Final exam in

Digital Communications

on October 23, 2008, 14–19.

Department of Electrical and Information Technology
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.
  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.
  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

Good Luck!

Problem 1: Determine for each of the five statements below if it is true or false.
Observe! As usual, motivations to your answers should be given.

a) Assume a conventional M-ary bandpass PAM system. Also assume that $g(t) = g_{rc}(t)$ with duration $4T_s/5$. "If the bit rate is 400 kbps and $M = 16$ then the width of the mainlobe is 0.5 MHz."

b) Assume a conventional M-ary QAM system that uses $g(t) = g_{hcs}(t)$ with duration T_s. "If the bandwidth efficiency is 3.39 bps/Hz, $M = 256$, and $R_b = 2$ Mbps then the 99% definition of bandwidth is used."

c) Assume a conventional M-ary QAM system that uses $g(t) = g_{hcs}(t)$ with duration T_s, a conventional AWGN channel, and ML receiver. "If $M = 256$ and $\mathcal{E}_b/N_0$ is 26.522 dB then $P_s \approx 1.6 \cdot 10^{-10}$."

d) "M-ary conventional bandpass PAM and M-ary conventional QAM have different bandwidth efficiency and different energy efficiency."

e) "With uncoded equally likely binary signals the bit error probability is always larger than $1.5 \cdot 10^{-22}$ if $\mathcal{E}_b/N_0$ is 16.7 dB."

(10 points)

Problem 2: Assume a binary communication system that uses equally likely signal alternatives, a conventional AWGN channel, and an ML receiver. It is known that if $\mathcal{E}_b/N_0$ is 6.7709 dB then $P_b = 5.4799 \cdot 10^{-2}$.

Determine P_b if instead $\mathcal{E}_b/N_0$ is 17.174 dB.

Determine also the energy efficiency for this case, compared with if antipodal signal alternatives were used instead.

The error probability performance results above can be obtained if the signal alternative $z_0(t) = g_{rec}(t)$ with amplitude A and duration $T = T_b$, and if the signal alternative $z_1(t) = g_{rec}(t)$ with amplitude A and duration $T = xT_b$ where the parameter x has a specific value in the interval $0 < x < 1$.

Determine this specific value of the parameter x.

(10 points)

Problem 3: Assume that the received signal alternatives $\{z_\ell(t)\}_{\ell=0}^{M-1}$ are conventional M-ary PSK signals. Also assume that a hcs pulse with amplitude A and duration $T = 3T_s/4$ is used. It is given that the width of the mainlobe equals 800 kHz. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. It is also given that the symbol error probability in this case can be upper bounded by $2Q(\sqrt{d_{\min}^2 \mathcal{E}_b/N_0})$.

It is a requirement that the symbol error probability must not exceed the value $2 \cdot 10^{-9}$.

i) Determine requirements on the ratio $\bar{P}_z/N_0$ if $M = 8$.
ii) Determine a relationship between the parameter M and the ratio $\bar{P}_z/N_0$ for the M-ary PSK case considered above. Conclusions?
iii) Describe differences and similarities between M-ary PSK and M-ary bandpass PAM.

(10 points)

Problem 4:

a) Here we consider a three-user digital communication system. The information carrying user signals are denoted $u_1(t)$, $u_2(t)$ and $u_3(t)$, respectively, and they are baseband (low-frequency) signals. The width of the mainlobes of the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are: 400 kHz, 800 kHz and 1600 kHz, respectively.

The transmitted multi-user signal is denoted $s(t)$, and
$s(t) = u_1(t)\cos(2\pi f_1 t) + u_2(t)\cos(2\pi f_2 t) + u_3(t)\cos(2\pi f_3 t)$

The received multi-user signal is denoted $r(t)$, and
$r(t) = \alpha s(t) + n(t)$
where α is a given channel parameter, and $n(t)$ denotes a disturbance.

The receiver first constructs the signal denoted $y(t)$ as
$y(t) = r(t)\cos(2\pi f_4 t)$
and the desired information carrying signal is then obtained by filtering $y(t)$ in a properly designed low-pass filter.

It is known that $f_1 = 12$ MHz, $f_2 = 12.8$ MHz, and $f_3 = 14.4$ MHz. The disturbance is $n(t) = \cos(2\pi f_A t) + \cos(2\pi f_B t)$ where $f_A = 12.6$ MHz and $f_B = 13.4$ MHz.

Note that detailed calculations are not required below. However, the frequency content must be clearly seen in the figures.

i) Sketch the frequency content in $r(t)$.

ii) Sketch the frequency content in $y(t)$ if $f_4 = 13.6$ MHz.

iii) The choice of $f_4 = 13.6$ MHz is not a correct choice but it can occurr due to a malfunction in the receiver. Determine the correct choice of f_4 if the receiver should recover the information in $u_3(t)$.

b) Assume an application where three equally likely signal alternatives are used. It is given that the received signal alternatives are $z_0(t) = 4g(t)$, $z_1(t) = 8g(t)$ and $z_2(t) = 12g(t)$, where $g(t)$ denotes an arbitrary received pulse shape with duration $T = T_s$ and with energy denoted E_g.

It is also here assumed that the ratio E_z/N_0 is fixed and given, where E_z denotes the average received symbol energy.

i) Determine an expression of the union bound that contains the ratio E_z/N_0.
ii) Above, the three amplitudes $(4, 8, 12)$ are used. Suggest another set of three amplitudes that will decrease the union bound at the same ratio E_z/N_0.

(10 points)

Problem 5: Here we consider some consequences of a two-ray multi-path channel. Assume binary communication with equally likely signal alternatives $s_0(t)$ and $s_1(t)$. It is given that $s_0(t) = g_{rec}(t)$ with amplitude A and duration $T = 0.4T_b$, and $s_1(t) = -s_0(t - 0.4T_b)$. Hence, $s_0(t)$ and $s_1(t)$ are orthogonal.

Due to the multipath channel the received signal alternatives are

$$z_i(t) = \alpha s_i(t) + \beta s_i(t - 0.2T_b),\ i = 0, 1$$

where α and β are parameters of the two-ray channel. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used.

i) Determine in detail an ML receiver that uses only one correlator. Consider the specific cases: $\beta = 0$, $\beta = -\alpha/2$, and $\beta = \alpha/2$. Conclusions?

ii) Calculate the bit error probability for the three cases in i) if $\mathcal{E}_b/N_0$ is 9.6 dB. Conclusions?

(10 points)