Answers/short solutions to
Examination in Digital Communications - October 15, 2007

Problem 1:

a) True:

$$\frac{W_{lobe}}{2} = \frac{1}{2} \cdot \frac{4}{T} = \frac{10}{3T_b} = \frac{10 \cdot 300 \cdot 10^3}{3} = 1\text{MHz}$$

b) False: The bandwidth is constant since T is constant.

c) True:

$$P_s \approx \frac{2 \cdot 63}{64} Q\left(\sqrt{\frac{36}{64^2-1} \cdot 5011.9}\right) = 1.9688 Q(6.638) \approx 4 \cdot 10^{-11}$$

d) True:

$$E_0 = E_1 = E_b = A^2 \frac{T_b}{2} = \frac{P}{R_b} \; , \; P = \frac{A^2}{2}$$

e) True:
32-PAM:

$$d^2_{\min} = \frac{6 \cdot 5}{32^2 - 1} = \frac{30}{1024 - 1} \; ,$$

1024-QAM:

$$d^2_{\min} = \frac{3 \cdot 10}{1024 - 1}$$

Problem 2:

$$\mathcal{E}_b/N_0 = 19.95$$

$$P_b = Q\left(\sqrt{d^2 \frac{\mathcal{E}_b}{N_0}}\right) = 10^{-6}$$

$$\frac{d^2 \cdot \alpha^2 \bar{P}_{sent}}{R_b N_0} = 4.7534^2$$

$$\frac{\alpha^2}{R_b} = \text{constant} = \frac{\alpha^2}{96 \cdot 10^3}$$

a) i)

$$\frac{(\alpha/5)^2}{R_b'} = \frac{\alpha^2}{96 \cdot 10^3} = \frac{\alpha^2}{25 R_b'}$$

$$\Rightarrow R_b' = \frac{1}{25} \cdot 96 \cdot 10^3 = 3.84\text{kbps}$$

ii)

$$\frac{(5\alpha)^2}{R_b''} = \frac{\alpha^2}{96 \cdot 10^3} = \frac{25\alpha^2}{R_b''}$$

$$R_b'' = 25 \cdot 96 \cdot 10^3 = 2.4\text{Mbps}$$

b)

$$d^2 = 4.7534^2/19.95 = 1.133$$

$$10 \log_{10} \frac{2}{1.133} = 2.47\text{dB worse}$$

c) See Figure 4.10 in the compendium.

Problem 3:

$$W_{99.5} = \frac{3}{T} = \frac{6}{T_s} = 10^6, \quad T_s = \frac{6}{10^6} = kT_b$$

$$\bar{P}_z/N_0 = 4 \cdot 10^7$$

a) From Table 5.1 (also from Laboration 2):

$$P_s = 2\,\frac{M-1}{M}\,Q\left(\sqrt{d_{\min}^2\,\frac{\mathcal{E}_b}{N_0}}\right) \le 2Q\left(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0}\right)$$

b) i)

$$d_{\min}^2\,\frac{\mathcal{E}_b}{N_0} \ge 3.0902^2$$

$$d_{\min}^2\,\frac{\mathcal{E}_b}{N_0} = \frac{6T_s}{M^2-1} \cdot \frac{P_z}{N_0} \ge 3.0902^2$$

$$M^2 - 1 \le \frac{6T_s}{3.0902^2} \cdot \frac{P_z}{N_0} = \frac{6 \cdot 6}{3.0902^2 \cdot 10^6} \cdot 4 \cdot 10^7 = 150.8$$

$$M \le \sqrt{151.8} = 12.3$$

So, $M = 8$ and $R_b = \frac{3}{6} \cdot 10^6 = \frac{1}{2}\text{Mbps}$

ii)

$$d_{\min}^2\,\frac{\mathcal{E}_b}{N_0} \ge 7.0345^2$$

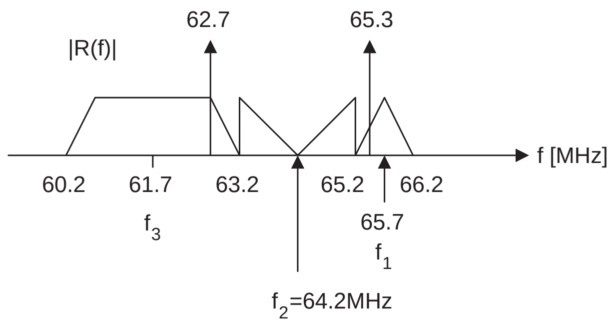
$$M^2 - 1 \le 29.1$$

$$M \le \sqrt{30.1} = 5.49$$

So, $M = 4$ and $R_b = \frac{2}{6} \cdot 10^6 = \frac{1}{3}\text{Mbps}$

Problem 4:

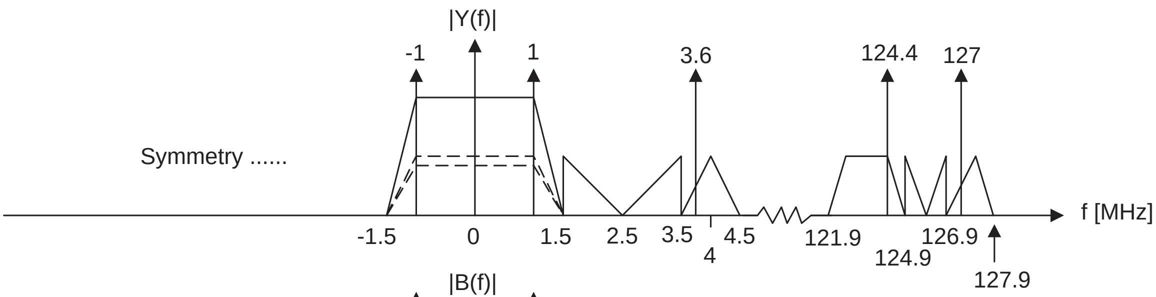
a)

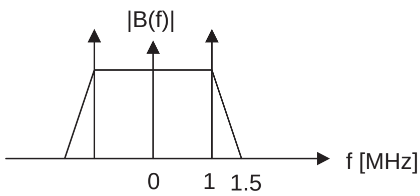


i) $f_2 = 64.2\text{MHz}$

ii) $|R(f)|$ see above.

iii)





b) i) See the compendium.

ii) $T_h =$ the largest delay in the N-ray channel.
No overlapping if $T_s = T + T_{zero} \geq T_z = T + T_h$, i.e.if $T_{zero} \geq T_h$.

Problem 5:

$$D_{0,1}^2 = E_0 + E_1 = A^2 0.4T_s + A^2 0.3T_s = 0.7A^2T_s = D_4^2$$
$$D_{0,2}^2 = E_0 + E_2 = 0.4A^2T_s + 0.2A^2T_s = 0.6A^2T_s = D_3^2$$
$$D_{0,3}^2 = E_0 + E_3 = 0.4A^2T_s + 0.1A^2T_s = 0.5A^2T_s = D_2^2$$
$$D_{1,2}^2 = E_1 + E_2 = 0.5A^2T_s = D_2^2$$
$$D_{1,3}^2 = E_1 + E_3 = 0.4A^2T_s = D_1^2$$
$$D_{2,3}^2 = E_2 + E_3 = 0.3A^2T_s = D_{\min}^2$$

$$2\mathcal{E}_b = \frac{1}{4}\,(0.4 + 0.3 + 0.2 + 0.1)A^2T_s = \frac{A^2T_s}{4}$$

i)

$$
\begin{aligned}
&d_{\min}^2 = \frac{0.3A^2T_s}{A^2T_s/4} = 1.2 \\
&c = \frac{1}{4}\,(0+0+1+1) = 1/2 \\
&\frac{\mathcal{E}_b}{N_0} = 15.85 \\
&\text{Union bound} \approx \frac{1}{2}\,Q(\sqrt{1.2\cdot 15.85}) = \frac{1}{2}\,Q(4.361) \approx 2.7\cdot 10^{-6}
\end{aligned}
$$

ii) 4-PPM $\Rightarrow d_{\min}^2 = 2$ is 2.22 dB better.
So, 4-PPM is to be preferred.

iii)

$$
\begin{aligned}
&d_1^2 = \frac{0.4A^2T_s}{A^2T_s/4} = 1.6, \quad d_2^2 = \frac{0.5A^2T_s}{A^2T_s/4} = 2 \\
&d_3^2 = \frac{0.6A^2T_s}{A^2T_s/4} = 2.4, \quad d_4^2 = \frac{0.7A^2T_s}{A^2T_s/4} = 2.8 \\
&c = 1/2 \\
&c_1 = \frac{1}{4}\,(0+1+0+1) = 1/2 \\
&c_2 = \frac{1}{4}\,(1+1+1+1) = 1 \\
&c_3 = \frac{1}{4}\,(1+0+1+0) = 1/2 \\
&c_4 = \frac{1}{4}\,(1+1+0+0) = 1/2 \quad (c+c_1+c_2+c_3+c_4 = 3 \text{ OK})
\end{aligned}
$$