Final exam in

Digital Communications

on October 16, 2006, 14–19.

Department of Information Technology
Lund University

- During this final exam, you are allowed to use a calculator, the textbook, and Tefyma (or equivalent).
- Each solution should be written on a separate sheet of paper.

  Please add Your name on each sheet.
- Show the line of reasoning clearly, and use the methods presented in the course.

  If You use results from the textbook, add a reference in Your solution.
- If any data is lacking, make reasonable presumptions.
- If You want or if You do not want Your result to appear on the department's web site, please write so on the cover page of the exam.

Good Luck!

Problem 1: Assume a binary communication system using equally likely received signal alternatives $z_0(t)$ and $z_1(t)$. The received signal $r(t)$ is

$$r(t) = \begin{cases} z_1(t) + N(t), & \text{if message “1” is sent} \\ z_0(t) + N(t), & \text{if message “0” is sent} \end{cases}$$

where $N(t)$ is AWGN with $R_N(f) = N_0/2$. It is also known that the ML receiver is used. It is given that the received signal alternative $z_1(t)$ is a rectangular pulse ($g_{rec}(t)$) with duration T_b and energy E_{z_1}. The received signal alternative $z_0(t)$ is designed such that,

$$z_0(t) = x \cdot z_1(t)$$

where the parameter x is a constant. It is known that if $x = 0$ then the bit error probability equals 10^{-3}.

a) Calculate the bit error probability if instead:

i) $x = -1$ ii) $x = 4$

b) Determine the average received energy per bit for the two cases i) and ii) in a) and compare with the $x = 0$ case.

(10 points)

Problem 2: Determine for each of the five statements below if it is true or false.

Observe! As usual motivations to your answers should be given.

1. "A 16-ary PPM signal constellation with equally likely signal alternatives is 6 dB more energy efficient than binary FSK."
2. "Binary FSK is 15.3 dB more energy efficient than 1024-QAM."
3. "Assume 64-ary baseband PAM with equally likely signal alternatives, and that the hcs pulse-shape $g_{hcs}(t)$ with duration $T = 5T_s/6$ is used. It is also assumed that the amplitudes $\pm 1, \pm 3, \pm 5, \ldots, \pm 63$ are used. In this case the bandwidth W is $W = 120$ kHz if the bit rate is 400 kbps. The bandwidth is defined as half the width of the mainlobe."
4. "With $M = 2$ and equally likely signal alternatives $s_1(t) = -s_0(t) = g_{rc}(t)$, with amplitude A and duration $T = 5T_s/6$, the average signal power $\bar{P}$ is $\bar{P} = 5A^2/16$."
5. "Assume that the largest delay in a five-ray multipath channel equals $0.25 \cdot 10^{-6}$ [s]. If signals with duration $T = 0.75 \cdot 10^{-6}$ [s] are sent from the transmitter then the symbol rate 1 Msymbol/s can be used without any overlapping signals at the output of the channel."

(10 points)

Problem 3: Assume an adaptive conventional M-ary QAM system. The possible values of M, together with the corresponding bit rates, are given below:

M :	4	16	64	256	1024
R_b (kbps) :	192	384	576	768	960

The carrier frequency is $f_c = 3$ GHz. The communication is disturbed by AWGN $N(t)$ with power spectral density $R_N(f) = N_0/2$, and the ML receiver is used. It is required that the symbol error probability must not exceed $4 \cdot 10^{-6}$.
As usual, the input signal $r(t)$ to the ML receiver can be expressed as

$$r(t) = z(t) + N(t)$$

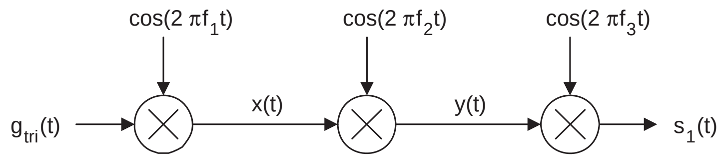
a) Which bit rate do you recommend to use if it is known that the ratio $P_z/N_0 = 6 \cdot 10^8$?

b) Determine the 99% bandwidth if a $g_{rc}(t)$ pulse with duration $T = 3T_s/4$ is used.

(10 points)

Problem 4:

a) In a communication application a binary signal constellation is used. The signal alternative $s_1(t)$ is constructed according to the block diagram below,



The triangular pulse $g_{tri}(t)$ has duration $T = 80\ \mu$s and the frequencies are: $f_1 = 200$ kHz, $f_2 = 400$ kHz, $f_3 = 2$ MHz.

Sketch the frequency content in $x(t)$, $y(t)$ and $s_1(t)$.

Observe! Detailed calculations are not required. However, the frequency content should be clearly seen in the sketches.

b) In an 8-ary PAM system it has been found that if the bit rate is 384 kbps then the equivalent discrete time impulse response $x[i]$ (notation from compendium) is;

i :	0	1	2	3	4	5	6	7	8	9	...
$x[i]$:	a	$-a/10$	$a/10$	0	0	$-a/20$	$a/20$	0	0	0	...

It is also known that $x[-i] = x[i]$

i) Determine bit rates for which ISI = 0 if $M = 256$.

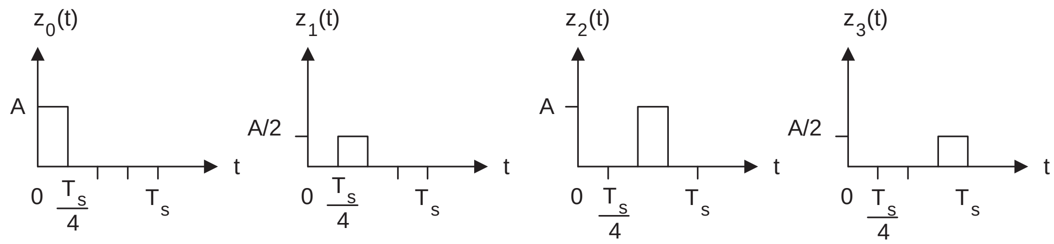
ii) Calculate the worst case ISI if $M = 64$ and the bit rate is 256 kbps.

(10 points)

Problem 5: In a 4-ary communication system the input signal $r(t)$ to an ML receiver is

$$r(t) = z_\ell(t) + N(t)$$

if the ℓ:th message is sent in $0 \le t \le T_s$, $\ell = 0, 1, 2, 3$. The messages are equally likely, and $N(t)$ is AWGN with $R_N(f) = N_0/2$. The signal alternatives are given below:



a) Compare the energy efficiency of this scheme with 4-ary PAM.

b) Determine the union bound and calculate its value if $\mathcal{E}_b/N_0$ is 10.51 dB.

(10 points)