Extra problems – ETIN15
Batch 2

2015-04-21

1. A digital transmission system uses, for forward error-correction, a $R = 1/2$ recursive convolutional code with memory depth 3. The channel encoder is placed between a data source with an output data rate of 64 kbit/sec and a 4QAM modulator.

   (a) What is the coded bit rate after the channel encoder?

   (b) At what rate are the 4QAM symbols transmitted over the channel?

   (c) Assume that the modulation basis pulses are rectangular and that we have used scrambling to remove any (noticeable) dependence between the code bits. How large a bandwidth is required to contain 99% of the energy of the radio (modulation) spectrum?

   (d) If the channel between transmitter and receiver can be modeled as a simple additive white Gaussian noise (AWGN) channel without time dispersion, how many states would we need in a Viterbi decoder to perform maximum-likelihood detection?

   (e) We would like to compensate for the "increased number of bits", due to coding, and make the bandwidth similar to what we would have had without coding. To do this, we change the modulation to something else. Propose a different modulation and give a simple explanation of why it is a suitable choice.

2. The Viterbi algorithm (VA) can be applied both for decoding error-correcting codes and performing channel equalization. In principle it is the same problem being solved in both cases – under the right circumstances the algorithm traveses a trellis generated by a *finite state machine* and finds the *best* path.

   (a) The VA must know the structure of the *finite state machine* to operate correctly. What is it that determines this structure, for the above two applications?

   (b) Assume that the channel over which transmission takes place is an additive white Gaussian (AWGN) channel and that the Euclidean metric is used by the VA. What do we mean when we say that the VA finds the *best* path – in what sense is it the *best* path?

3. Assume that we have a single carrier digital communication system, operating with 4QAM modulation, and is intended to carry a data-rate of 100 kbit/sec. The communication

channel is affected by Rayleigh fading and the average bit error rate (BER) required is 10^{-2} (by using ARQ on top of this the error rate can be made much lower, but this is beyond the scope of this problem).

(a) What average C/N_0 is required at the receiver side, if no channel coding is used?

(b) What average C/N_0 is required at the receiver side, if a rate 1/2 error correcting code and proper interleaving are added? The code has a 6 dB coding gain (for independent Rayleigh fading at BER = 10^{-2}).

(c) How much larger is the transmission bandwidth in (b) as compared to (a)?

4. A mobile radio link, subjected to Rayleigh fading, is carrying an information data-rate of 10 kbit/sec. The specified quality is expressed as an average bit-error probability of 10^{-4}. The noise temperature of the receiver system is 1000 Kelvin (the noise source placed at the receiver input) and 2ASK is used.

   The first two of these were addressed in the previous batch of extra problems – now, focus on (c) and (d):

   (a) What signal strength (average power) is required at the receiver input, if no countermeasures are used against the fading?

   (b) To improve the system, two antennas and RSSI selection diversity is employed. The distance between the antennas is chosen so that their fading is uncorrelated. How large is the diversity gain and what is the required signal strength (average power) at the receiver input?

   (c) Instead of selection diversity, the system is improved by the use of channel coding. The code employed is a (23,12) Golay code with $d_{\min} = 7$. Interleaving is also introduced, with a depth sufficient to make the errors at the decoder input appear uncorrelated. What coding gain is obtained and what is the required signal strength (average power) at the receiver input?

   (d) The diversity arrangement in b) and the channel coding in c) is combined. How large is the total (selection diversity and coding) gain and what is new the required signal strength (average power)?