LUND
UNIVERSITY
Electrical and Information Technology

Exam in
Digital Communications, EITG05

October 31, 2019

- During this exam you are allowed to use
  - the course compendium
  - a printed version of the lecture slides
  - your own handwritten notes: limited to 2 sheets (= 4 pages), no copy
  - a pocket calculator (but no device that can connect to the internet)
- Please use a new sheet of paper for each solution. Write your anonymized assessment code + a personal identifier on each paper.
- Solutions should clearly show the line of reasoning and follow the methods presented in the course. If you use results from the compendium or lecture slides, please add a reference in your solution.
- If any data is lacking, make reasonable assumptions.

Good luck!

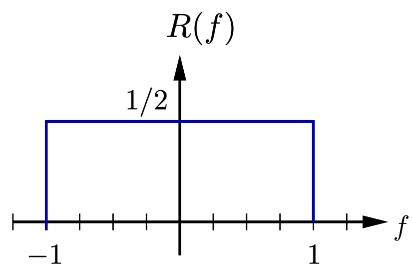
Problem 1

Determine for each of the five statements below if it is true or false.
Give a motivation for each of your answers.

(a) Consider binary PSK signaling, using a triangular pulse $g_{tri}(t)$ with amplitude A and duration T.

"If $T = 3/10\ T_s$, then the average signal power is $A^2/10$."

(b) The power spectral density of a signal $s(t)$ is given in the following figure:



"The average power of the signal $s(t)$ is equal to $\bar{P} = 1/2$."

(c) *"M-ary QAM is about $M/2$ times more energy efficient than M-ary PAM if M is large."*

(d) *"Two signals $s_i(t)$ and $s_j(t)$ are orthogonal if and only if their squared Euclidean distance $D_{i,j}^2$ is equal to the sum of their energies E_i and E_j."*

(e) Consider the discrete-time model of a binary PAM communication system, as shown in Fig. 6.3 on page 439. The overall discrete impulse response is equal to

$$x[i] = 0.5\,\delta[i+1] + \delta[i] - 0.5\,\delta[i-1]\ .$$

Assume that the transmitted sequence of amplitudes is $A[i] = +1$ if i is even and $A[i] = -1$ if i is odd (i.e., $A[0] = +1, A[1] = -1, A[2] = +1, A[3] = -1, \dots$).

"Under the above conditions the decision variable is equal to $\xi[i] = A[i]$ for all i."

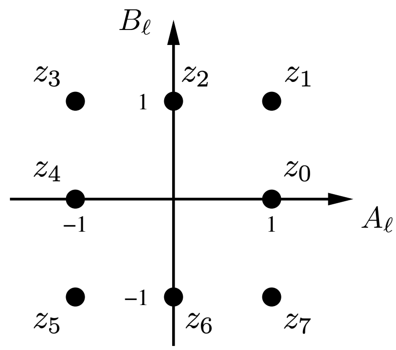
(10p)

Problem 2

Consider a QAM signal constellation with rectangular pulse shape of duration $T = T_s$ and amplitude $A = 1$. The signal alternatives can be written as

$$z_\ell(t) = A_\ell \, g_{rec}(t) \cos(2\pi \, f_c \, t) - B_\ell \, g_{rec}(t) \sin(2\pi \, f_c \, t), \quad \ell = 0, \dots, 7 \, ,$$

where the amplitude pairs are given in the following constellation diagram:



(a) For the carrier frequency $f_c = 2/T_s$ and the transmitted message sequence

$$\mathbf{m} = (m[0] \; m[1] \; m[2] \; m[3]) = (4 \; 2 \; 1 \; 6) \, ,$$

draw the signal $z(t)$ within the time interval $0 \leqslant t \leqslant 4T_s$.

(b) Choose some Gray mapping to assign bits to the different signal alternatives. Determine the bit sequence $\mathbf{b}$ corresponding to the message from (a).

(c) The union bound on the symbol error probability can be written as

$$P_s \leqslant c \, Q\left(\sqrt{\frac{D^2_{\text{min}}}{2N_0}}\right) + c_1 \, Q\left(\sqrt{\frac{D^2_1}{2N_0}}\right) + \cdots + c_x Q\left(\sqrt{\frac{D^2_{\text{max}}}{2N_0}}\right) \, .$$

Determine the parameters of the first two terms: D^2_{min}, c, D^2_1, and c_1.
You can assume that all signal alternatives are transmitted with equal probability.

(d) For large signal-to-noise ratio, P_s can be approximated by

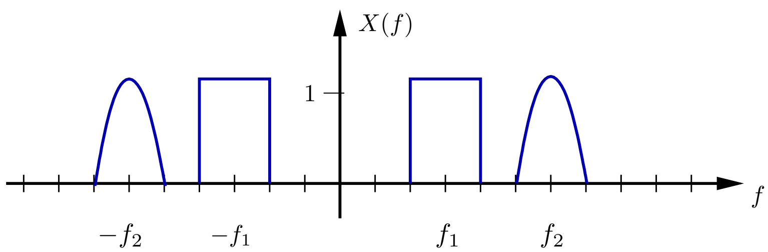
$$P_s \approx c \, Q\left(\sqrt{d^2_{\text{min}} \frac{\mathcal{E}_b}{N_0}}\right) \, .$$

Compare the constellation above with a standard 8-PSK constellation: which of the two constellations achieves a lower error probability if $\mathcal{E}_b/N_0$ is large?

(10p)

Problem 3

Two users transmit at carrier frequencies $f_1 = 300\,\text{MHz}$ and $f_2 = 600\,\text{MHz}$, respectively. The bandpass signal $x(t) = x_1(t) + x_2(t)$ has the following frequency spectrum $X(f)$:



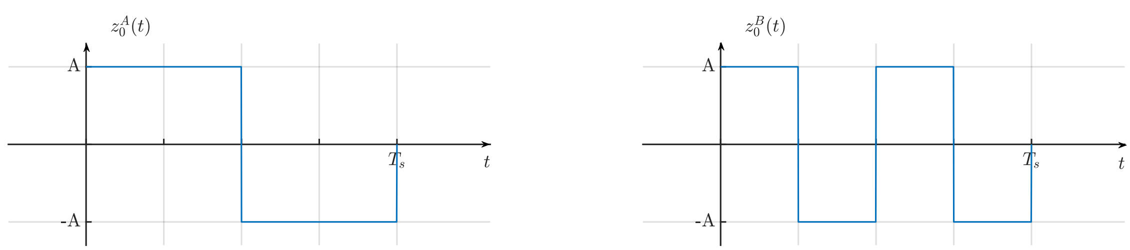
The baseband signal of user 2 is recovered by multiplication with the carrier signal, followed by an (ideal) low-pass filter:

$$x_{\text{LP}}(t) = y(t) * h_{\text{LP}}(t)\,, \quad \text{where} \quad y(t) = x(t) \cdot \cos\big(2\pi\, f_2\, t\big)$$

(a) Draw the spectrum $Y(f)$ of the unfiltered signal $y(t)$.

(b) Choose a suitable value for the bandwidth W_{LP} of the low-pass filter. Draw the frequency response $H_{\text{LP}}(f)$ of the filter and the spectrum $X_{\text{LP}}(f)$ of the signal after filtering.

Consider now another system, in which user 1 and user 2 transmit within the same frequency band. Each user applies binary antipodal signaling, but with different signal alternatives $z_0^A(t) = -z_1^A(t)$ and $z_0^B(t) = -z_1^B(t)$, respectively:



Both users implement the standard binary minimum Euclidean distance receiver in Fig. 4.10 on p. 247. The influence of the other user is ignored, i.e., treated as noise $N(t)$.

(c) Assume that user 2 receives the signal $r(t) = z_1^B(t) + N(t)$, where $N(t) = 0.5 \cdot z_0^A(t)$. Determine the decision variable ξ computed by the receiver.

(d) Does the receiver in (c) make a correct decision? What happens if the other user gets closer to the receiver? Explain.

Remark: Parts (c) and (d) can be solved independently from (a) and (b).

(10p)

Problem 4

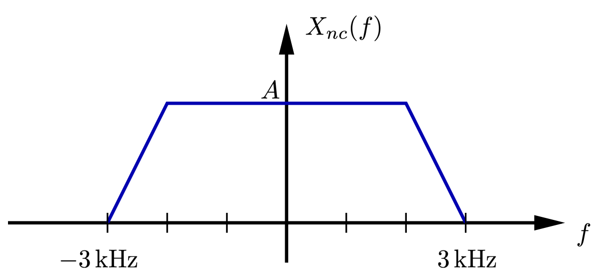
Assume a communication system employing binary PAM modulation with equally likely signal alternatives. The combination of the transmit pulse $g(t)$, channel filter $h(t)$, and receiver filter $v(t)$ can be written as $x(t) = g(t) * h(t) * v(t)$. The signal is sampled in the receiver at time instants $\mathcal{T} + i\,T_s$, $i = 0, 1, 2, \ldots$.

Consider first transmission with a rectangular pulse $g(t) = g_{rec}(t)$ of duration $T = 1\,\mu s$ and a multipath channel with $h(t) = \delta(t) + 0.5 \cdot \delta(t - T)$.

(a) Assume that $s_1(t) = g(t)$ is transmitted and that $N(t) = 0$ (no noise). Draw the signal $z_1(t) = g(t) * h(t)$ at the output of the channel.

(b) Let the impulse response of the receiver filter $v(t)$ be matched to the pulse, i.e., $v(t) = g(T - t)$. Draw the signal $x(t)$ at the output of the receiver filter.

Consider now another scenario, in which the Fourier transform $X_{nc}(f)$ of the non-causal pulse $x(\mathcal{T} + t)$ is given as follows:



(c) Determine the maximum possible symbol rate R_s that can be achieved so that there is no intersymbol interference (ISI). How should A be chosen in this case?

(d) Assume now that $R_s = 4000$ symbols per second and draw the signal

$$\sum_{n=-\infty}^{\infty} X_{nc}(f - nR_s)$$

within the range $-9\,\mathrm{kHz} < f < 9\,\mathrm{kHz}$. Is the Nyquist condition for ISI-free reception satisfied in this case?

Remark: Parts (c) and (d) can be solved independently from (a) and (b).

(10p)

Problem 5

Consider a communication system with bandpass M-PPM signaling using a pulse $g(t)$ with duration $T = T_s/M$. The signal alternatives are transmitted with equal probability and the system bandwidth W is measured by the main-lobe.

A requirement is that the symbol error probability satisfies $P_s \leqslant 10^{-6}$.

(a) Assume first a binary transmission ($M = 2$) with a rectangular pulse $g(t) = g_{rec}(t)$. What is the minimum $\mathcal{SNR}_r$ at the receiver that fulfills the requirements?

(b) You are now told that the $\mathcal{SNR}_r$ of the low-power link should be below 5 dB. Furthermore, at least 99.5% of the power of the pulse should be contained within its double-sided mainlobe.

Which pulse shapes fulfill these additional conditions if $M = 2$?

(c) In order to further improve the system, you consider using higher order signaling with $M > 2$. For general values of M, determine the parameters d^2_{min} and c that are used in the approximate expression for P_s.

(d) For achieving best data rates, the system should adaptively select between modulation orders $M = 2$, $M = 4$, and $M = 8$ depending on the $\mathcal{SNR}_r$ at the receiver.

Determine for each M in which range of $\mathcal{SNR}_r$ in dB it should be used.

(10p)