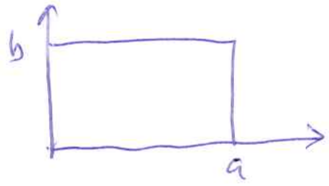


TM-waves $H_z = 0$

$$E_z = v(\bar{\rho})\, e^{i k_z z}$$

$$\begin{cases} \nabla_T^2 v(\bar{\rho}) + k_t^2 v = 0 & \text{in } \Omega \\ v(\bar{\rho}) = 0 & \text{on } \Gamma \end{cases}$$

$$\Rightarrow \quad v(\bar{\rho}) = \frac{2}{\sqrt{ab}} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

$$k_{tmn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2, \qquad m = 1, 2, 3 \ldots \quad n = 1, 2 \ldots$$

$f_c = \frac{c}{2\pi} k_{tmn}$ v and f_c are the same as for a membrane. A ~~drum~~ rectangular drum. v = displacement

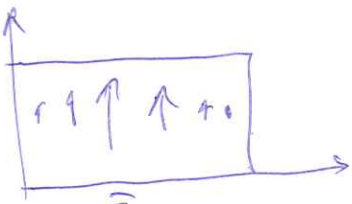
TE$_{10}$ the fundamental mode

$$\bar{E} = \bar{E}_T$$

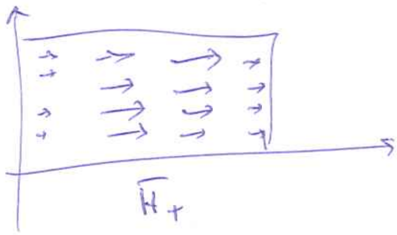
$$\bar{H} = \bar{H}_T + \hat{z}\, w(\bar{\rho})\, e^{i k_z z}$$

$$\text{Eq (15)} \Rightarrow \quad \bar{E}_T = \hat{y}\, \frac{i\omega\mu_0}{\pi} \sqrt{\frac{2a}{b}} \sin\left(\frac{\pi x}{a}\right) e^{i k_z z}$$

$$\bar{H}_T = -\hat{x}\, i\, \frac{k_z}{\pi} \sqrt{\frac{2a}{b}} \sin\left(\frac{\pi x}{a}\right) e^{i k_z z}$$

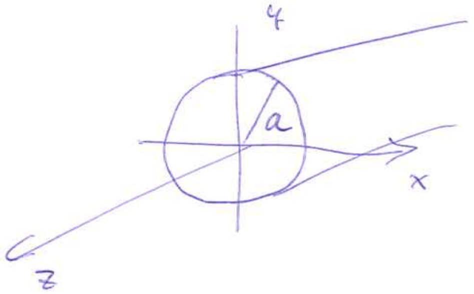


$\bar{E}_T$



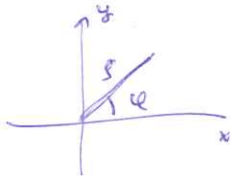
$\bar{H}_T$

Circular waveguides



Cylindrical coordinates (ρ, φ, z)

$\Rightarrow \quad \vec{r} = \rho \hat{\rho} + z \hat{z}$



TM-modes

$H_z = 0 \qquad E_z = \psi(\rho, \varphi)\, e^{i k_z z}$

Eigenvalue problem

$$\begin{cases} \nabla_T^2 \psi + k_t^2 \psi = 0 \\ \psi(a, \varphi) = 0 \\ |\psi(0, \varphi)| < \infty \\ \psi(\rho, \varphi) = \psi(\rho, \varphi + 2\pi) \end{cases}$$

Cylindrical coord. (see appendix in P. B.) 3

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial v}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 v}{\partial\varphi^2} + k_t^2 v = 0$$

Separation of variables

$$v = f(\rho)\cdot g(\varphi)$$

$$\Rightarrow \quad g\cdot\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial f}{\partial\rho} + \frac{f}{\rho^2}\frac{\partial^2 g}{\partial\varphi^2} + k_t^2 f\cdot g = 0$$

Divide by $\frac{f\cdot g}{\rho^2}$

$$\Rightarrow \quad \frac{1}{f}\rho\frac{\partial}{\partial\rho}\rho\frac{\partial f}{\partial\rho} + k_t^2\rho^2 = -\frac{1}{g}\frac{\partial^2 g}{\partial\varphi^2} \qquad (1)$$

$\Rightarrow$ LHS = RHS = constant $= \lambda$

$$\Rightarrow \quad g'' + \lambda g = 0 \qquad (2)$$
$$g(\varphi) = g(\varphi + 2\pi) \qquad (3)$$

$\Rightarrow$

$(2) \Rightarrow g(\varphi) = A\cos\sqrt{\lambda}\,\varphi + B\sin\sqrt{\lambda}\,\varphi$

$(3) \Rightarrow \lambda = m^2 \qquad m = 0, 1, 2 \ldots$

$$(1) \Rightarrow \quad \rho\frac{\partial}{\partial\rho}\rho\frac{\partial f}{\partial\rho} + k_t^2\rho^2 f - m^2 f = 0$$

Bessel's diff. eq.
see App. in P.B.

$$\Rightarrow \quad f = C_m J_m(k_t\rho) + D_m N_m(k_t\rho)$$

Bessel fcn

Neumann fcn $|N_m(k_t\rho)| \to \infty$ as $k_t\rho \to 0$

$\Rightarrow D_m = 0$

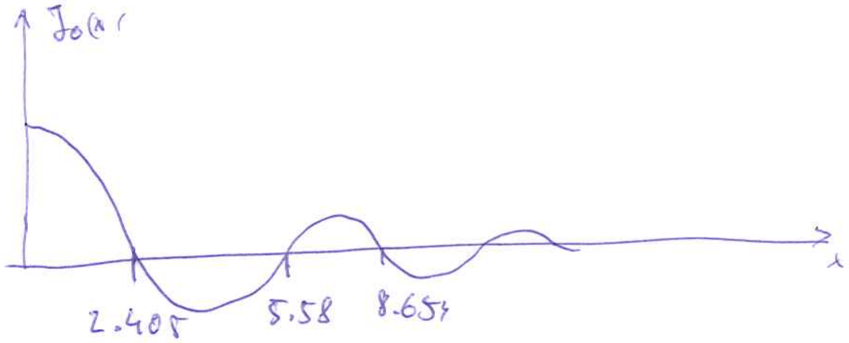
$\Rightarrow f(\rho) = C_m J_m(k_t \rho)$

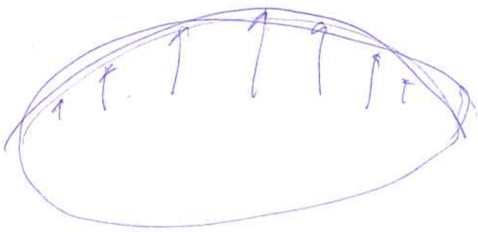
$f(a) = 0 \Rightarrow \quad k_t \cdot a = \xi_{mn}, \quad m = 0, 1, 2 \ldots \quad n = 1, 2 \ldots$

where ξ_{mn} = n:th zero of $J_m(x)$

$$\therefore \boxed{\begin{aligned} E_{zmn} &= J_m(k_{tmn}\rho)\,(A_{mn}\cos m\varphi + B_{mn}\sin m\varphi)\, e^{ik_{zmn}z} \\ k_{tmn} &= \frac{\xi_{mn}}{a} \end{aligned}}$$

Ex $\quad m = 0$





TM

Table 4 in App. 1 gives the lowest zero

$\xi_{01} = 2.405$

$\Rightarrow$ Fundamental TM-mode is TM_{01}

Cut off for $f_c = \frac{c}{2\pi} \cdot k_{t01} = \frac{c \cdot 2.405}{2\pi \cdot a}$

<u>Ex</u> $a = 1$ cm $\Rightarrow$ $f_{c_{01}} = 11.48$ GHz

<u>TE-modes</u>

$E_z = 0 \quad H_z = w(\rho, \varphi)\, e^{ik_z z}$

$$\begin{cases} \nabla_T^2 w + k_t^2 w = 0 \\ \frac{\partial w}{\partial \rho}(a, \varphi) = 0 \qquad |w(0, \varphi)| < \infty \\ w(\rho, \varphi) = w(\rho, \varphi + 2\pi) \end{cases}$$

$$\Rightarrow \quad w(\rho, \varphi) = J_m(k_{tmn}\rho)\left(C_m \cos m\varphi + D_m \sin m\varphi\right)$$

$m = 0, 1, 2 \ldots$
$n = 1, 2 \ldots$

where $k_{tmn} = \frac{\eta_{mn}}{a}$

and $\eta_{mn} =$ n:th zero of $J_m'(x)$

Table 5 in App. 1 says that the lowest η_{mn} is

$\eta_{11} = 1.841$

$\Rightarrow$ Fundamental TE mode is TE_{11}

Cut off: $f_{c_{11}} = \frac{c}{2\pi} \cdot \frac{1.841}{a}$

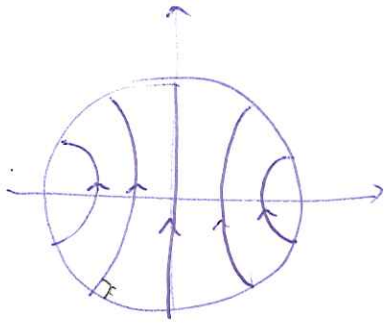
<u>Ex</u> $a = 1$ cm $\Rightarrow$ $f_c = 8.79$ GHz

TE_{11}

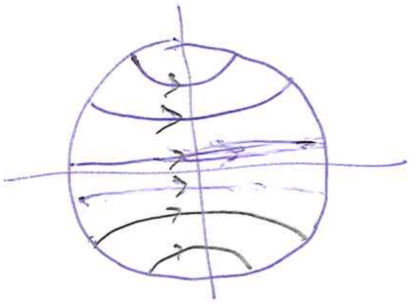
$$5.24 \Rightarrow \bar{E}_{10}(\bar{r}) = -\frac{i\omega}{k_{t10}^2}\,\mu_0\,\hat{z} \times \bar{\nabla}_T W_{11}(\rho, \varphi)$$

where $\nabla_T = \hat{\rho}\,\frac{\partial}{\partial \rho} + \hat{\varphi}\,\frac{1}{\rho}\,\frac{\partial}{\partial \varphi}$

$\Rightarrow$



or



Compare with TE_{10}