

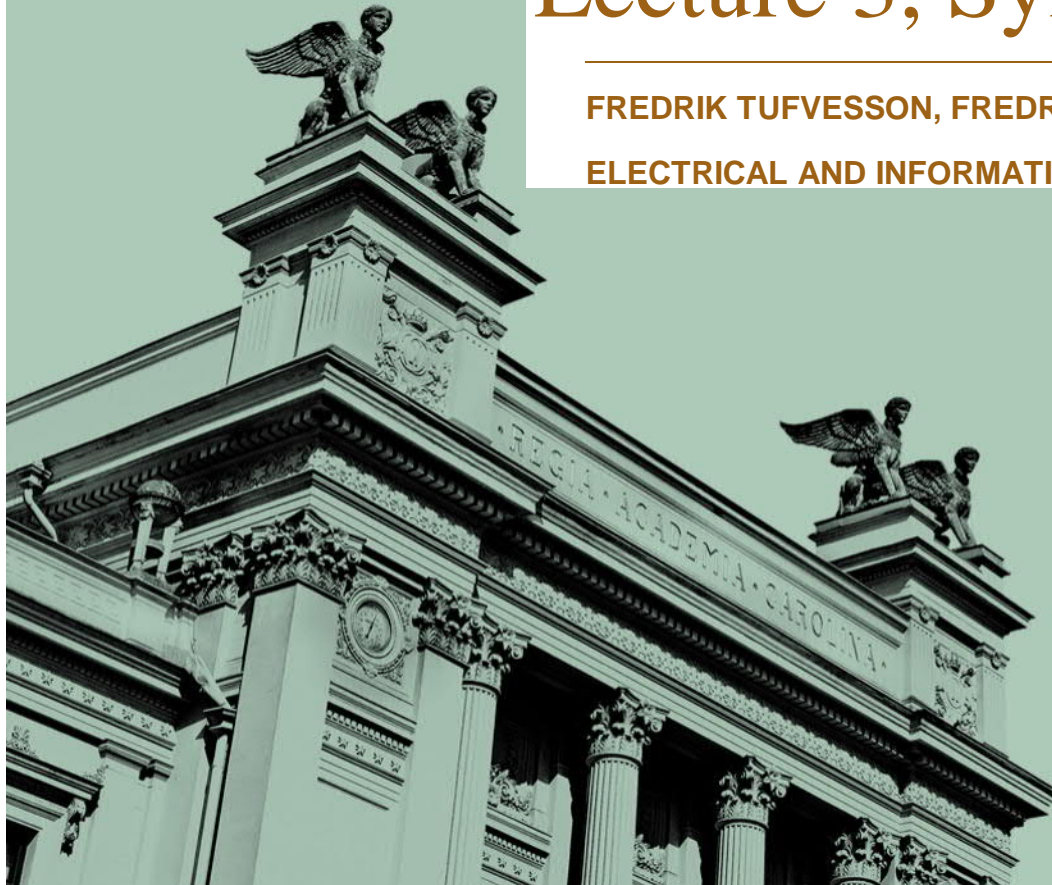


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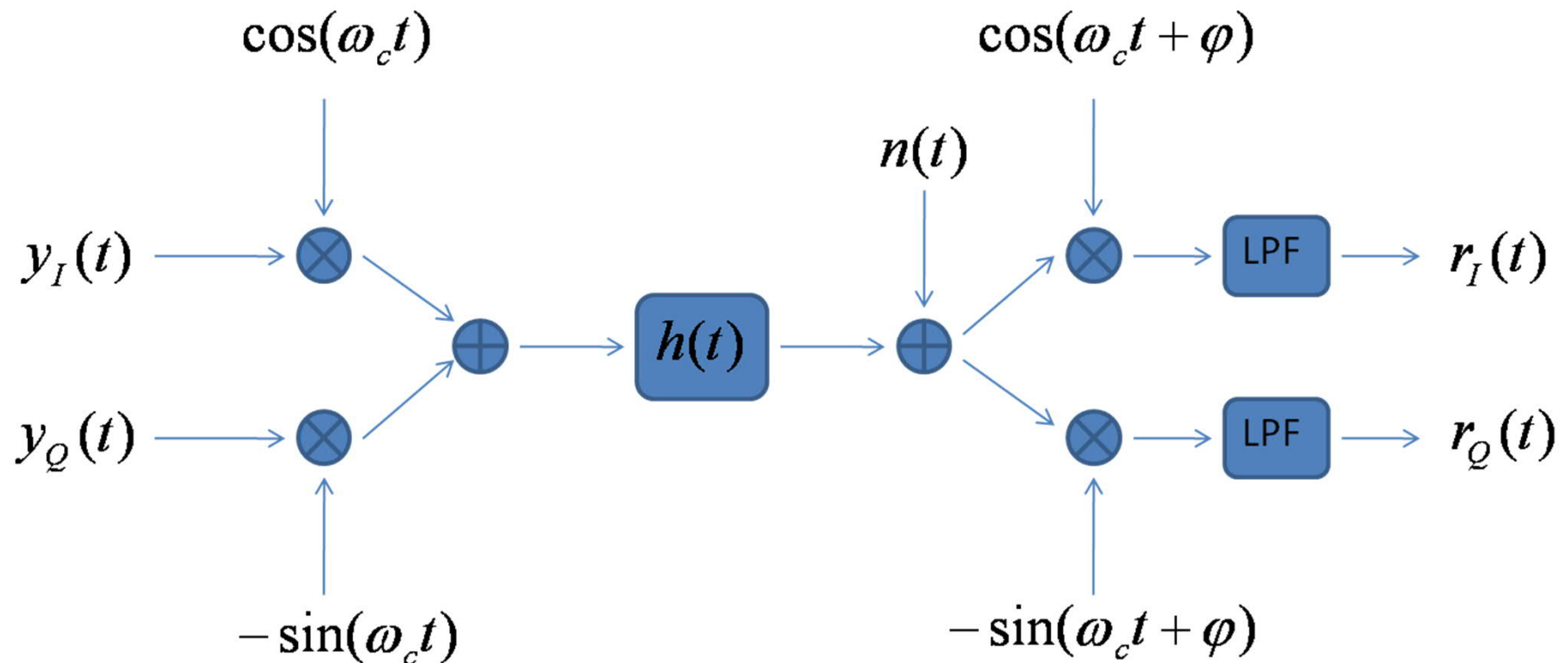
# Project in Wireless Communication Lecture 3, Synchronization

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ELECTRICAL AND INFORMATION TECHNOLOGY



# Final result, complex representation



This can be modeled in the complex baseband as

# Final result, complex representation

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$$h^c(t) \triangleq h_I(t) \cos(\phi) + h_Q(t) \sin(\phi) + j(h_Q(t) \cos(\phi) - h_I(t) \sin(\phi)).$$



$$r^c(t) = y^c(t) \star h^c(t)$$



# Conclusion

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We can always work in the complex baseband domain with the input/output relation

$$r^c(t) = y^c(t) \star h^c(t) + n^c(t)$$

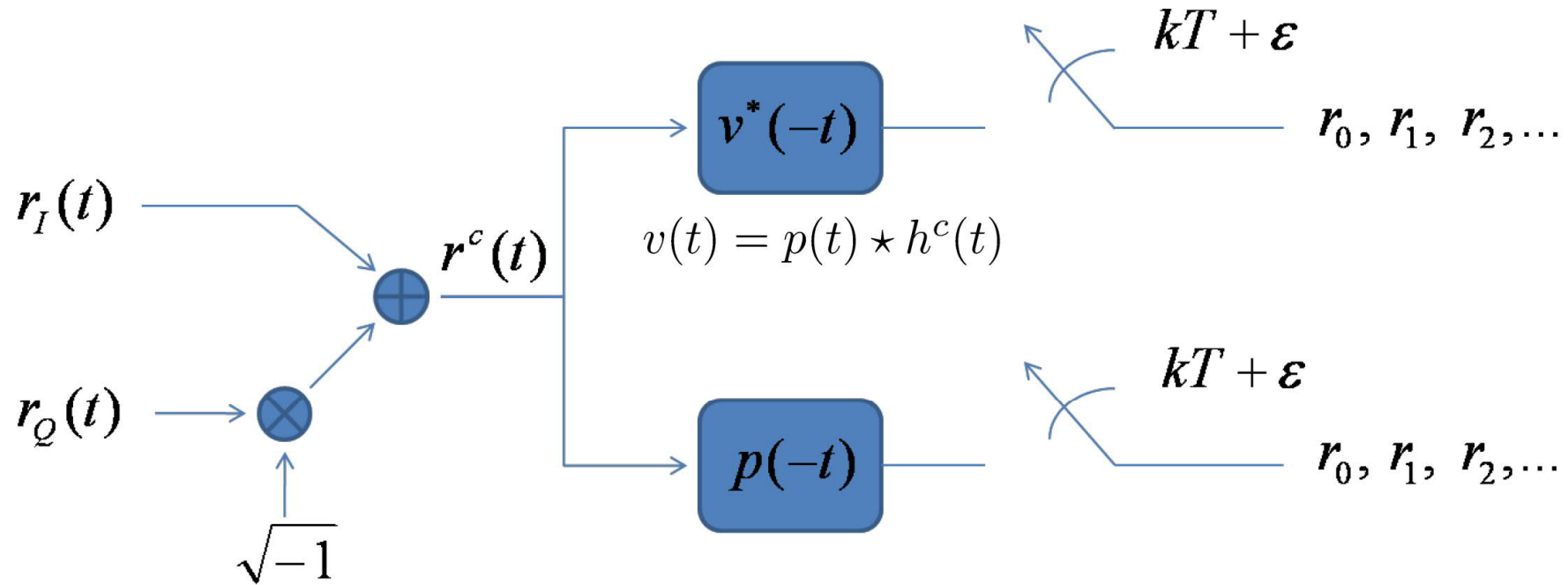
where

$$y^c(t) = \sum_k a_k p(t - kT_s)$$



# Matched filtering

*optimal but not possible*



*suboptimal but possible*



# Matched filter output

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Let  $z(t)$  denote the matched filter. The signal part of each sample  $r_k$  equals

$$\begin{aligned} r_k &= \int_{-\infty}^{\infty} r^c(\tau)z(t - \tau)d\tau \Big|_{t=kT+\epsilon} \\ &= \int_{-\infty}^{\infty} r^c(\tau)p(\tau - t)d\tau \Big|_{t=kT+\epsilon} \\ &= \int_{-\infty}^{\infty} \left( \sum_{\ell} a_{\ell}v(\tau - \ell T) \right) p(\tau - kT - \epsilon)d\tau \\ &= \sum_{\ell} a_{\ell} \int_{-\infty}^{\infty} v(\tau - \ell T)p(\tau - kT - \epsilon)d\tau \\ &= \sum_{\ell} a_{\ell} \int_{-\infty}^{\infty} v(\tau)p(\tau + (\ell - k)T - \epsilon)d\tau \\ &= \sum_{\ell} a_{\ell}h_{k-\ell}, \end{aligned}$$

where

$$h_n \triangleq \int_{-\infty}^{\infty} v(\tau)p(\tau - nT - \epsilon)d\tau.$$

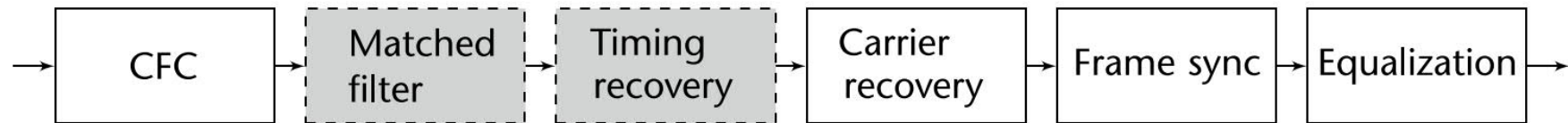


# Examples

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In "Software Defined Radio for Engineers", by Collins et al, different aspects of synchronization is dealt with in chapter 6-9.

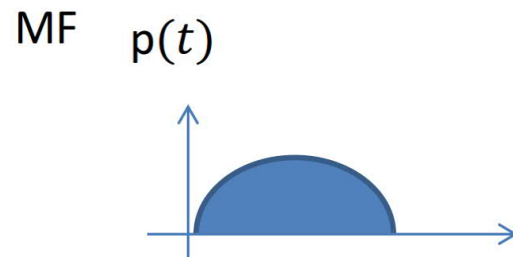
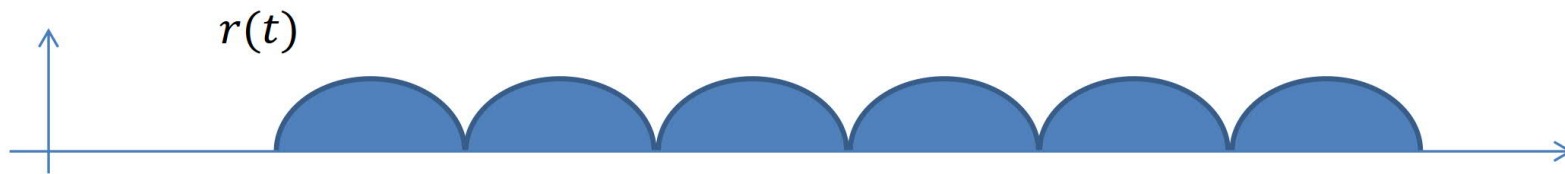
Let's now consider timing recovery after the matched filter.



Source: Software Defined Radio for Engineers, Collins et al

# Synch in the project

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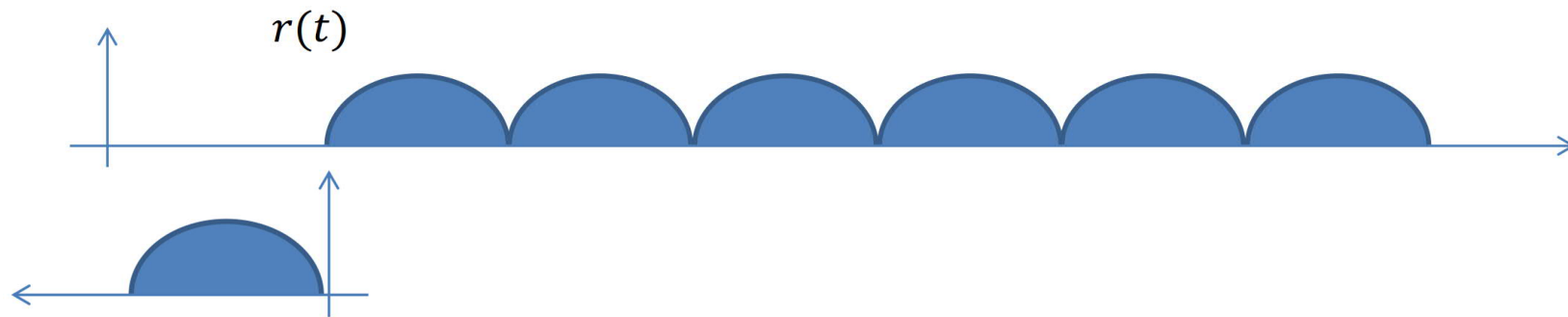


Operation of the MF:  $\int p(\tau)r(t - \tau)d\tau$



# Synch in the project

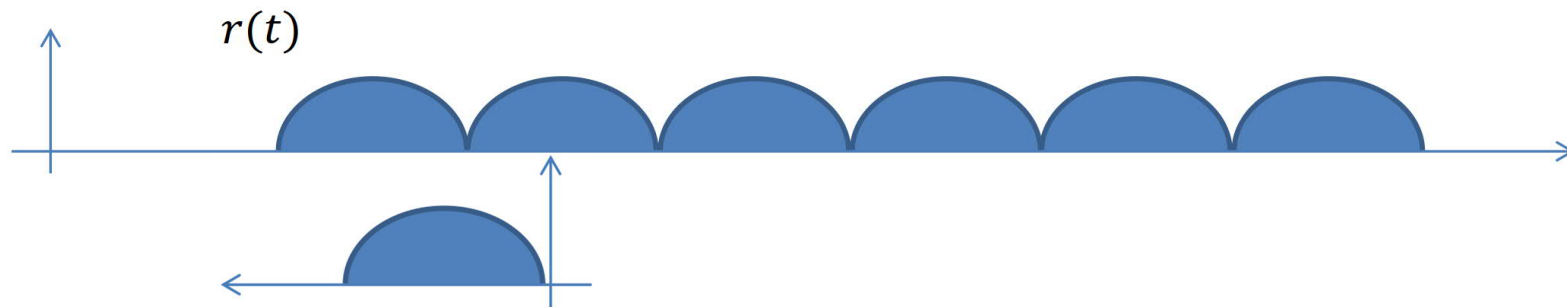
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Operation of the MF:  $\int p(\tau)r(t - \tau)d\tau$

# Synch in the project

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Operation of the MF:  $\int p(\tau)r(t - \tau)d\tau$

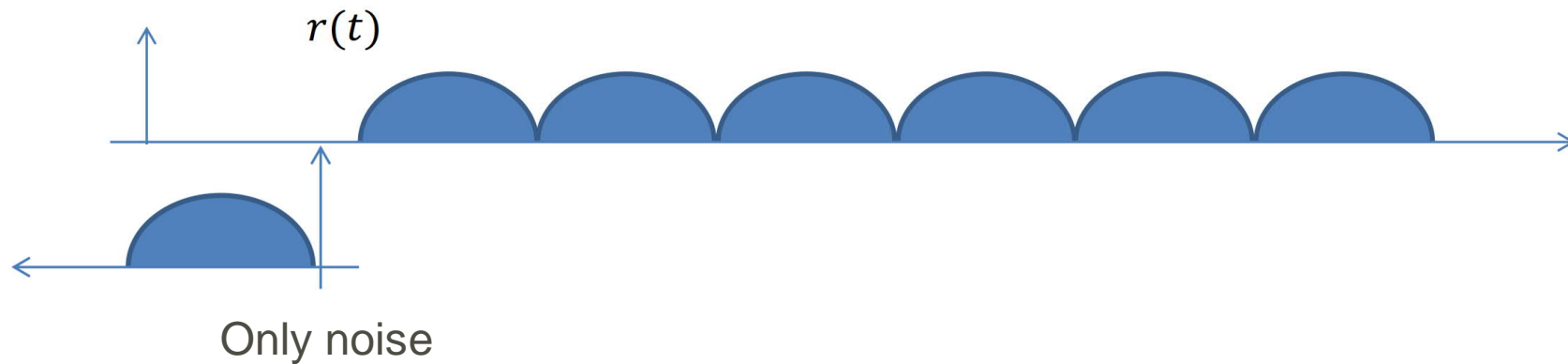
**Graphical interpretation:**

- 1. Let  $p(-\tau)$  slide along the  $x$ -axis**
- 2. At each position, multiply  $p(-\tau)$  and  $r(\tau)$**
- 3. Integrate the product**



# Synch in the project

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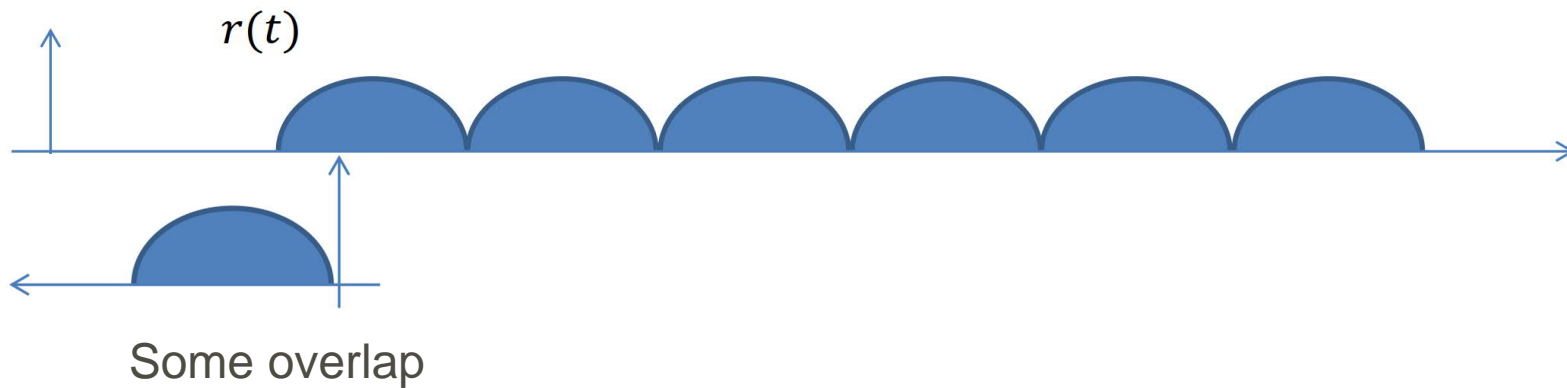
$$\text{Operation of the MF: } \int p(\tau)r(t - \tau)d\tau$$

## Graphical interpretation:

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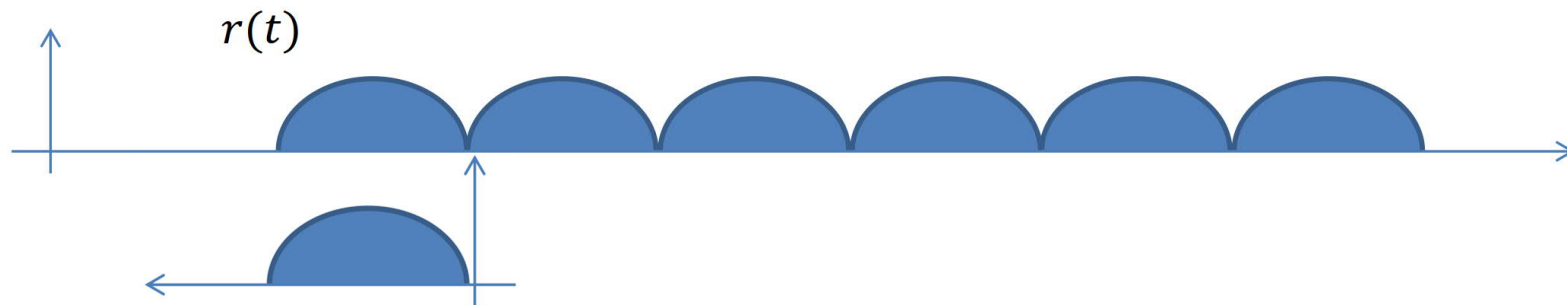
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# Synch in the project

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Full overlap, max value

$$\text{Operation of the MF: } \int p(\tau)r(t - \tau)d\tau$$

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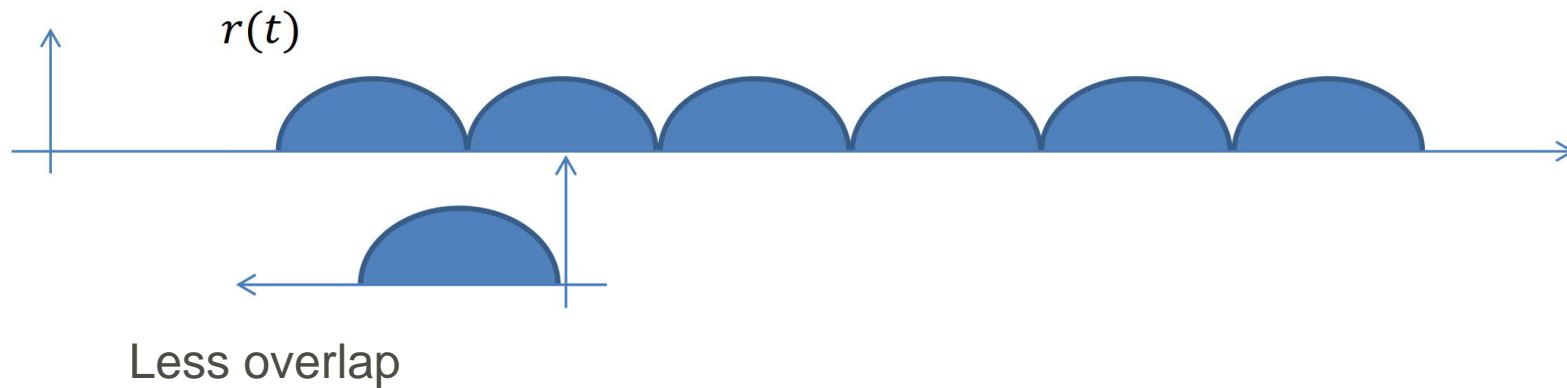
$$\text{Recall Cauchy-Schwarz: } \left| \int f(x)g(x)dx \right| \leq \sqrt{\int (f(x))^2 dx} \sqrt{\int (g(x))^2 dx}$$



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# Synch in the project

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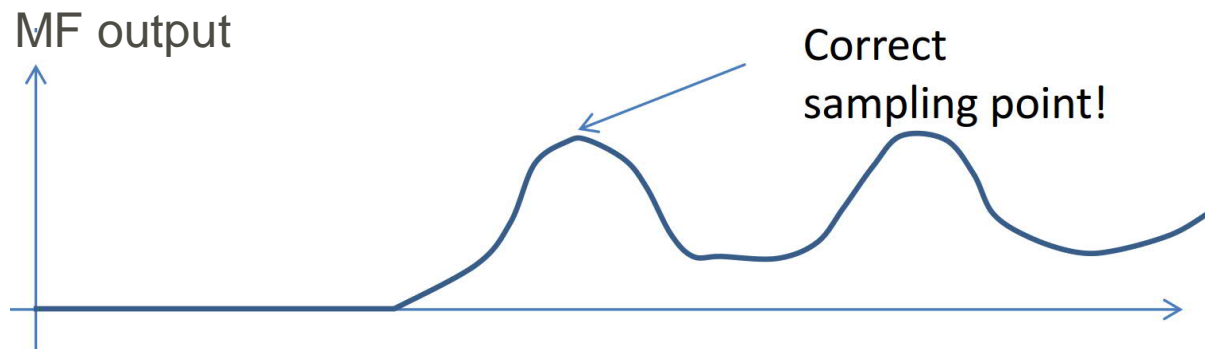
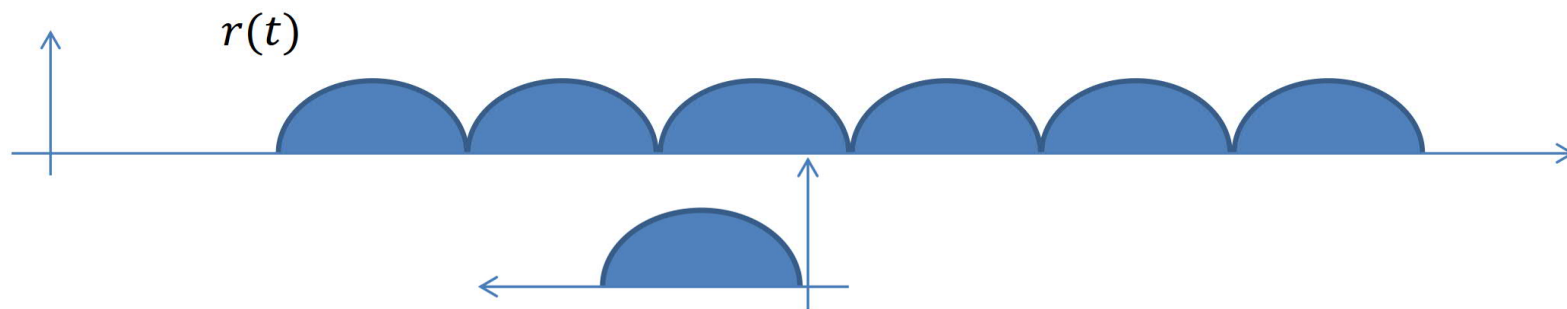
$$\text{Operation of the MF: } \int p(\tau)r(t - \tau)d\tau$$

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# Synch in the project

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