Electrodynamics, lecture 10, 2019

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Electrical and information technology

Last lecture

- Special relativity
- Time dilatation
- Length contraction
- Lorentz transformation

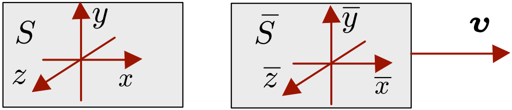
Einstein's postulates

Inertial system: A system that moves with constant velocity.

1. The principle of relativity. The laws of physics apply to all inertial systems.
2. The universal speed of light. The speed of light in vacuum is the same for all inertial observers, regardless of the motion of the source.

Lorentz transformation

Let S and $\bar{S}$ be two inertial systems. At $t = \bar{t} = 0$ their origins coincide. $\bar{S}$ moves with velocity $\boldsymbol{v} = v\hat{\boldsymbol{x}}$ relative S.



An event that occurs at (x, y, z, t) in S occurs at $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ in $\bar{S}$ where

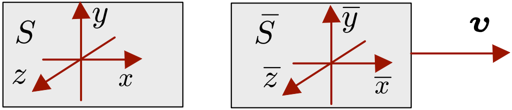
$$\bar{x} = \gamma(x - vt) \tag{1}$$

$$\bar{y} = y \tag{2}$$

$$\bar{z} = z \tag{3}$$

$$\bar{t} = \gamma\left(t - \frac{v}{c^2}x\right) \tag{4}$$

Lorentz transformation



An event $(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ in $\bar{S}$ occurs at (x, y, z, t) in S where

$$x = \gamma(\bar{x} + v\bar{t}) \tag{5}$$

$$y = \bar{y} \tag{6}$$

$$z = \bar{z} \tag{7}$$

$$t = \gamma\left(\bar{t} + \frac{v}{c^2}\bar{x}\right) \tag{8}$$

Today

- Four vectors. Contravariant and covariant.
- Invariants under Lorentz transformation
- Relativistic energy and momentum
- Relativistic dynamics

Four vector

$$x^\mu = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \tag{9}$$

Four vector

The Lorentz transformation

$$\bar{x}^\mu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu \tag{10}$$

where Λ is the matrix

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{11}$$

Four vector

Any vector a^μ; with four components, that transforms as

$$\bar{a}^\mu = \sum_{\nu=0}^{3} \Lambda^\mu_\nu a^\nu \tag{12}$$

is a four vector

Invariants under Lorentz transformations

The contravariant vector is a^μ whereas the covariant is a_μ where

$$a_\mu = (a_0, a_1, a_2, a_3) \equiv (-a^0, a^1, a^2, a^3) \tag{13}$$

Einstein introduced a summation rule to get rid of the summation sign

$$a^\mu b_\mu = a_\mu b^\mu = \sum_{n=0}^{3} a^n b_n \tag{14}$$

The scalar product $a^\mu a_\mu$ is an invariant under Lorentz transformation!

$$-(\bar{a}^0)^2+(\bar{a}^1)^2+(\bar{a}^2)^2+(\bar{a}^3)^2 = -(a^0)^2+(a^1)^2+(a^2)^2+(a^3)^2 \tag{15}$$

Invariants under Lorentz transformations

For x^μ we have the invariant

$$x^\mu x_\mu = -c^2 t^2 + x^2 + y^2 + z^2$$

The distance between two events x_A^μ and x_B^μ is $\Delta x^\mu = x_A^\mu - x_B^\mu$. The distance is

1. Spacelike if $\Delta x^\mu \Delta x_\mu > 0$
2. Lightlike if $\Delta x^\mu \Delta x_\mu = 0$
3. Timelike if $\Delta x^\mu \Delta x_\mu < 0$