Electrodynamics, lecture 9, 2019

Anders Karlsson, anders.karlsson@eit.lth.se

Electrical and information technology

Last lectures

- Liénard-Wiechert potentials, E and B for charged particle
- Radiated power from charged particle
- Larmor formula for non-relativistic case
- Relativistic particle: Liénard's generalization of the Larmor formula
- Radiated power per solid angle

Potentials

Vector potential $\boldsymbol{A}$ and scalar potential V

$$\boldsymbol{A}(\boldsymbol{r},t) = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{r}', t_\mathrm{r})}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}v'$$

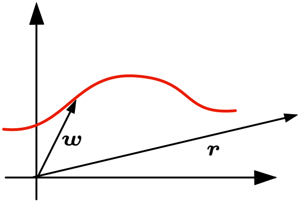
$$V(\boldsymbol{r},t) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\boldsymbol{r}', t_\mathrm{r})}{|\boldsymbol{r}-\boldsymbol{r}'|}\,\mathrm{d}v'$$

where

$$t_\mathrm{r} = t - \frac{|\boldsymbol{r}-\boldsymbol{r}'|}{c} \tag{1}$$

is the retarded time.

$\boldsymbol{E}$ and $\boldsymbol{B}$ from moving charge



Let $\boldsymbol{\mathscr{r}} = \boldsymbol{r} - \boldsymbol{w}(t_\mathrm{r})$, $|\boldsymbol{\mathscr{r}}| = |\boldsymbol{r} - \boldsymbol{w}| = c(t - t_\mathrm{r})$, and $\boldsymbol{u} = c\hat{\boldsymbol{\mathscr{r}}} - \boldsymbol{v}$, then

$$\boldsymbol{E}(\boldsymbol{r}, t) = \frac{q}{4\pi\varepsilon_0(\boldsymbol{\mathscr{r}} \cdot \boldsymbol{u})^3}[(c^2 - v^2)\boldsymbol{u} + \boldsymbol{\mathscr{r}} \times (\boldsymbol{u} \times \boldsymbol{a})]$$

$$\boldsymbol{B}(\boldsymbol{r}, t) = \frac{1}{c}\hat{\boldsymbol{\mathscr{r}}} \times \boldsymbol{E}(\boldsymbol{r}, t)$$

Charge with constant velocity

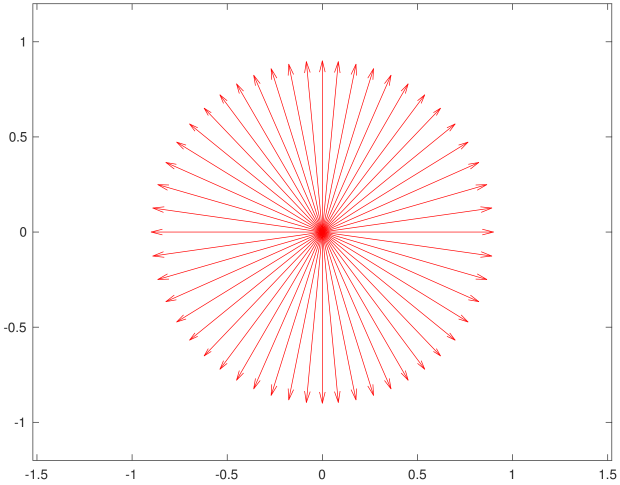
When the velocity is constant, $\boldsymbol{v} = v\hat{\boldsymbol{z}}$ then $\boldsymbol{a} = \boldsymbol{0}$ and

$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{|\boldsymbol{\imath}|(c^2-v^2)\boldsymbol{u}}{(\boldsymbol{\imath}\cdot\boldsymbol{u})^3}$$

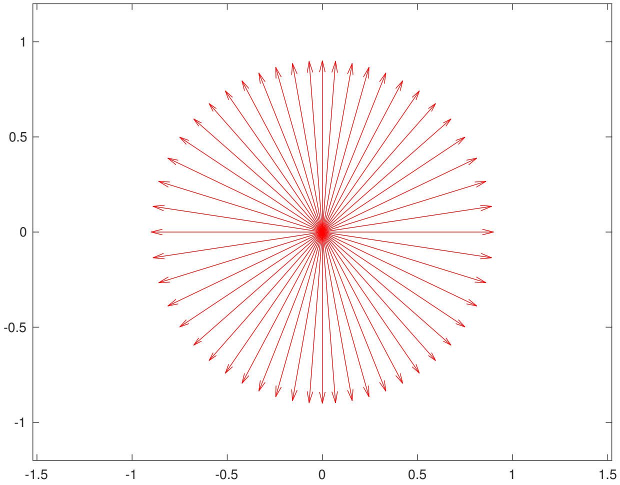
$$\boldsymbol{E}(\boldsymbol{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{1-\beta^2}{(1-\beta^2\sin\theta)^{3/2}}\frac{\hat{\boldsymbol{R}}}{R^2}$$

where $\boldsymbol{R}$ is the vector from the charge to the field point, $\beta = \dfrac{v}{c}$ and θ is the polar angle.

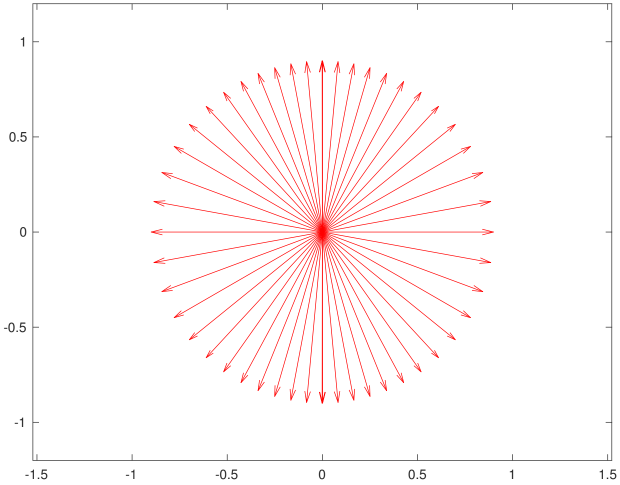
Charge with constant velocity $\beta = 0.5$



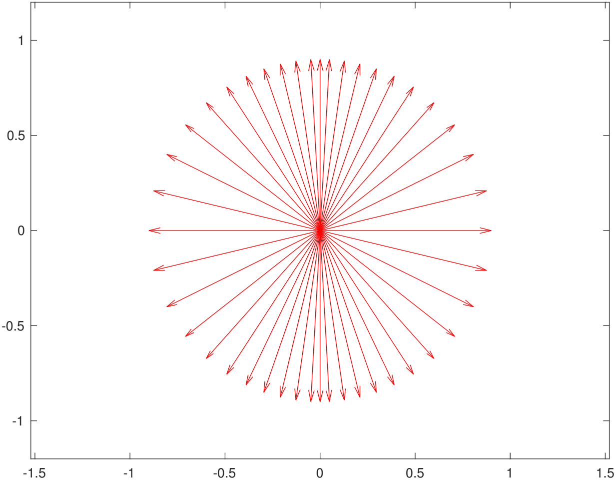
Charge with constant velocity $\beta = 0.6$



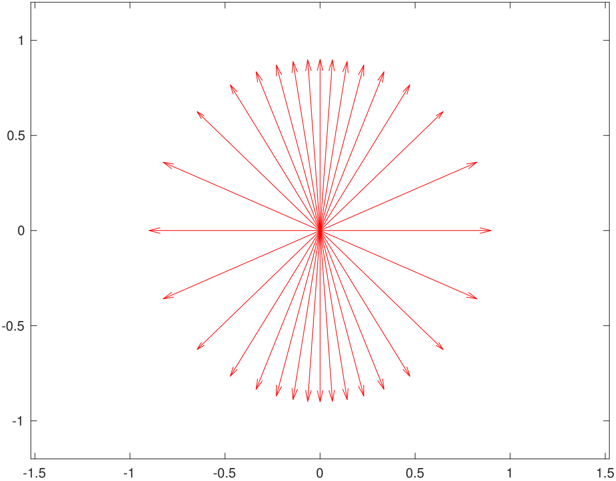
Charge with constant velocity $\beta = 0.7$



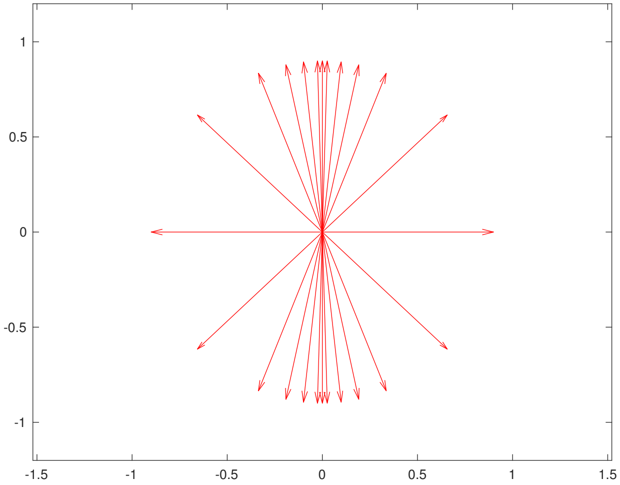
Charge with constant velocity $\beta = 0.8$



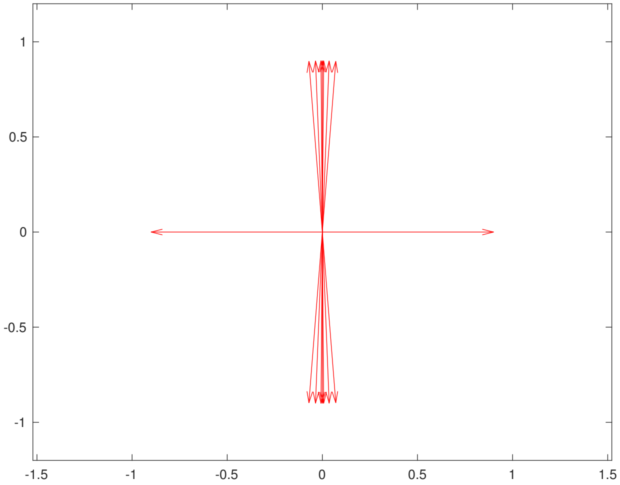
Charge with constant velocity $\beta = 0.9$



Charge with constant velocity $\beta = 0.95$



Charge with constant velocity $\beta = 0.99$



Far-field from charge

Non-relativistic velocities: The Larmor formula gives the radiated power at time t from a charge q with acceleration $a(t)$.

$$P(t) = \frac{\mu_0 q^2 a(t)^2}{6\pi c}$$

Far-field from charge

Relativistic velocities: The Liénard generalization of the Larmor formula.

$$P(t) = \frac{\mu_0 q^2 \gamma(t)^6}{6\pi c} \left(a(t)^2 - \left(\frac{|\boldsymbol{v}(t) \times \boldsymbol{a}(t)|}{c} \right)^2 \right)$$

where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Far-field from charge

Relativistic velocities: The Liénard generalization of the Larmor formula. Intensity

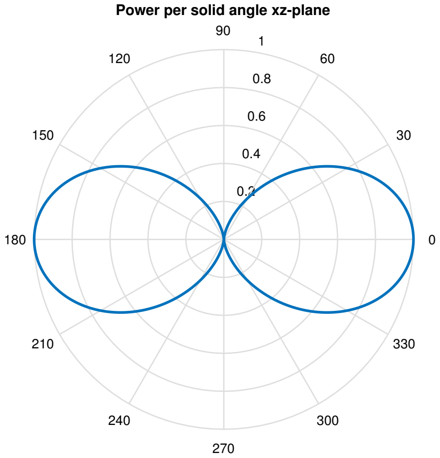
$$\frac{dP(t)}{d\Omega} = \frac{q^2}{16\pi^2\varepsilon_0}\frac{|\boldsymbol{r}\times(\boldsymbol{u}\times\boldsymbol{a})|^2}{|\hat{\boldsymbol{r}}\cdot\boldsymbol{u}|^5}$$

Far-field from charge

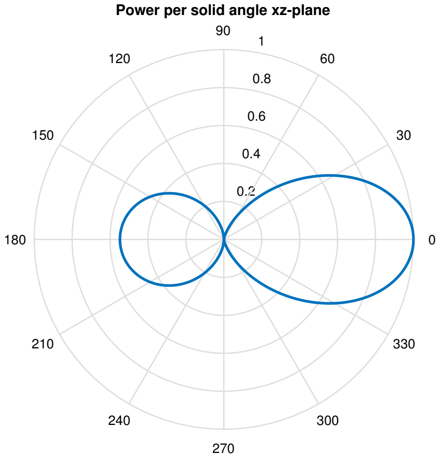
Relativistic velocities: Special case $\boldsymbol{v} = v\hat{\boldsymbol{z}}$ $\boldsymbol{a} = a\hat{\boldsymbol{x}}$.

$$\frac{dP(t)}{d\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \frac{(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\phi}{(1-\beta\cos\theta)^5}$$

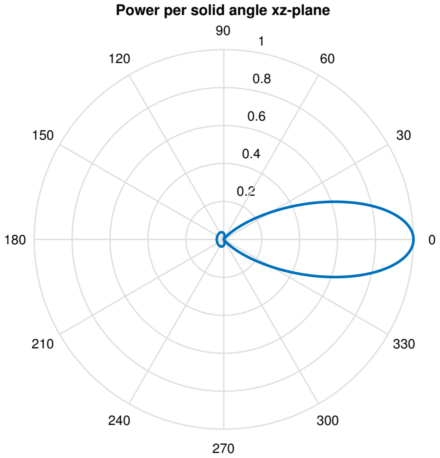
Radiation from from q with $\boldsymbol{v} = v\hat{\boldsymbol{z}}$, $\boldsymbol{a} = a\hat{\boldsymbol{x}}$ and $\beta = 0$



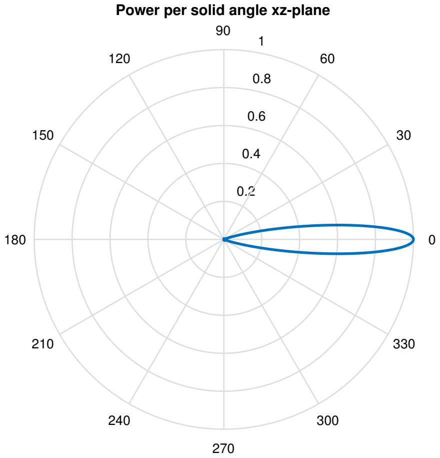
Radiation from from q with $\boldsymbol{v} = v\hat{\boldsymbol{z}}$, $\boldsymbol{a} = a\hat{\boldsymbol{x}}$ and $\beta = 0.1$



Radiation from from q with $\boldsymbol{v} = v\hat{\boldsymbol{z}}$, $\boldsymbol{a} = a\hat{\boldsymbol{x}}$ and $\beta = 0.5$



Radiation from from q with $\boldsymbol{v} = v\hat{\boldsymbol{z}}$, $\boldsymbol{a} = a\hat{\boldsymbol{x}}$ and $\beta = 0.9$



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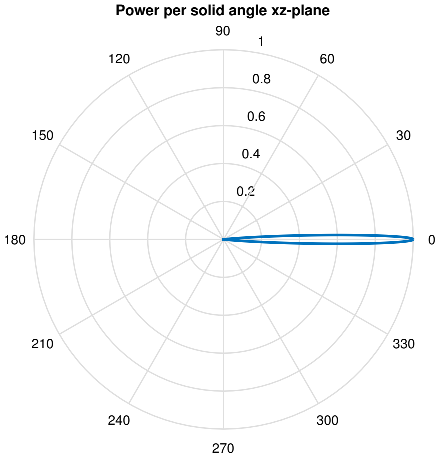

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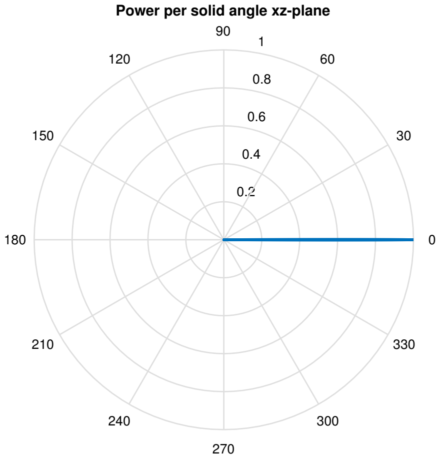
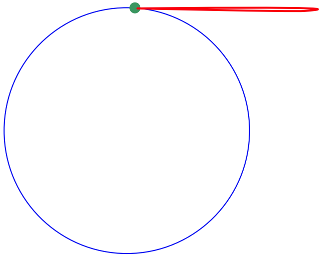


Radiation from electron in circular motion with $\beta \approx 1$



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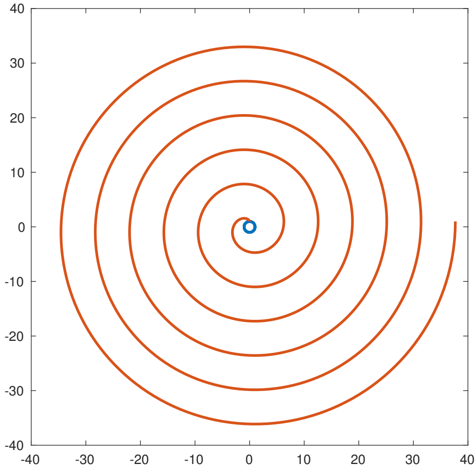

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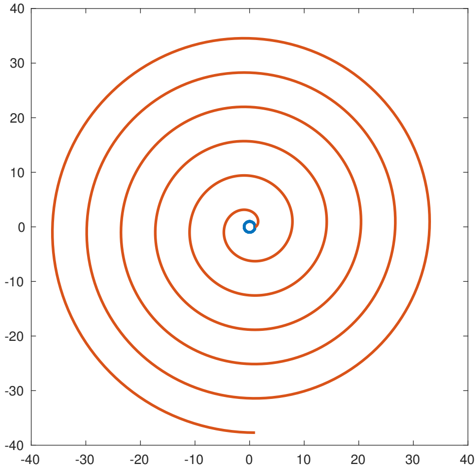

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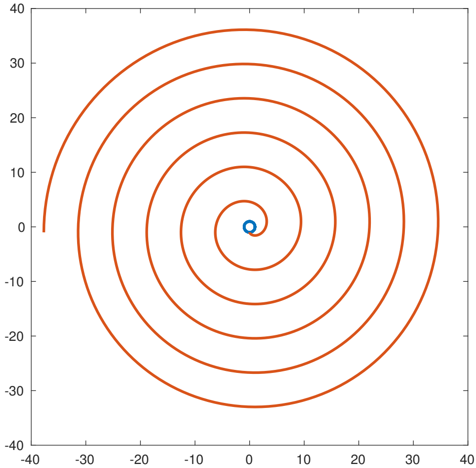

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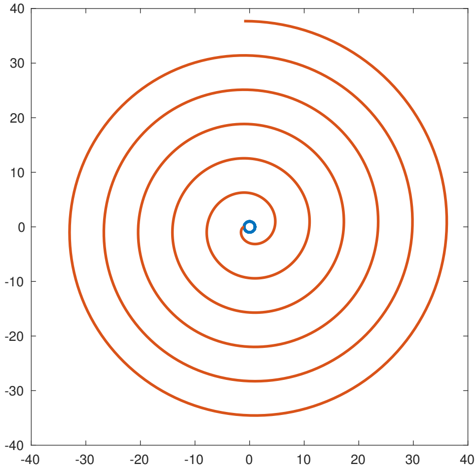

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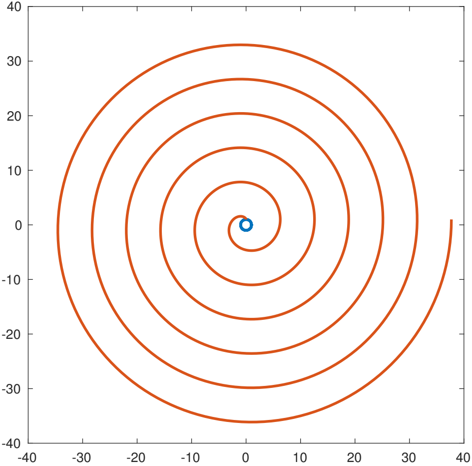

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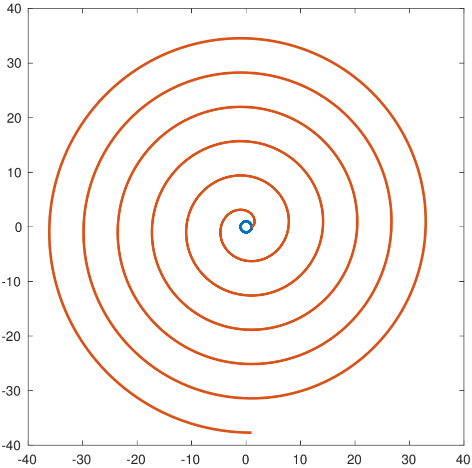

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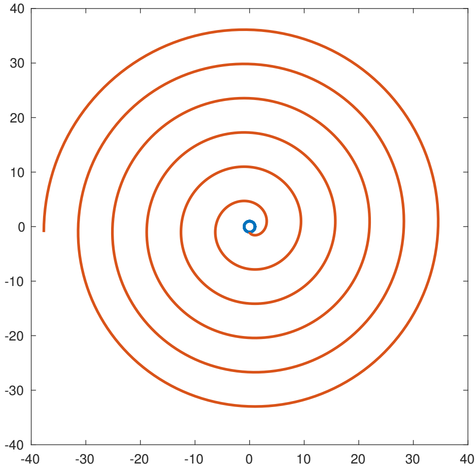

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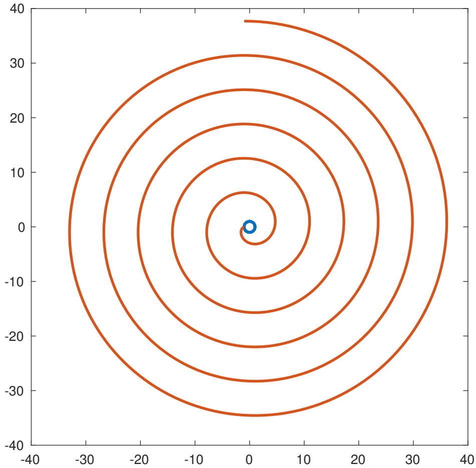

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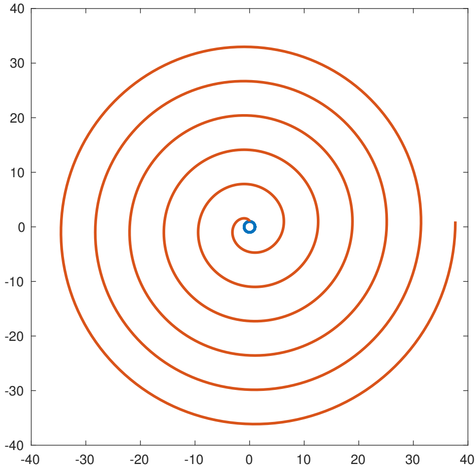

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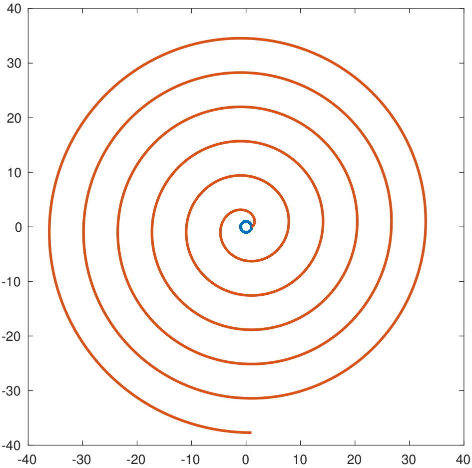

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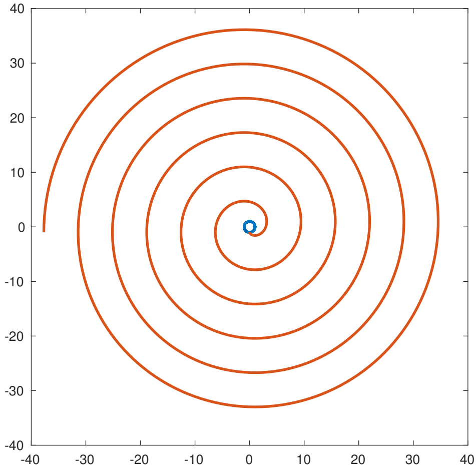

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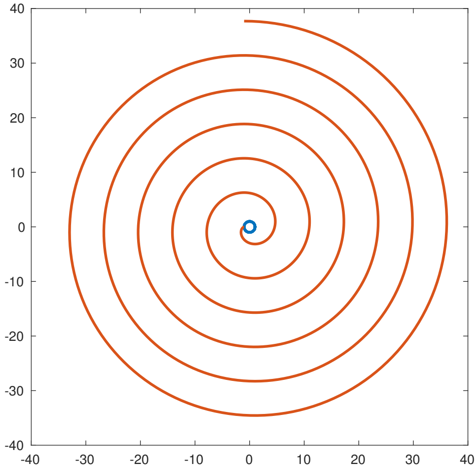

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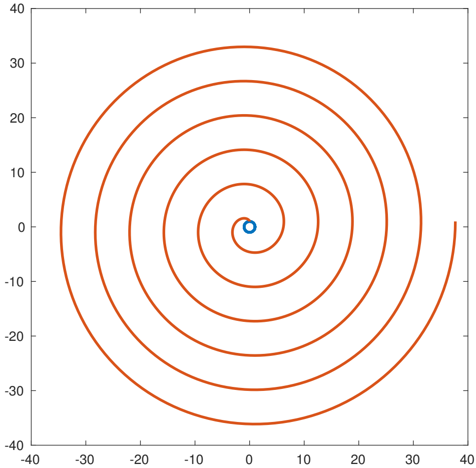

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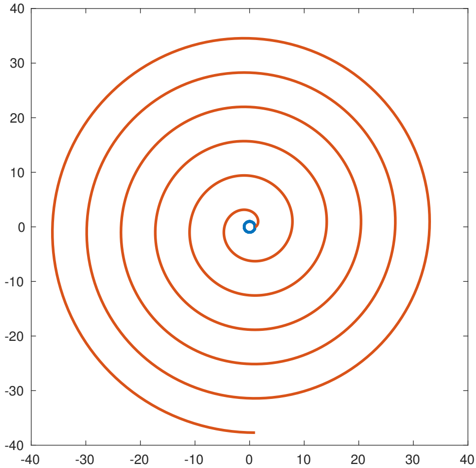

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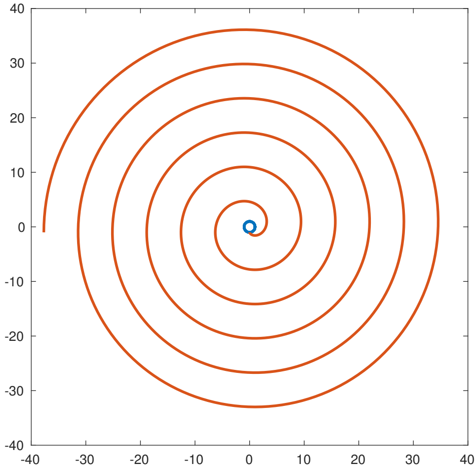

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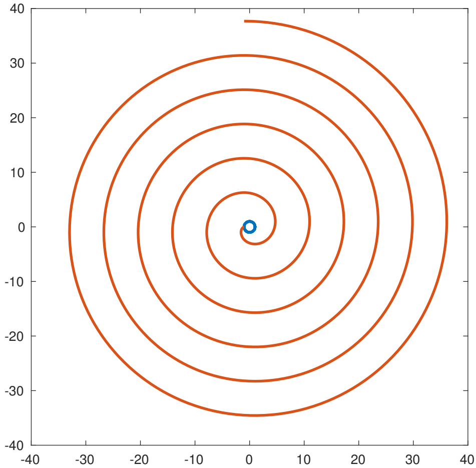

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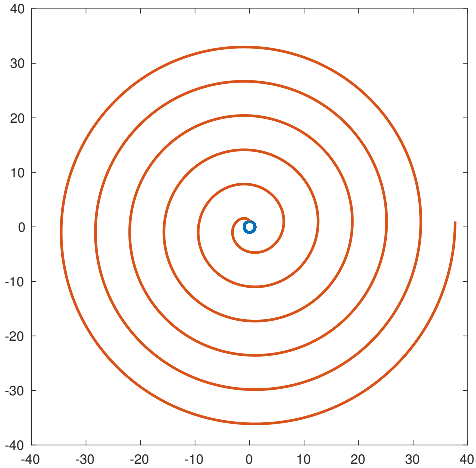


Today

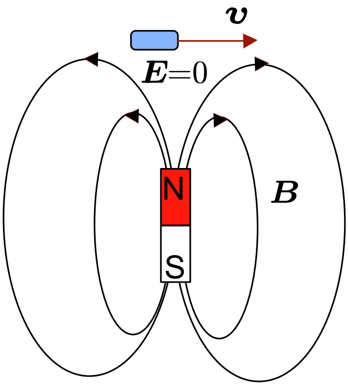
Special relativity

Special relativity

ON THE ELECTRODYNAMICS OF MOVING BODIES By A. Einstein June 30, 1905

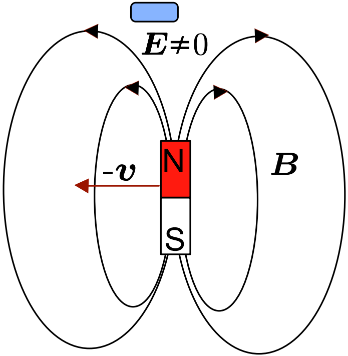
It is known that Maxwell's electrodynamics–as usually understood at the present time–when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. . . .

The asymmetry in Maxwell's equations



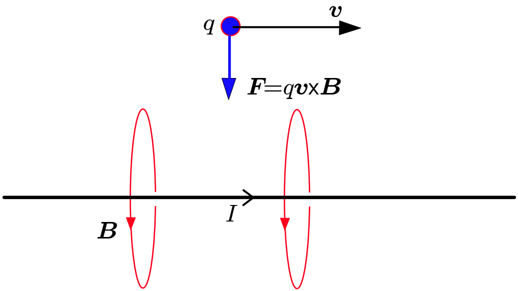
Magnet system: The conductor moves with velocity $\boldsymbol{v}$. The Lorentz force $\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B}$ induces currents in conductor.

The asymmetry in Maxwell's equations



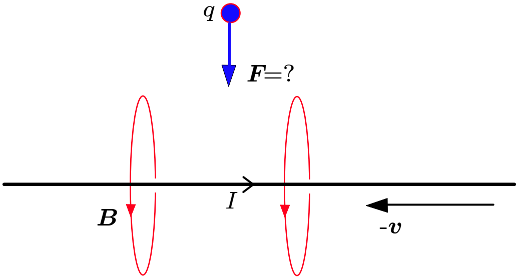
Conductor system: The magnet moves with velocity $-\boldsymbol{v}$. The electric force $\boldsymbol{F} = q\boldsymbol{E}$ induces currents in conductor.
Where does $\boldsymbol{E}$ come from?

The asymmetry in Maxwell's equations



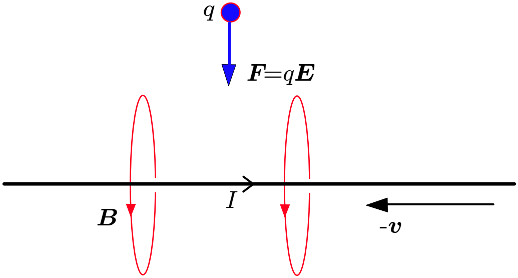
Wire system: The wire has no charge but current I. No electric force on q

The asymmetry in Maxwell's equations



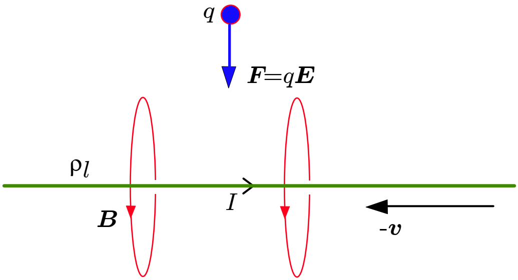
Charge system: Velocity $\boldsymbol{v} = \mathbf{0}$. No magnetic force on q. What mysterious force acts on q?

The asymmetry in Maxwell's equations



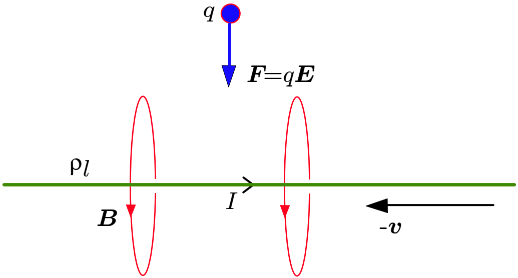
Charge system: Velocity $\boldsymbol{v} = \mathbf{0}$. No magnetic force on q. The force must be electric!

The asymmetry in Maxwell's equations



Charge system: The conductor must have a line charge ρ_ℓ!

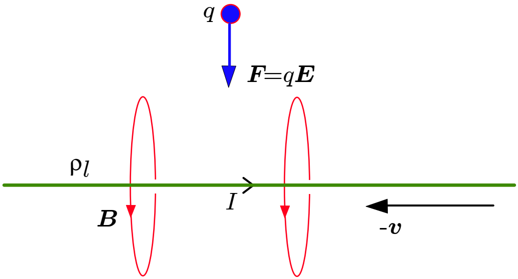
The asymmetry in Maxwell's equations



Wire system: Magnetic force $\boldsymbol{F} = -q\mu_0 \dfrac{vI}{2\pi r_c}\hat{\boldsymbol{z}}$

Charge system: Electric force $\boldsymbol{F} = q\dfrac{\rho_\ell}{2\pi\varepsilon_0 r_c}\hat{\boldsymbol{z}}$

The asymmetry in Maxwell's equations



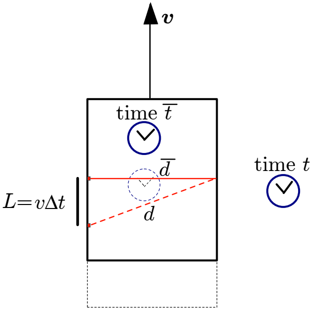
$\rho_\ell = -\frac{vI}{c^2}$. But where does the charges come from?

Einstein's postulates

Inertial system: A system that moves with constant velocity.

1. The principle of relativity. The laws of physics apply to all inertial systems.
2. The universal speed of light. The speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.

Einstein's gedanken experiment

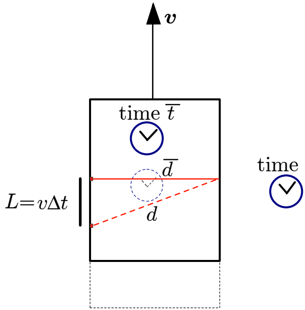


Elevator system: $\bar{d} = c\Delta\bar{t}$, $\bar{L} = v\Delta\bar{t}$
House system: $d = c\Delta t$, $L = v\Delta t$.
But $d^2 = (\bar{d})^2 + (v\Delta t)^2$. Then $(c\Delta t)^2 = (c\Delta\bar{t})^2 + (v\Delta t)^2$.

$$\Delta\bar{t} = \Delta t\sqrt{1-\beta^2} \quad (2)$$

$$\bar{L} = L\sqrt{1-\beta^2} \quad (3)$$

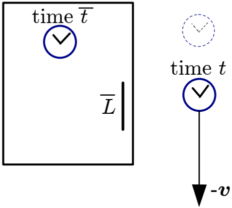
Einstein's gedanken experiment



A person in the house sees that the elevator clock is slowed down with a factor $\sqrt{1-\beta^2}$.

The bar with length L in the house system has the length $\bar{L} = L\sqrt{1-\beta^2}$ in the elevator system.

Einstein's gedanken experiment

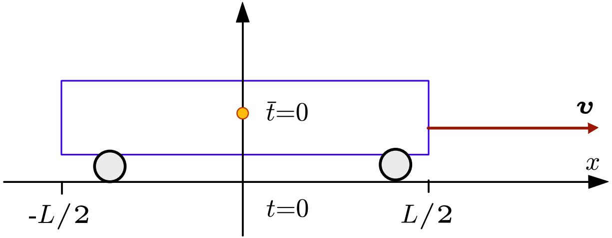


A person in the elevator sees that the house clock is slowed down with a factor $\sqrt{1-\beta^2}$.
The bar with length $\bar{L}$ in the elevator system has the length $L = \bar{L}\sqrt{1-\beta^2}$, measured in the house system.
The width of the elevator is the same in both systems.

Special relativity

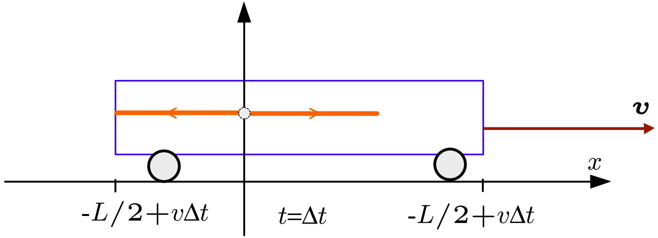
Time is not invariant. Time dilatation.
Length parallell with velocity is not invariant.
Length perpendicular to velocity is invariant.
Two events that are simultaneous in one inertial system do not have to be simultaneous in another inertial system.

Simultaneous events



Train car passing by at velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$.
At time $t = 0$ we see a light bulb turn on.
The person in the middle of the car sees the light turn on at $\bar{t} = 0$.

Simultaneous events



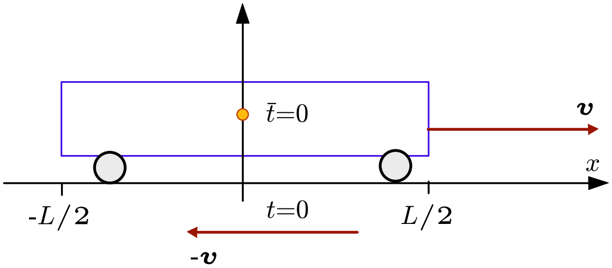
At time $t = \Delta t$ we see the light from the bulb reach the end of the car.

$$\Delta t = \frac{0.5L - v\Delta t}{c}$$

$$\Delta t = \frac{0.5L}{1 + v/c}$$

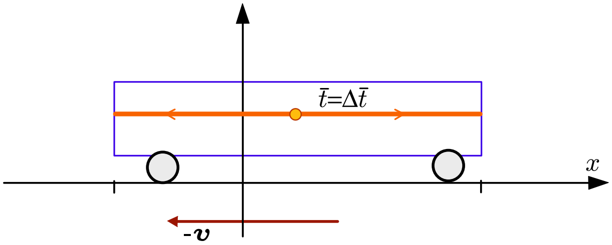
The light has not reached the front of the car yet.

Simultaneous events



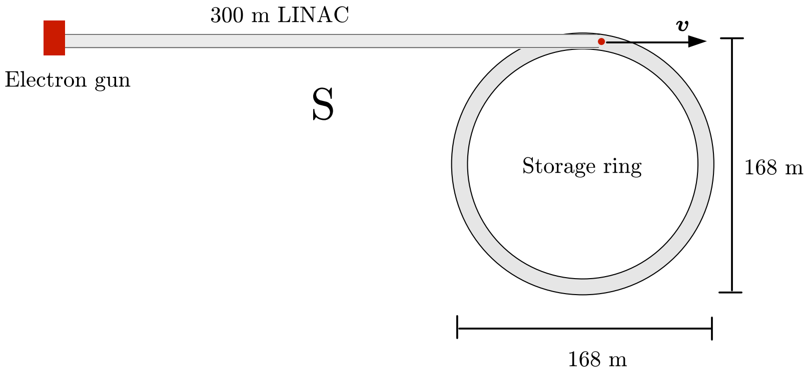
We ask the person in the car what he has seen.
At time $\bar{t} = 0$ the light turned on.

Simultaneous events



We ask the person in the car what he has seen.
At time $\bar{t} = 0$ the light turns on.
At time $\Delta\bar{t}$ the light reaches the end and the front of the car simultaneously!
$\Delta\bar{t} = \dfrac{0.5\bar{L}}{c}$, where $\bar{L}$ is the length of the car measured by the person in the car.

Example: Electron in MAX IV



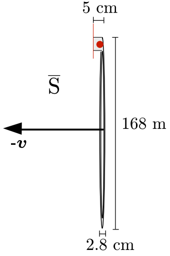
MAX IV seen from system S:

When the electron exits the LINAC it has energy 3 GeV and speed $v = 0.99999998c$.

The length of the LINAC is 300 m

The diameter of the ring is 168 m.

Example: Electron in MAX IV



Storage ring

MAX IV seen from system $\bar{S}$:
The LINAC and storage ring travels with speed $v = 0.99999998c$ seen from the system $\bar{S}$ of the electron.
$\sqrt{1-\beta^2} = 1.69 \cdot 10^{-4}$
The length of the LINAC is 5 cm
The ring is now an ellipse with width 168 m and length 2.8 cm.

Example: Flat earth society

The flat earth society claims that the earth is flat.
They are right, if they travel with a speed of $0.999999999999c$!
Then earth is an oblate spheroid with half axes $a = 6371$ km and $b = 9$ m.

Example: Reaching c

Assume that you manage to travel with speed of light.
How long does it take you to go to the end of the Universe as we see it today (13.6 billion lightyears long in our system)?
What will the Universe look like for you?
Assume that you are reflected from a mirror at the end of the Universe. How long does the reflection take you when you look at the stationary clock at rest in the Universe?