LUND UNIVERSITY
Electrical and Information Technology

Exam in
Digital Communications, EITG05

April 13, 2018

- During this exam you are allowed to use a calculator, the compendium, a printout of the lecture slides, and Tefyma (or equivalent).
- Please use a new sheet of paper for each solution. Write your anonymized assessment code + a personal identifier on each paper.
- Solutions should clearly show the line of reasoning and follow the methods presented in the course. If you use results from the compendium or lecture slides, please add a reference in your solution.
- If any data is lacking, make reasonable assumptions.

Good luck!

Problem 1

Determine for each of the five statements below if it is true or false.
Give a motivation for each of your answers.

(a) Consider an M-ary QAM system with pulse $g(t) = g_{rc}(t)$ of duration $T_s/4$.

"If the bit rate is 300 kbps and $M = 64$ then the width of the mainlobe is 800 kHz."

(b) *"The energy-efficiency of a QAM system with matched filter receiver is independent of the shape of the transmit pulse $g(t)$."*

(c) *"4-ary QAM is about 3 dB more energy efficient than 4-ary PAM."*

(d) Consider binary antipodal signaling with an ML receiver. Usually, the signal alternatives $s_0(t)$ and $s_1(t)$ are transmitted with equal probability $P_0 = P_1 = 1/2$.

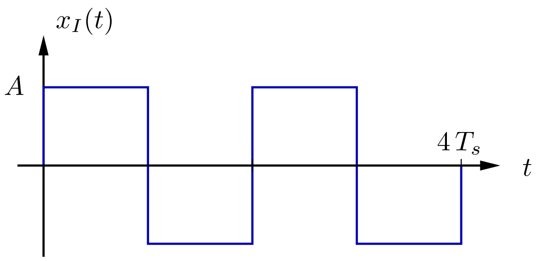
"If $s_0(t)$ is transmitted more often than $s_1(t)$, the bit error probability becomes smaller."

(e) *"The two signals $s_0(t) = \sin(2\pi\, t/T_s)$ and $s_1(t) = \sin(2\pi\, 3\, t/(2T_s))$ are orthogonal."*

(10p)

Problem 2

Consider a QAM signal $x(t)$ for which the baseband quadrature components $x_I(t)$ and $x_Q(t)$ are given as below.

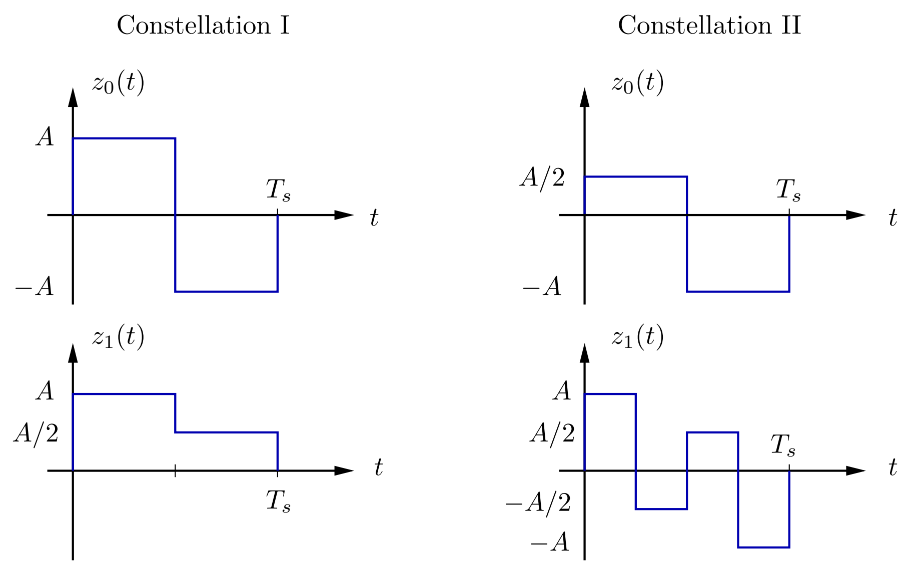


(a) Draw the I-Q diagram of the signal $x(t)$, including the constellation points and the transitions for the different time intervals.

(b) Assuming a carrier frequency of $f_c = 2/T_s$, draw the bandpass signal $x(t)$.

(c) Suppose that the rectangular pulse is replaced by a triangular pulse of same duration and amplitude. Discuss how this change affects the I-Q diagram.

Two binary signal constellations are given below.



(d) Compare the energy efficiencies of the two constellations on the basis of $d_{0,1}^2$.

Remark: Part (d) can be solved independently from (a), (b), or (c).

(10p)

Problem 3

Consider a 2-ray multi-path channel with impulse response

$$h(t) = \sum_{i=1}^{2} \alpha_i \, \delta(t - \tau_i) \;, \quad \text{where } \alpha_1 = 1, \alpha_2 = 0.5, \tau_1 = 0\,\mu\text{s}, \tau_2 = 1\,\mu\text{s} \;.$$

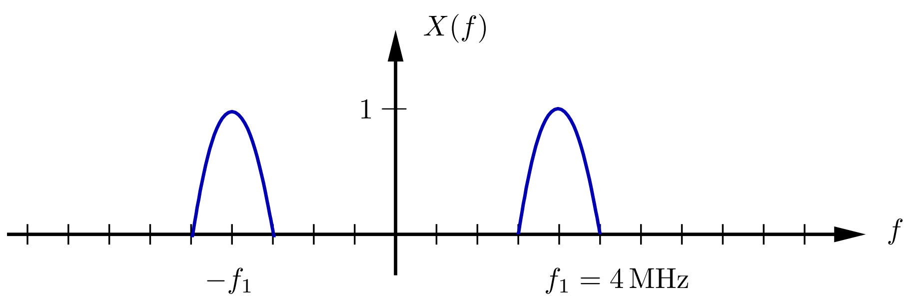
A binary PAM signal with triangular pulse $g_{tri}(t)$ of amplitude A and duration $T = 2\,\mu$s is transmitted over this channel.

(a) Determine the largest bit rate R_b for which no overlap of signal alternatives will occur after the channel.

(b) Draw the signal $z_1(t)$ at the output of the channel for the input $s_1(t) = A\, g_{tri}(t)$.

(c) Your task is to implement an ML receiver for the given system by means of a matched filter. Determine the impulse response $v(t)$ of the matched filter.

The frequency spectrum $X(f)$ of a bandpass signal $x(t)$ is given as in the figure below:



(d) Before the signal is digitized at the receiver it is converted to an intermediate frequency as follows:

$$y(t) = x(t) \cdot \cos\left(2\pi \left(f_1 + f_2\right) t\right) \;, \quad \text{where } f_2 = 1\,\text{MHz} \;.$$

Draw the spectrum $Y(f)$ of the signal.

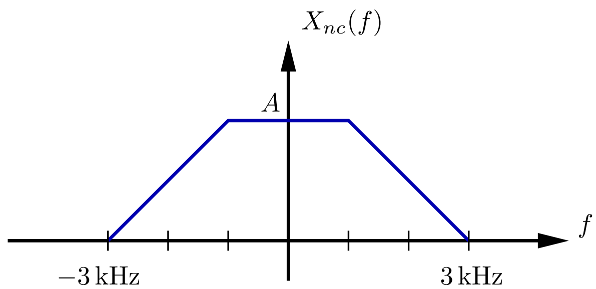
Remark: Part (d) can be solved independently from (a), (b), or (c).

(10p)

Problem 4

Assume a communication system employing binary PAM modulation with equally likely signal alternatives. The combination of the transmit pulse $g(t)$, channel filter $h(t)$, and receiver filter $v(t)$ can be written as $x(t) = g(t) * h(t) * v(t)$. The signal is sampled in the receiver at time instants $\mathcal{T} + i\,T_s$, $i = 0, 1, 2, \ldots$.

The Fourier transform $X_{nc}(f)$ of the non-causal pulse $x(\mathcal{T} + t)$ is given as follows:



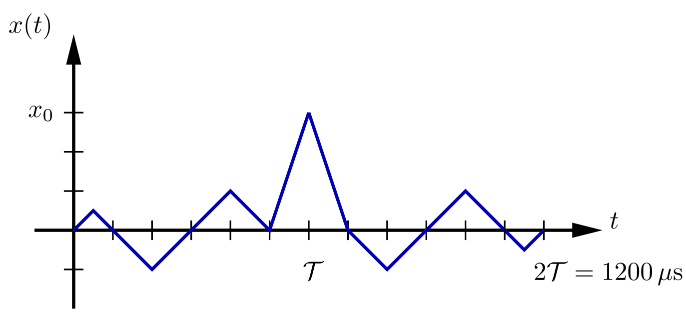
(a) Determine the maximum possible symbol rate R_s that can be achieved so that there is no intersymbol interference (ISI). How should A be chosen in this case?

(b) Assume now that $R_s = 2000$ symbols per seconds and draw the signal

$$\sum_{n=-\infty}^{\infty} X_{nc}(f - nR_s)$$

within the range $-9\,\text{kHz} < f < 9\,\text{kHz}$. The amplitude A is chosen as in (a). Is the Nyquist condition for ISI-free reception satisfied in this case?

The transmit pulse of the system is changed, and $x(t)$ is now given as follows:



(c) What is now the maximum possible symbol rate for ISI-free reception?

(d) Let us now tolerate some ISI and choose $T_s = 200\,\mu\text{s}$. Draw the discrete impulse response $x[i]$ of the system for this particular case.

(e) Is there a risk for erroneous decisions, assuming noise-free transmission with $T_s = 200\,\mu\text{s}$? In order to answer this question, give an example of an information sequence $A[i]$ for which the worst case ISI occurs.

(10p)

Problem 5

Your task is to design a short-range reliable communication link for a medical application with low-power transmit signals. A requirement is that the symbol error probability P_s is below 10^{-10}, and the power P_z at the receiver is at most $10^{-13}\,W$. The parameter of the additive white Gaussian noise is equal to $N_0 = 8 \cdot 10^{-20}$ [W/Hz].

(a) For ensuring high energy-efficiency, you consider using BPSK transmission. What is the maximum bit rate R_b that can be achieved under these requirements?

(b) Since you realize that at least a rate of $100\,\text{kbps}$ should be achieved, you decide to use M-ary FSK transmission instead. Determine the smallest alphabet size M that fulfills your goal.

(c) Compare the required bandwidth of the two solutions, assuming in both cases a rectangular pulse of duration $T = T_s$, and a frequency separation of $f_\Delta = R_s$.

(d) Explain why FSK modulation is a good choice in this scenario. Could you have used any other kind of M-ary modulation instead?

(10p)