# Problem 1

(a) False.
With $\bar{E}_s = E_g/2$ (PSK) we obtain $\bar{P} = R_s \bar{E}_s = A^2/20$.

(b) False.
From the figure we get

$$\bar{P} = \int_{-\infty}^{\infty} R(f)\, df = 1 \; .$$

(c) True.
For the two constellations we compute

$$\frac{d^2_{\text{min,QAM}}}{d^2_{\text{min,PAM}}} = \frac{M^2 - 1}{2(M-1)} = \frac{M+1}{2} \; .$$

For large values of $M$ this is about the same as $M/2$.

(d) True.
On page 28 we see that

$$D^2_{i,j} = E_i + E_j - 2 \int_0^{T_s} s_i(t) \cdot s_j(t)\; dt \; ,$$

where the last term is equal to zero if and only if the two signals are orthogonal.

(e) True. (assuming there is no noise)
For the given amplitude sequence we obtain

$$\xi[i] = 0.5\, A[i+1] + A[i] - 0.5 A[i-1] = A[i] \; ,$$

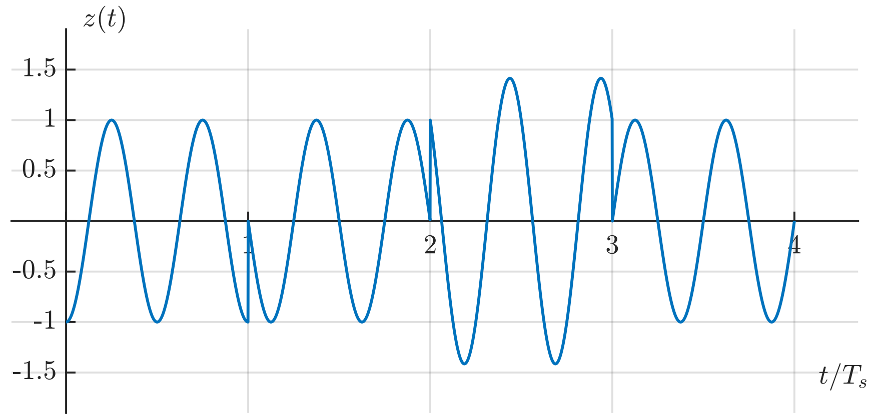
since $A[i+1] = A[i-1]$ for all $i$.

*Answers without explanation do not give any points.*

(10p)

## Problem 2

(a) We can compute the magnitudes and phases using (2.89). For example, for $z_1$ the magnitude is equal to $\sqrt{2}$ and the phase is equal to $\pi/4$.




(b) An example for Gray mapping is

| $z_0$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ | $z_6$ | $z_7$ |
|---|---|---|---|---|---|---|---|
| 000 | 001 | 011 | 010 | 110 | 111 | 101 | 100 |

This corresponds to the bit sequence $\mathbf{b} = 110\ 011\ 001\ 101$.

(c) From the diagram we can identify $D^2_{\text{min}} = D^2_{0,1} = E_g/2$. Each signal alternative has two neighbors at that distance, resulting in

$$c = c_0 = 1/8\,\big(2+2+2+2+2+2+2+2\big) = 2$$

Likewise, $D_1^2 = D^2_{0,2} = E_g$, which occurs twice for half of the signal alternatives, resulting in

$$c_1 = 1/8\,\big(2+0+2+0+2+0+2+0\big) = 1\ .$$

(d) With

$$\mathcal{E}_b = 1/3 \cdot 1/8\,\big(1+2+1+2+1+2+1+2\big)E_g/2 = E_g/4$$

we obtain

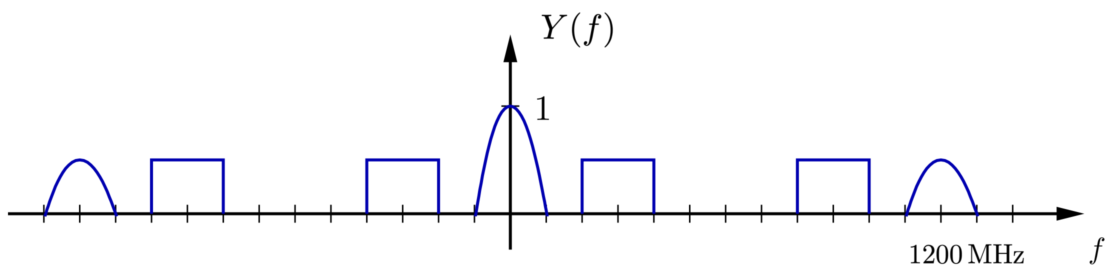
$$d^2_{\text{min}} = \frac{D^2_{\text{min}}}{2\mathcal{E}_b} = 1$$

For a standard 8-PSK constellation we get $d^2_{\text{min}} = 2\,\log_2 M\ \sin^2(\pi/M) \approx 0.87867$ and $c = 2$.
Both constellations have equal $c$, but $d^2_{\text{min}}$ is larger for the given QAM constellation. It follows that the QAM constellation achieves a lower error probability.

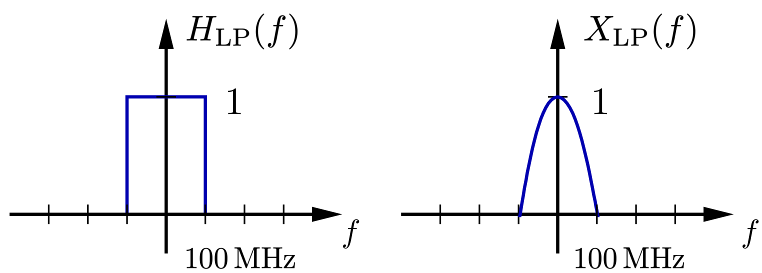
(10p)

## Problem 3

(a)




(b) Choosing $W_{LP} = 100\,\mathrm{MHz}$:




$W_{LP}$ should be between $100\,\mathrm{MHz}$ and $200\,\mathrm{MHz}$ to recover the baseband signal of user 2. Recall: we always use positive frequencies to measure bandwidth.

(c) The decision variable of user 2 is equal to

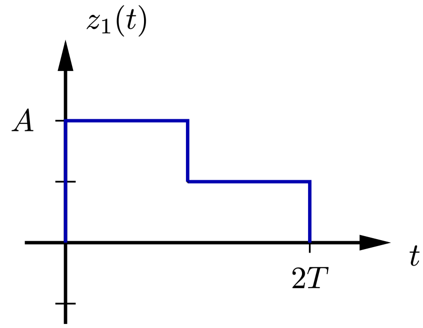
$$\xi = \int r(t) \cdot \left(z_1^B(t) - z_0^B(t)\right)\, dt = 2\, A^2\, T_s\ .$$

(d) Since $\xi > 0$ the receiver will decide for signal $z_1^B(t)$, which is a correct decision. If the other user gets closer, the signal $z_0^A(t)$ at the receiver will have a larger amplitude. However, repeating the calculations will show that the decision variable will not change in this case. The reason is that the signals $z_0^A(t)$ and $z_0^B(t)$ are orthogonal to each other. User 2 is therefore not disturbed by user 1.

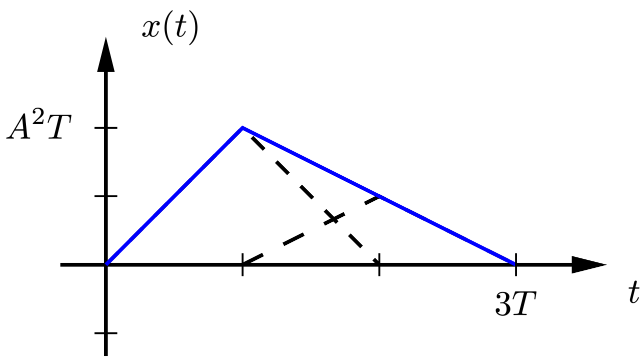
(10p)

## Problem 4

(a)

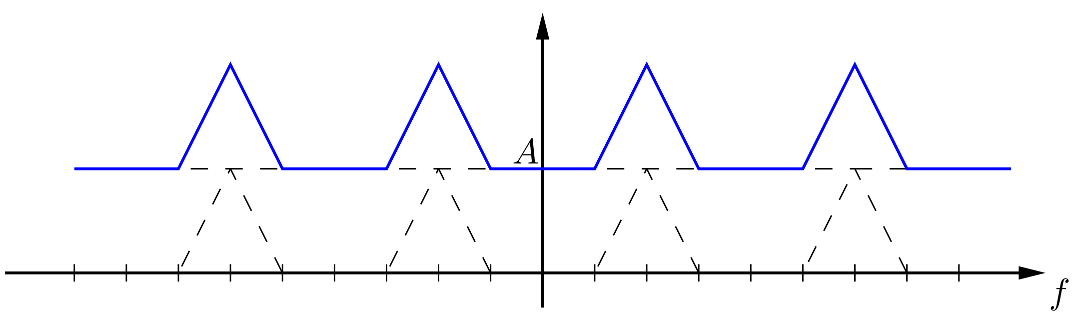



(b)




(c) We need to identify the largest $R_s$ for which the shifted signals add to a constant. We obtain $R_s = 5000$ symbols per second, and $A$ has to be chosen as $x_0/R_s$.

(d)




We can see that the shifted signals do not add up to a constant, which means that the ISI-free condition cannot be satisfied.

(10p)

## Problem 5

(a) From

$$\frac{d_{\min}^2}{\rho}\,\mathcal{SNR}_r \geqslant 4.7534^2 \approx 22.59$$

with $d_{\min}^2 = 1$ and $\rho = 1/4$ we obtain $\mathcal{SNR}_r \geqslant 7.51\text{dB}$.

(b) To ensure at least 99.5% of the power in the main lobe we can choose $g_{hcs}(t)$, $g_{tri}(t)$, or $g_{rc}(t)$. Note that the Nyquist pulse does not have a finite duration $T = T_s/M$. We can now check the condition

$$\mathcal{SNR}_r \geqslant 4.7534^2\,\frac{1/T_s}{W}$$

for the different pulse shapes. For $g_{hcs}(t)$ we have $W = 3/T$ and the required $\mathcal{SNR}_r$ is above 5 dB. For $g_{tri}(t)$ and $g_{rc}(t)$ the conditions are fulfilled.

(c) For orthogonal signaling we have $d_{\min}^2 = \log_2 M$ and $c = M - 1$.

(d) Using $g_{tri}(t)$ or $g_{rc}(t)$ as a pulse, we get $d_{\min}^2/\rho = 4M$ and

$$\mathcal{SNR}_r \geqslant \frac{\left(Q^{-1}\left(\frac{10^{-6}}{M-1}\right)\right)^2}{4M}$$

This leads to $\mathcal{SNR}_r \geqslant 4.509\,\text{dB}$ for $M = 2$, $\mathcal{SNR}_r \geqslant 1.938\,\text{dB}$ for $M = 4$ and $\mathcal{SNR}_r \geqslant -0.731\,\text{dB}$ for $M = 8$.

We observe that with PPM, the largest data rate can be obtained with $M = 8$ for any $\mathcal{SNR}_r \geqslant -0.731\,\text{dB}$. Adaptive modulation is only required if we want to save bandwidth in case of better channel quality.

(10p)