Electrodynamics 2019: Lecture 4

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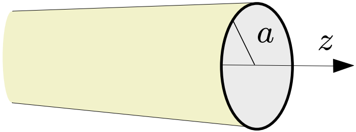
Electrical and information technology

Last lecture

- TM-modes Circular cylindric waveguides
- Comsol

Circular waveguide



Cylindrical coordinates (ρ, ϕ, z). Then $\boldsymbol{r} = \rho\hat{\boldsymbol{\rho}} + z\hat{\boldsymbol{z}}$.
TM-modes $H_z = 0$ and

$$\boldsymbol{E} = E_\rho\hat{\boldsymbol{\rho}} + E_\phi\hat{\boldsymbol{\phi}} + \hat{\boldsymbol{z}}E_z, \quad (1)$$

$$\boldsymbol{H} = H_\rho\hat{\boldsymbol{\rho}} + H_\phi\hat{\boldsymbol{\phi}} \quad (2)$$

$E_z(\boldsymbol{r}) = J_m(k_{tmn}\rho)\,(A_{mn}\cos m\phi + B_{mn}\sin m\phi)\,e^{\mathrm{i}k_z z}$
$m = 0,\, 1,\, 2\ldots,\; n = 1,\, 2\ldots$

Circular waveguide

$k_{tmn} = \dfrac{\xi_{mn}}{a}$, $J_m(\xi_{mn}) = 0$,

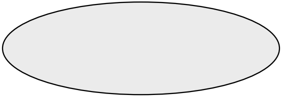
Cut-off frequencies TM: $k_z = 0 \Rightarrow k = k_{tmn} \Rightarrow f_{mn} = \dfrac{c}{2\pi}\dfrac{\xi_{mn}}{a}$

Fundamental TM-mode: TM_{01} with $f_{01} = \dfrac{c}{2\pi}\dfrac{2.405}{a}$

Example: $a = 15$ mm gives $f_{01} = 7.65$ GHz

Circular waveguide

Comparison with membrane:



Resonance frequencies$=f_{mn} = \dfrac{v}{2\pi}\dfrac{\xi_{mn}}{a}$

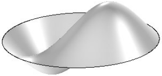
Shape$=\sum_{m=0}^{\infty}\sum_{n=1}^{\infty} J_m(k_{tmn}\rho)\left(A_{mn}\cos m\phi + B_{mn}\sin m\phi\right)$

Circular waveguide

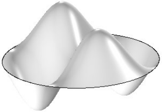
Guitarr string

Drum head

f_1



$1.59 f_1$



$2.13 f_1$

$2.30 f_1$

Circular waveguide

TE-modes, $E_z = 0$

$$\boldsymbol{E} = E_\rho \hat{\boldsymbol{\rho}} + E_\phi \hat{\boldsymbol{\phi}}, \quad (3)$$

$$\boldsymbol{H} = H_\rho \hat{\boldsymbol{\rho}} + H_\phi \hat{\boldsymbol{\phi}} + \hat{\boldsymbol{z}} H_z \quad (4)$$

$$H_z(\boldsymbol{r}) = J_m(k_{tmn}\rho)\left(A_{mn}\cos m\phi + B_{mn}\sin m\phi\right) e^{\mathrm{i}k_z z}$$

$$k_{tmn} = \frac{\eta_{mn}}{a},\ J'_m(\eta_{mn}) = 0,\ \begin{cases} m = 0, 1, 2 \ldots \\ n = 1, 2 \ldots \end{cases}$$

Circular waveguide

Cut-off frequencies, $f_{mn} = \dfrac{c}{2\pi}\dfrac{\eta_{mn}}{a}$

Fundamental mode: TE_{11} with $f_{11} = \dfrac{c}{2\pi}\dfrac{1.841}{a}$

Example: $a = 15$ mm gives $f_{11} = 5.860$ GHz

Transverse components $\boldsymbol{E}_{\mathrm{T}}$ and $\boldsymbol{H}_{\mathrm{T}}$

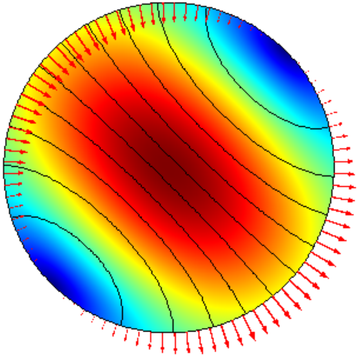
From Maxwell's equations it follows that the transverse components can be expressed in E_z and H_z:

$$\begin{cases} \boldsymbol{E}_{\mathrm{T}}(\boldsymbol{r}) = \dfrac{\mathrm{i}}{k_t^2}\left\{k_z \nabla_T E_z(\boldsymbol{r}) - \omega\mu_0 \hat{\boldsymbol{z}} \times \nabla_{\mathrm{T}} H_z(\boldsymbol{r})\right\} \\ \boldsymbol{H}_{\mathrm{T}}(\boldsymbol{r}) = \dfrac{\mathrm{i}}{k_t^2}\left\{k_z \nabla_{\mathrm{T}} H_z(\boldsymbol{r}) + \omega\varepsilon_0 \hat{\boldsymbol{z}} \times \nabla_{\mathrm{T}} E_z(\boldsymbol{r})\right\} \end{cases}$$

(Equation (15) in the exercise book)

Transverse electric field $\boldsymbol{E}_T$.

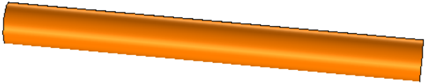
Fundamental mode TE_{11}.



Today

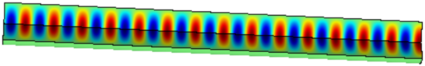
- Traveling wave cavity
- Cavities
- The pillbox cavity

Traveling wave cavity

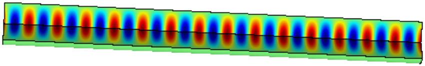


Traveling wave cavity

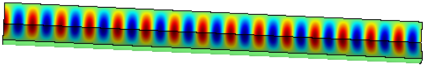
Traveling wave cavity



Traveling wave cavity



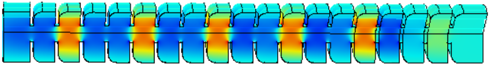
Traveling wave cavity



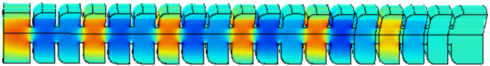
Traveling wave cavity

Traveling wave cavity

Traveling wave cavity

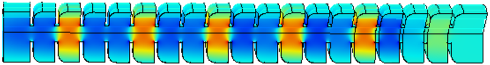


Traveling wave cavity

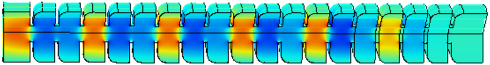


Traveling wave cavity

Traveling wave cavity

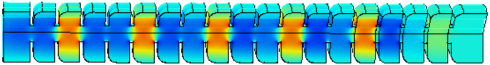


Traveling wave cavity

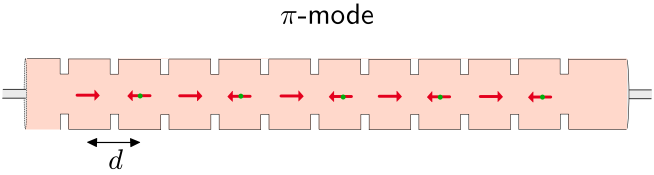


Traveling wave cavity

Traveling wave cavity



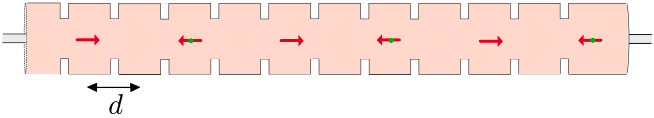
Design of traveling wave cavity for MAX IV



$$d = \frac{cT}{2}$$

Design of traveling wave cavity for MAX IV

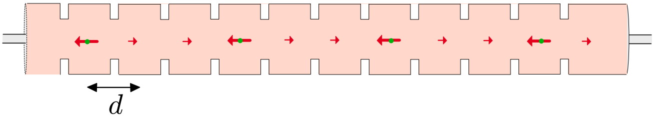
$\frac{\pi}{2}$-mode



$$d = \frac{cT}{4}$$

Design of traveling wave cavity for MAX IV

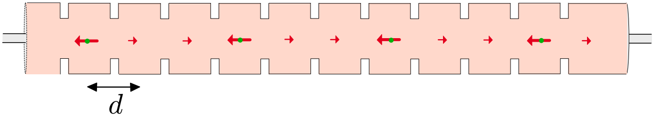
$\frac{2\pi}{3}$-mode



$$d = \frac{cT}{3}$$

Design of traveling wave cavity for MAX IV

$\frac{2\pi}{3}$-mode



Frequency$=3$ GHz $\Rightarrow T=333$ ps and $d=3.33$ cm

The $\frac{2\pi}{3}$-mode