Electrodynamics 2019: Lecture 2

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Electrical and information technology

Today's lecture covers pages 7–15 in the Exercise book.

Last lecture

- The Maxwell equations in time domain.
- The Maxwell equations in frequency domain.
- The Vector Helmholtz equation.
- Introduction to waveguides.
- Helmholtz equation and boundary conditions for E_z.
- Helmholtz equation and boundary conditions for H_z.

Maxwell in time domain

The Maxwell equations in vacuum are

$$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t} \quad (1)$$

$$\nabla \times \boldsymbol{B}(\boldsymbol{r},t) = \mu_0 \boldsymbol{J}(\boldsymbol{r},t) + \frac{1}{c^2}\frac{\partial \boldsymbol{E}(\boldsymbol{r},t)}{\partial t} \quad (2)$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r},t) = \frac{\rho(\boldsymbol{r},t)}{\varepsilon_0} \quad (3)$$

$$\nabla \cdot \boldsymbol{B}(\boldsymbol{r},t) = 0. \quad (4)$$

Maxwell in time domain

Since $\boldsymbol{B} = \mu_0 \boldsymbol{H}$.

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}, t) = -\mu_0 \frac{\partial \boldsymbol{H}(\boldsymbol{r}, t)}{\partial t} \tag{5}$$

$$\nabla \times \boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{J}(\boldsymbol{r}, t) + \varepsilon_0 \frac{\partial \boldsymbol{E}(\boldsymbol{r}, t)}{\partial t} \tag{6}$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}, t) = \frac{\rho(\boldsymbol{r}, t)}{\varepsilon_0} \tag{7}$$

$$\nabla \cdot \boldsymbol{H}(\boldsymbol{r}, t) = 0. \tag{8}$$

Maxwell in frequency domain

Time harmonic signals:
$\boldsymbol{E}(\boldsymbol{r})$ is the complex representation of $\boldsymbol{E}(\boldsymbol{r},t)$. The rule is
$\boldsymbol{E}(\boldsymbol{r},t) = \mathrm{Re}\{\boldsymbol{E}(\boldsymbol{r})e^{-\mathrm{i}\omega t}\}$

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = \mathrm{i}\omega\mu_0 \boldsymbol{H}(\boldsymbol{r}) \tag{9}$$

$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{J}(\boldsymbol{r}) - \mathrm{i}\omega\varepsilon_0 \boldsymbol{E}(\boldsymbol{r}) \tag{10}$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = \frac{\rho(\boldsymbol{r})}{\varepsilon_0} \tag{11}$$

$$\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) = 0. \tag{12}$$

Vector Helmholtz equation

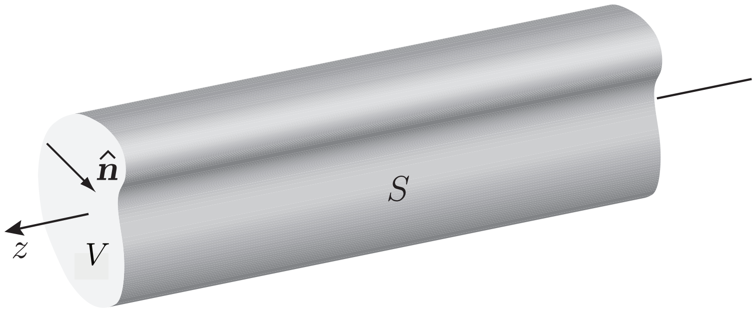
Consider a source free region ($\boldsymbol{J} = \boldsymbol{0}$, $\rho = 0$) with vacuum. Then:

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + k^2 \boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{0}$$
$$\nabla^2 \boldsymbol{H}(\boldsymbol{r}) + k^2 \boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{0}$$

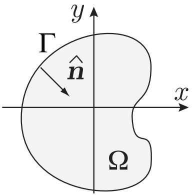
Also

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{0}$$
$$\nabla \cdot \boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{0}$$

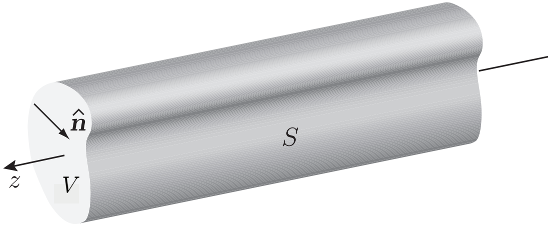
Hollow waveguide



Hollow waveguide



Hollow waveguide

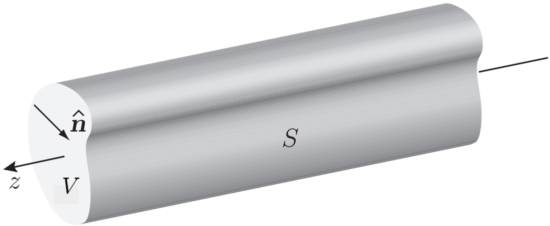


$$\nabla^2 E_z(\boldsymbol{r}) + k^2 E_z(\boldsymbol{r}) = 0, \;\; \boldsymbol{r} \in V \tag{13}$$

Boundary condition

$$E_z(\boldsymbol{r}) = 0, \;\; \boldsymbol{r} \in S \tag{14}$$

Hollow waveguide

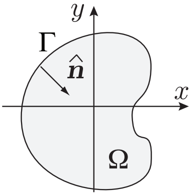


$$\nabla^2 H_z(\boldsymbol{r}) + k^2 H_z(\boldsymbol{r}) = 0, \;\; \boldsymbol{r} \in V \tag{15}$$

Boundary condition

$$\hat{\boldsymbol{n}} \cdot \nabla H_z(\boldsymbol{r}) = 0, \;\; \boldsymbol{r} \in S \tag{16}$$

Hollow waveguide



$$\boldsymbol{r} = \boldsymbol{\rho} + z\hat{\boldsymbol{z}}$$

$$\boldsymbol{E} = \boldsymbol{E}_{\mathrm{T}} + E_z\hat{\boldsymbol{z}}$$

$$\boldsymbol{H} = \boldsymbol{H}_{\mathrm{T}} + H_z\hat{\boldsymbol{z}}$$

$$\nabla = \nabla_{\mathrm{T}} + \frac{\partial}{\partial z}\hat{\boldsymbol{z}}$$

$$\nabla^2 = \nabla_{\mathrm{T}}^2 + \frac{\partial^2}{\partial z^2}$$

Today

- $z-$dependence: $E_z(\boldsymbol{r}) = v(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
- Eigenvalue problem for $v(\boldsymbol{\rho})$
- $z-$dependence: $H_z(\boldsymbol{r}) = w(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
- Eigenvalue problem for $w(\boldsymbol{\rho})$
- Find $v(\boldsymbol{\rho})$ and k_z for rectangular waveguide.
- Find $w(\boldsymbol{\rho})$ and k_z for rectangular waveguide.
- Introduction to Comsol.