nabla_T E_z = \frac{i}{k_t^2} k_z \hat{\rho} A \frac{\partial J_0(k_t \rho)}{\partial \rho} e^{i k_z z}$$

$$\Rightarrow \quad \bar{E}_T = i \frac{k_z}{k_t} A\, J_0'(k_t \rho)\, e^{i k_z z} \cdot \hat{\rho}$$

But $J_0'(k_t \rho) = -J_1(k_t \rho)$ see page 78 Appendix 1.

$$\Rightarrow \quad \bar{E}_T = -i \frac{k_z}{k_t} A\, J_1(k_t \rho)\, e^{i k_z z}\, \hat{\rho}$$

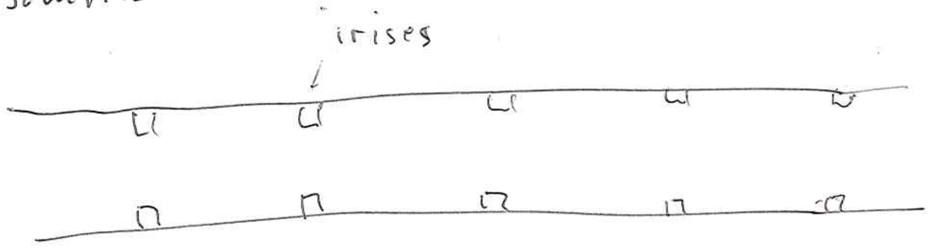




phase speed $\quad v_p = \frac{k}{k_z} c = \frac{k}{\sqrt{k^2 - \left(\frac{2.405}{a}\right)^2}}\, c$

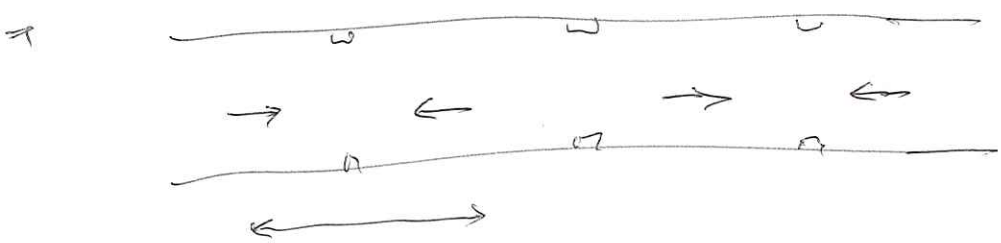
The wave needs to be slowed down!

Solution



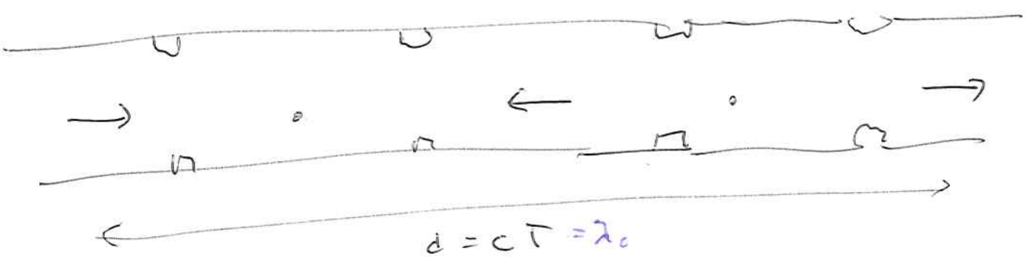
Different types of waves can be used

π-mode ⇒ π difference in phase between two cells



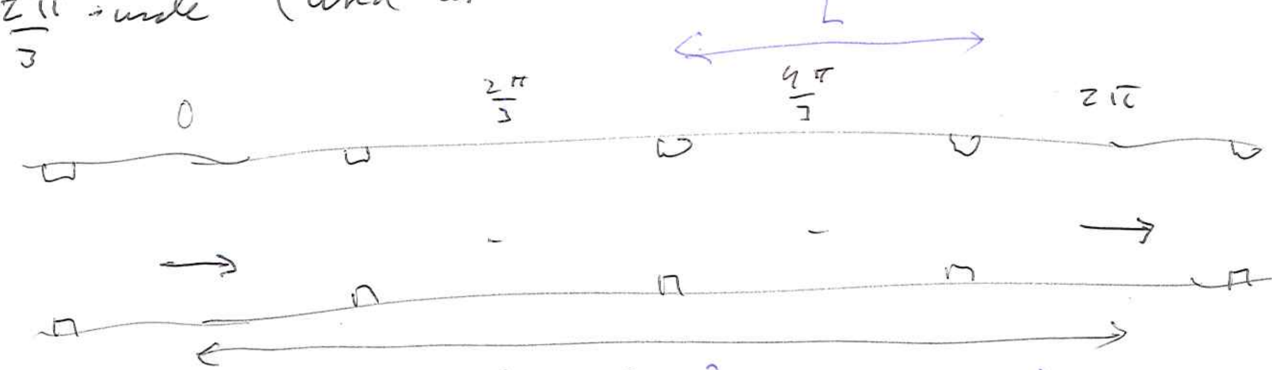
⇒ $d = c \cdot T = \lambda_c$ T = period

π/2-mode



$d = cT = \lambda_c$

$\frac{2\pi}{3}$-mode (used in MAX IV)

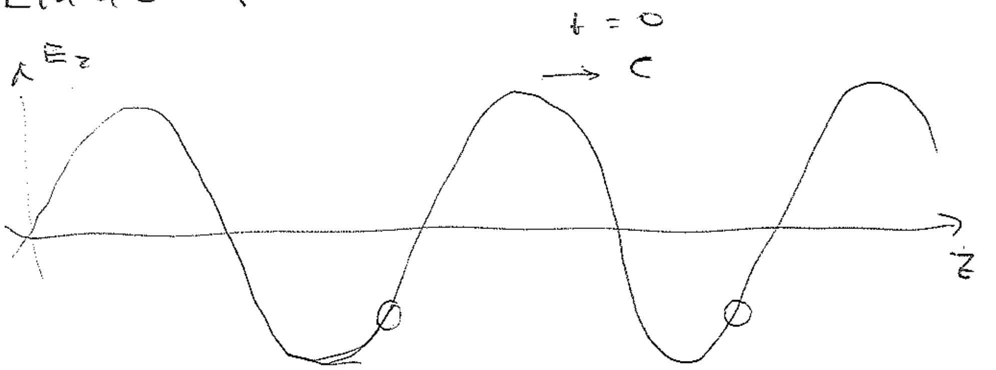


$d = cT = \lambda_c$ L

Ex MAX IV $f = 3 \cdot 10^9$ Hz ⇒ $\lambda = 10$ cm ⇒ $d = 10$ cm

⇒ $L = 3.33$ cm

Linac for e^-



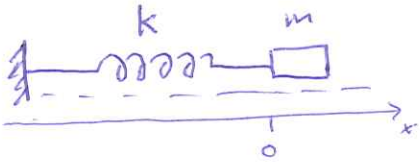
where should we put the bunches of electrons?

Answer at ~~[illegible]~~ $\emptyset$

$\Rightarrow$ The slow ones will catch up and the fast ones will be slowed down $\rightarrow$ compressed bunch

Resonance cavities

Ex



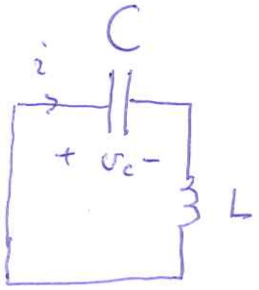
spring energy $\longleftrightarrow$ kinetic energy

$$\frac{1}{2}kx^2 \longleftrightarrow \frac{1}{2}m\dot{x}^2$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\left(\begin{array}{ll} x = A\sin\omega t & W_{s\,max} = \frac{1}{2}kA^2 \\ \dot{x} = A\omega\cos\omega t & W_{k\,max} = \frac{1}{2}mA^2\omega^2 \end{array} \right)$$

Ex



electric energy $\longleftrightarrow$ magnetic energy

$$\frac{1}{2}Cv_c^2 \longleftrightarrow \frac{1}{2}Li^2$$

$$\Rightarrow v_c = V_0\sin\omega t$$

$$i = C\frac{dv_c}{dt} = C\omega\cos\omega t$$

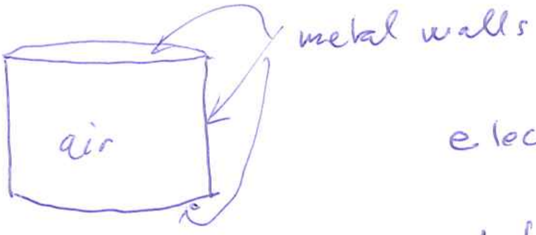
$$\Rightarrow W_{e\,max} = \frac{1}{2}CV_0^2$$

$$W_{m\,max} = \frac{1}{2}L(C\omega)^2V_0^2$$

$$\Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

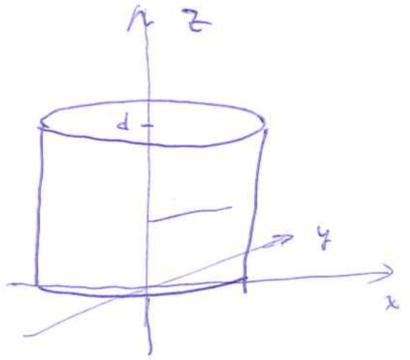
Ex



electric energy $\longleftrightarrow$ magnetic energy

$$\frac{1}{2}\int_V \bar{E}\cdot\bar{D}\,dv \longleftrightarrow \frac{1}{2}\int_V \bar{B}\cdot\bar{H}\,dv$$

Cylindrical cavities



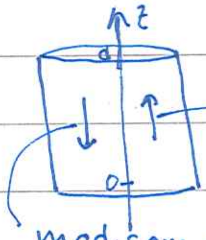
- Microwave filters narrowband
- Cavities in accelerators

Boundary conditions at z=0 and d

$$\frac{\partial E_z}{\partial z}(x,y,0) = \frac{\partial E_z}{\partial z}(x,y,d) = 0$$

$$H_z(x,y,0) = H_z(x,y,d) = 0$$

Fältet i kaviteten



mod som propagerar i positiv z-led

mod som propagerar i negativ z-led

TM-mod $\Rightarrow$ $E_z(\bar r) = \left(a_n^+ e^{ik_z z} - a_n^- e^{-ik_z z}\right) v_n(\bar\rho)$

$\Rightarrow \frac{\partial E_z}{\partial z} = ik_z\left(a_n^+ e^{ik_z z} + a_n^- e^{-ik_z z}\right) v_n(\bar\rho)$

$z=0 \Rightarrow \quad a_n^+ + a_n^- = 0 \Rightarrow a_n^+ = -a_n^- \Rightarrow E_z(\bar r) = 2a_n^+ \cos k_z z \cdot v_n(\bar\rho)$

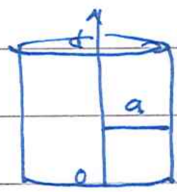
$z=d \Rightarrow \quad -2a_n^+ k_z \sin k_z d = 0 \Rightarrow k_z \cdot d = \ell\cdot\pi \quad \ell = 0,1,2\ldots$

$\therefore E_z(\bar r) = a_n^+ \cdot 2 \cdot \cos\frac{\ell\cdot\pi\cdot z}{d} \cdot v_n(\bar\rho)$

Resonansfrekvensen ges av att $k^2 = k_{tn}^2 + k_z^2 = k_{tn}^2 + \left(\frac{\ell\pi}{d}\right)^2$

$\Rightarrow \left(\frac{\omega}{c}\right)^2 = k_{tn}^2 + \left(\frac{\ell\pi}{d}\right)^2 \Rightarrow \quad f_{rn\ell} = \frac{c}{2\pi}\sqrt{k_{tn}^2 + \left(\frac{\ell\pi}{d}\right)^2}$

Exempel Cirkulär cylindrisk kavitet



$f = \frac{c}{2\pi}\sqrt{\left(\frac{\xi_{mn}}{a}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2}$ $\quad m = 0,1,2\ldots$ $\quad n = 1,2\ldots$ $\quad \ell = 0,1,2\ldots$

Finn lägsta resonansen för TM. TM_{01} ger lägst k_t

$\ell = 0, \quad k_{t01} = \frac{\xi_{01}}{a} = \frac{2.405}{a}$

$\Rightarrow \quad f_r = \frac{c}{2\pi}\cdot\frac{2.405}{a}$

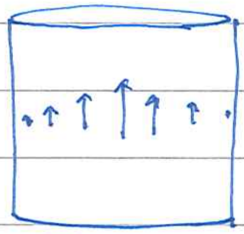
Vi kallar denna resonans för TM_{010} $\leftarrow \ell$

$TM_{010} \Rightarrow \quad E_z(\bar{r}) = a_a^+ \cdot 2 \cdot \cos\frac{0 \cdot \pi z}{d} \cdot v_{01}(\bar{\rho}) = A \cdot 1 \cdot J_0\left(\frac{\gamma_{01}\rho}{a}\right)$ oberoende av z

~~3.21~~ (15) ger $\quad \bar{E}_T(\bar{r}) = A \cdot \frac{i}{k_{t01}^2} \frac{l \cdot \pi}{d} J_0'(k_t \rho)\,\hat{\rho} = \bar{0}$ ty $l = 0$

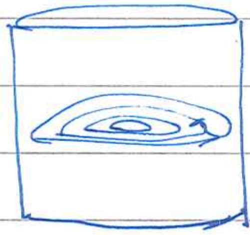
$$\bar{H}_T(\bar{r}) = \hat{\varphi}\,\frac{A}{\eta_0}\,\frac{i}{k_{t01}}\,\frac{\omega}{c_0}\,J_0'(k_t \rho)$$

$\Rightarrow$



$\bar{E}$

stående våg i ρ-led



$\bar{H}$

bildar cirklar

maximal amplitud vid $\rho = a$

TE - resonanser

$$TE \Rightarrow \quad H_z(\bar{r}) = \left(b_n^+ e^{ik_z z} + b_n^- e^{-ik_z z}\right) w_n(\bar{\rho})$$

$$z = 0 \Rightarrow H_z = 0 \Rightarrow \quad b_n^+ + b_n^- = 0 \Rightarrow \quad b_n^+ = -b_n^-$$

$$\Rightarrow H_z(\bar{r}) = b_n^+ \left(e^{ik_z z} - e^{-ik_z z}\right) w_n(\bar{\rho}) = 2i\, b_n^+ \sin k_z z \cdot w_n(\bar{\rho})$$

$$z = d \Rightarrow H_z = 0 \Rightarrow \quad 2i\, b_n^+ \sin k_z d \cdot w_n(\bar{\rho}) = 0$$

$$\Rightarrow \sin k_z d = 0 \Rightarrow \quad k_z \cdot d = l \cdot \pi \qquad l = 1, 2 \ldots$$

$$\therefore H_z(\bar{r}) = 2i \cdot b_n^+ \sin\frac{l \pi z}{d} \cdot w_n(\bar{\rho})$$

Exempel Finn lägsta resonansfrekvensen för TM-moder

$$k^2 = k_{tn}^2 + k_z^2 = \left(\frac{\xi_{mn}}{a}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2$$

$m = 0, 1, 2 \ldots$
$n = 1, 2 \ldots$
$\ell = 1, 2 \ldots$

$\Rightarrow$ välj $\ell = 1$, $m = 1$, $n = 1$ $\Rightarrow$ $\xi_{11} = 1.841$

$$f_{r_{111}} = \frac{c}{2\pi}\sqrt{\left(\frac{1.841}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2}$$

När blir $\underset{TM}{f_{r_{111}}} > \underset{TE}{f_{r_{010}}}$?

The TM_{010} mode in accelerators