# Problem 1

(a) True.
$d^2_{min} = 4/5$ for both cases.
16 QAM: $\rho = k \cdot \rho_{\text{BPSK}} = 4 \cdot \rho_{\text{BPSK}}$.
Baseband PAM: $\rho = 2 \cdot \rho_{\text{PAM,bp}} = 2 \cdot 2 \cdot \rho_{\text{BPSK}}$.

(b) False.
The signals are antipodal, which means that $d^2_{min} = 2$.

(c) False.
$\xi_1 = \int_0^{T_s} r(t) \cdot z_1(t) dt - \frac{E_1}{2}$, which is equal to $\int_0^{T_s} z_0(t) \cdot z_1(t) dt - \frac{E_1}{2}$ if $N(t) = 0$.
Since $z_0(t)$ and $z_1(t)$ are sine waves with frequency $f_0 = 1/T_s$ and $f_1 = 2/T_s$ they are orthogonal, which means that $\xi_1 = -E_1/2 = A^2 T_s/2 \neq 0$.

(d) False.
A general bandpass transmit signal $x(t)$ can be expressed as

$$x(t) = Re\left\{(x_I(t) + jx_Q(t))\, e^{j\,2\pi\, f_c\, t}\right\} = x_I(t)\, \cos(2\pi\, f_c\, t) - x_Q(t)\, \sin(2\pi\, f_c\, t)$$

Since $x_Q(t) \neq 0$ for PSK (if $M > 2$), the equivalent baseband signal $x_I(t) + jx_Q(t)$ is complex-valued with a spectrum that is not symmetric around $f = 0$. As a consequence, the bandpass signal spectrum $|X(f)|$ is not symmetric around $f_c$.

(e) True.
The one-sided bandwidth of a spectral raised cosine pulse is $W_{lp} = (1+\beta) \cdot R_s/2$. For $\beta = 1$ this corresponds to twice the bandwidth of a Nyquist pulse ($\beta = 0$).

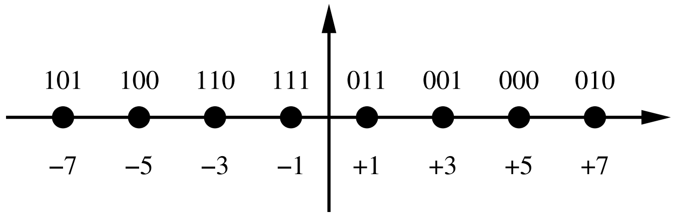
*Answers without explanation do not give any points.*

(10p)

## Problem 2

(a) The signal can be identified as PAM signal with amplitudes $-3, +1, -7, +3, +5$. With $A_\ell = -7, -5, -3, -1, +1, +3, +5, +7$ we obtain $\mathbf{m} = 2\ 4\ 0\ 5\ 6$. $f_c = 5$ MHz. From $M = 8$ we get $R_b = k/T_s = 3$ Mbps.

(b) With the Gray mapping below we obtain $\mathbf{b} = 110\ 011\ 101\ 001\ 000$.




(c) We obtain

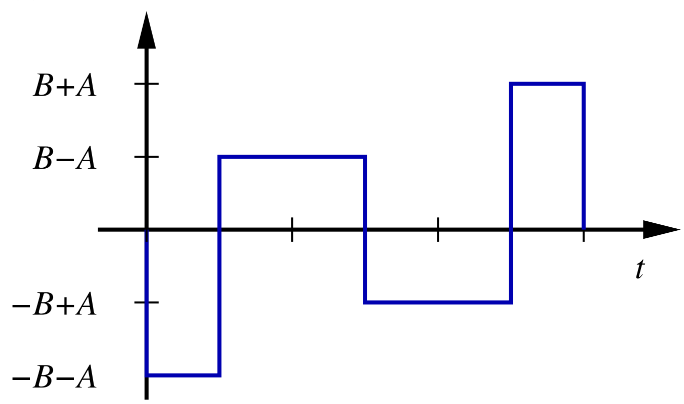
$$\xi_0 = \int_0^{T_s} r(t) \cdot z_0(t)\, dt - \frac{E_0}{2} = 0 - \frac{E_0}{2} = -\frac{A^2 T_s}{2}$$

and

$$\xi_1 = \int_0^{T_s} r(t) \cdot z_1(t)\, dt - \frac{E_1}{2} = \frac{ABT_s}{3} - \frac{A^2 T_s}{2}$$

Since $\xi_1 > \xi_0$ the receiver will choose $z_1(t)$.

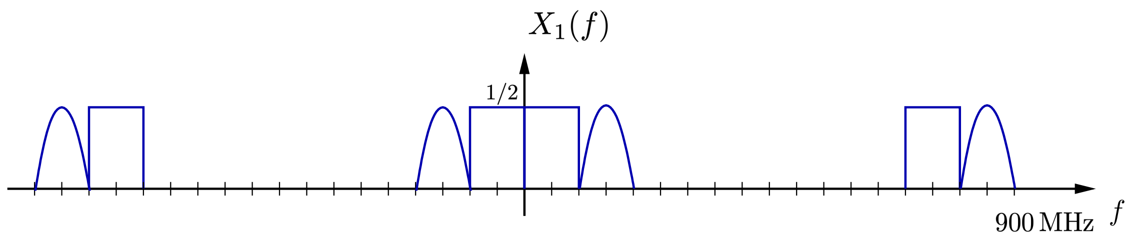
(d) $N(t) = r(t) - z_1(t)$

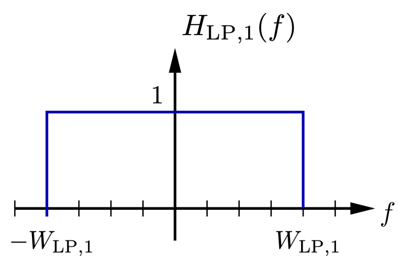


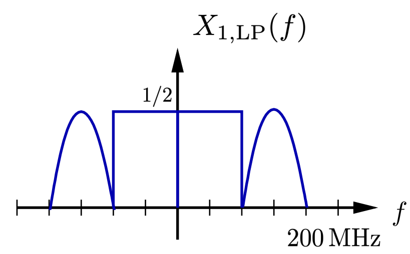

(10p)

# Problem 3

(a)

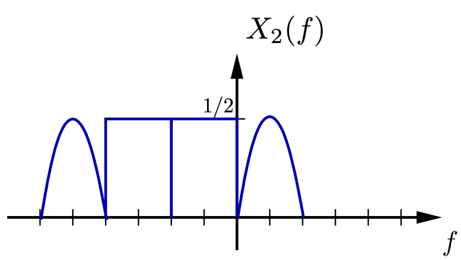


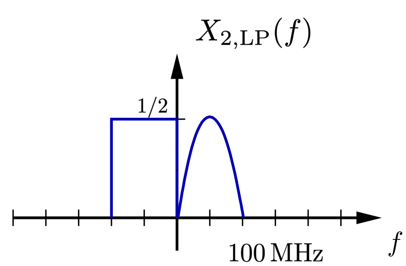




The bandwidth $W_{\text{LP},1}$ should be at least $200\,\text{MHz}$ and at most $700\,\text{MHz}$.

(b)




(c) The signals $x(t)$, $x_1(t)$, and $x_{1,\text{LP}}$ are all real-valued. Their spectrum is symmetric around $f = 0$ (even function).

The signals $x_2(t)$ and $x_{2,\text{LP}}$ are complex-valued. Their spectrum is not symmetric around $f = 0$.

(d) Coherent receiver: (see Lecture 9)

Using both inphase and quadrature components of the signal, the coherent receiver can recover the signal correctly despite of the presence of a phase error $\phi_{err}(t)$. An estimate of the phase error $\phi_{err}(t)$ is required for coherent reception.

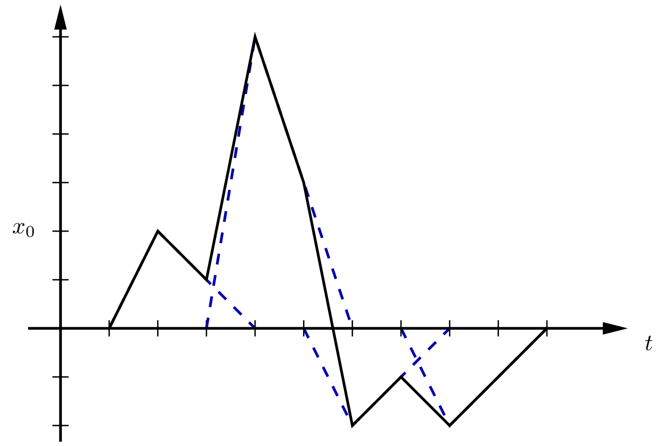
Non-coherent receiver: (see Lecture 7)

Can work well without the knowledge of the phase error. Example: differential PSK (DPSK). Disadvantage: variance of noise is increased.

(10p)

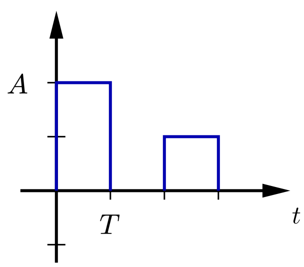
## Problem 4

(a) $y(t)$

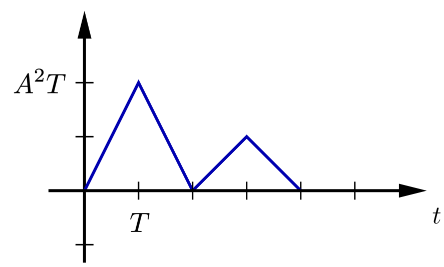


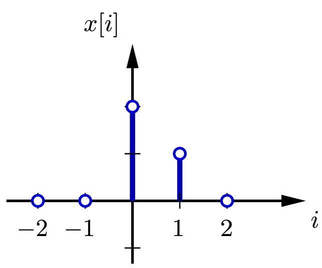

(b)

$u(t) = g(t) * h(t)$




$x(t) = u(t) * g(T - t)$

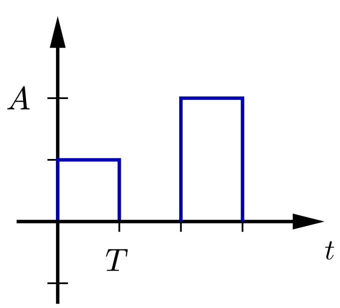



Yes, ISI occurs for $T_s = 2T$.

No ISI for $T_s = 3T \Rightarrow R_s = 1/3\,\text{Mbps}$

(c) $v(t) = u(T_u - t)$




With $T_s = T_u = 3T$ the transmitted signals do not overlap.

$\Rightarrow R_s = 1/T_s = 1/3\,\text{Mbps}$

(10p)

## Problem 5

(a) Since BPSK requires a carrier it is a bandpass signal and $\rho_{\text{BPSK}} = 1/2 \cdot \rho = 1/8$.

(b) Yes. From $\rho = R_b/W$ we get $W = 4\,R_b = 400\text{kHz} = 4/T_s = 4/(2T) = 2/T$. Baseband signaling implies that this is the *one-sided* bandwidth, which could correspond to a triangular or raised cosine pulse (see Table 2.1). Using a rectangular pulse this can be reduced to $W = 1/T = 200$ kHz, which gives $\rho = 1/2$. Choosing $T_s = T$ we can reduce the bandwidth to $W = 100$ kHz and achieve $\rho = 1$.

Alternatively, choosing a Nyquist pulse we have $W = R_s/2 = R_b/2$ and $\rho = 2$. Choosing a spectral raised cosine pulse with $\beta = 1$ this is reduced to $\rho = 1$.

(c) From $P_s = P_b = Q(\sqrt{2\,\mathcal{E}_b/N_0}) \leqslant 10^{-9}$ we obtain $\mathcal{X} > 6.0^2 = 36$.

With $\mathcal{E}_b = \alpha^2 E_g = \alpha^2 A^2 T$ this results in

$$A^2 > \frac{36 N_0}{\alpha^2 T_s} \quad \text{and} \quad A > 1.34 \cdot 10^{-3} [V]$$

(d) We have

$$d_{min}^2 = \frac{6\,\log_2 M}{M^2 - 1}$$

Compared to $d_{min}^2 = 2$, increasing $M$ leads to a reduction by $13.27$ dB for $M = 16$ (fine) and $18.34$ dB for $M = 32$ (too much). We can verify for $M = 16$ that

$$P_s = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{d_{min}^2\,\mathcal{E}_b/N_0}\right) < 10^{-9}\,,$$

i.e., the factor $2(1 - 1/M)$ is negligible. We can achieve $R_b = k/T_s = 400$ kbps.

(10p)