April 4, 2019

EITN80 Electrodynamics: Assignment 1

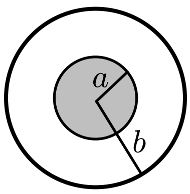
Hand in your solutions no later than 10.15 AM on April 11. You may hand in a paper copy, or mail a pdf file. If you have problems with Comsol you can ask Anders for advice.

1. Cut-off frequencies for coaxial waveguide

Coaxial waveguides are often used in particle accelerators. There are several reasons for this;

- The waveguide can receive, transport and transmit electromagnetic waves.
- There is no leakage of waves.
- The fundamental mode is the TEM mode, which propagates for all frequencies.
- Both the phase speed and group speed of the fundamental mode are equal to the speed of light for all frequencies.

The losses in coaxial waveguides comes from the currents running along the outer and inner conductor. The losses decrease with increasing radii of the conductors. If the radii increase they eventually reach values where higher order modes start to propagate.



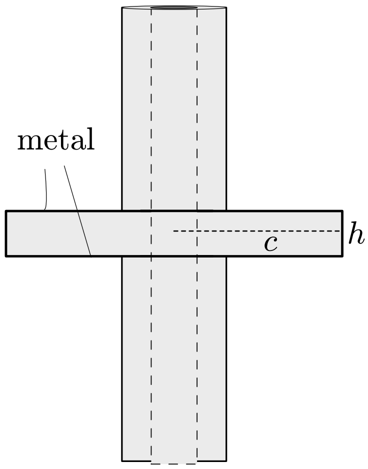
a) Consider a coaxial waveguide with vacuum between the conductors. Let the inner conductor have radius $a = 1$ cm and the outer conductor an inner radius of $b = 2.3$ cm. Find the frequency where the second mode can start to propagate.

b) Determine if the second mode is a TE- or TM-mode.

Hints: 2D. Electromagnetic waves. Eigenfrequency. Booleans and Partitions. Difference. The fundamental mode does not show up as a resonance in Comsol so you should look for the lowest resonance frequency that is larger than zero.

2. Bandstopp filter

Consider that you like to use the coaxial waveguide in problem 1 for transmitting signals with frequencies between 100 MHz and 2.2 GHz. Close to the transmitter there is a source that generates a signal with frequency 1.6 GHz. Also this signal is transmitted and you have to get rid of it. To do this you build a bandstop filter. You cut the coaxial waveguide in two pieces and put a short coaxial waveguide between the two pieces, as in the figure. The extra waveguide also has the radius 1 cm of the inner conductor. To find the proper radius c of the outer conductor and the height h you use Comsol. The flat surfaces between the outer conductors, marked in the figure, are also made of metal.



Determine a and h such that all frequencies except 1.6 GHz pass. The width of the stop band should be less than 50 MHz at the 3 dB level.

Hints: Axial symmetry. Type of port: Coaxial. S-parameters. What affects the frequency and what affects the bandwidth?

3. Resonance frequencies of pill-box cavity

Design a pill-box cavity, filled with air, such that the lowest axially symmetric TM-mode has the frequency 3.52 GHz and the second lowest axially symmetric TM-mode has the frequency 7.04 GHz. Give the radius and height of the cavity. Use the theory for cavities to solve the problem. Check with Comsol so that your design is correct.

4. Bandpass filter

You like to use the coaxial waveguide in problem 1 between a transmitting unit and a receiving unit. The transmitter now uses the two frequencies 352.0 MHz and 704.0 MHz and it is important that only signals with these two frequencies reach the receiver. Unfortunately the transmitter also generates noise with frequencies ranging from 0 Hz up to 750 MHz. To get rid of the noise you have to build a bandpass filter. You have no electronic components but you have tools to make pill box cavities of any size. You make the filter by cutting the coaxial waveguide and attach the two ends to the cavity.

First you might have some ideas on what length and radius the pill-box should have. Test your ideas in Comsol and then you can tune the design in Comsol.

It is enough if your filter has full transmission for one frequency in the interval 351.9 MHz$< f <$ 352.1 GHz and one frequency in the interval 703.9 MHz$< f <$ 704.1 MHz.

Give the dimensions of the pill-box cavity, describe how you attach the coaxial cables and provide a plot of S_{21}, in dB scale, in a region around each of the two frequencies.

Hints: Axial symmetry. Frequency domain. Port. Type of port: Coaxial. S-parameters.

The definition of the ´S-parameters is as follows:

- S_{11} =the reflection coefficient for port 1. It is the given by $|S_{11}| = \sqrt{P_\text{r}/P_\text{in}}$, where P_r is the reflected power for port 1 and P_in the incident power at port 1
- S_{21} =the transmission coefficient from port 1 to port 2. It is given by $|S_{21}| = \sqrt{P_\text{t}/P_\text{in}}$, where P_t is the transmitted power to port 2 and P_in the incident power at port 1.

The definition of the dB scale is that $|S_{11}|_\text{sB} = 20\log_{10}(|S_{11}|)$.

The TEM-mode for coaxial waveguides

The fundamental mode of the coaxial waveguide is the TEM-mode. The electric and magnetic fields of this mode is, in cylindrical coordinates (ρ, ϕ, z),

$$\boldsymbol{E}(\boldsymbol{r}) = E_0 \frac{a}{\rho} e^{\text{i}kz} \hat{\boldsymbol{\rho}} \tag{0.1}$$

$$\boldsymbol{H}(\boldsymbol{r}) = \eta_0^{-1} E_0 \frac{a}{\rho} e^{\text{i}kz} \hat{\boldsymbol{\phi}} \tag{0.2}$$

where a is the radius of the inner conductor and E_0 is an amplitude. Thus the magnetic and electric fields are related by the same right hand rule, $\boldsymbol{H} = \eta_0^{-1} \hat{\boldsymbol{z}} \times \boldsymbol{E}$, as we use for plane waves.