Problem 1

(a) False.
Energy efficiency reduces with increasing M for PAM, QAM and PSK. But this is not true for FSK, where $d_{min} = \log_2(M)$.

(b) True.
With $T = T_s$ being fixed, we see from $R_b = k\, R_s = k/T$ that the information bit rate increases proportionally with k. Since W is fixed if T is fixed, this is also true for the bandwidth efficiency $\rho = R_b/W$. From $M = 16$ to $M = 64$, k increases from 4 to 6, which corresponds to an increase of R_b and ρ by a factor $6/4 = 1.5$.

(c) True.
$P_s \approx c\, Q\left(\sqrt{d_{min}^2\, \frac{E_b}{N_0}}\right) \approx 1.158 \cdot 10^{-7}$
with $c = 2$ and $d_{min}^2 = 2\log_2(M)\sin^2(\pi/M) \approx 0.30448$.

(d) True.
All real-valued signals have an even spectrum, i.e., $|X(f)| = |X(-f)|$ is symmetric around $f = 0$. But not all bandpass signals have to be symmetric around f_c, which means that the corresponding baseband signal may be complex.
A general bandpass transmit signal $x(t)$ can be expressed as

$$x(t) = Re\left\{(x_I(t) + j x_Q(t))\, e^{j\, 2\pi\, f_c\, t}\right\} = x_I(t)\, \cos(2\pi\, f_c\, t) - x_Q(t)\, \sin(2\pi\, f_c\, t)$$

(e) False.
The bandwidth W of an OFDM system increases with the number N of carriers. A single carrier system with the same bandwidth can thus use a smaller pulse duration, which results in approximately the same bandwidth efficiency. In our example, since the single carrier system uses $k = 6$ it actually achieves a larger bit rate than the OFDM system with $k = 4$.

Answers without explanation do not give any points.

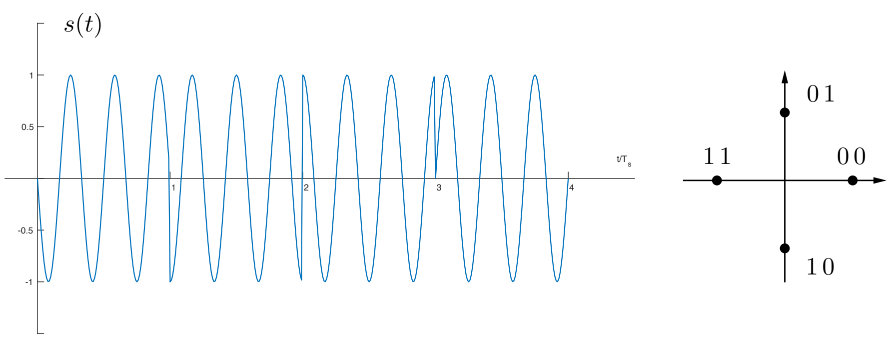
(10p)

Problem 2

(a)

- A $\rightarrow$ III (PSK): equal amplitude but different phases. Looking closer at the phase transition we see that it is III and not IV.
- B $\rightarrow$ I (bandpass PAM): we can identify amplitudes $-3, -1, +1, +3$. It cannot be III because we have two different magnitudes and no phase of $+\pi/2$ or $-\pi/2$.
- C $\rightarrow$ II (asymmetric QAM): BPSK in inphase and on-off keying in quadrature component. Three different phases and one signal with zero amplitude.
- D$\rightarrow$ I (baseband PAM): we can identify amplitudes $-3, -1, +1, +3$. Assuming the shape is sinusoidal it could alternatively be interpreted as bandpass PAM with a specific (very low) carrier frequency.

(b) $f_c = 3/T_s$



(c) We can write $z_\ell(t) = z_\ell \frac{g(t)}{\sqrt{E_g}} = z_\ell\, \phi_1(t)$. Then

$$\mathcal{E}_b = \frac{1}{k} \sum_{\ell=0}^{M-1} \frac{1}{M} \int_0^{T_s} z_\ell^2(t)\, dt = \frac{1}{k} \sum_{\ell=0}^{M-1} \frac{1}{M}\, z_\ell^2 \int_0^{T_s} \phi_1^2(t)\, dt = \frac{1}{k \cdot M} \sum_{\ell=0}^{M-1} z_\ell^2$$
$$= \frac{1}{2} \cdot \frac{1}{4} \left(1^2 + 3^2 + 5^2 + 7^2\right) \frac{5}{21} E_g = 2.5\, E_g$$

and

$$D_{min}^2 = (z_1 - z_0)^2 \int_0^{T_s} \phi_1^2(t)\, dt = \left((3-1) \sqrt{\frac{5\, E_g}{21}} \right)^2 = \frac{20}{21}\, E_g$$

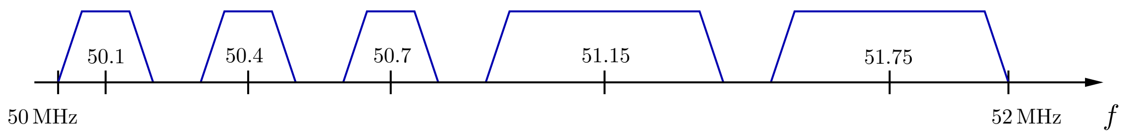
$$\Rightarrow\ d_{min}^2 = \frac{D_{min}^2}{2\, \mathcal{E}_b} = \frac{20/21\, E_g}{5\, E_g} = \frac{4}{21}$$

Conventional PAM: $\mathcal{E}_b = 2.5\, E_g$ (same) and $d_{min}^2 = 4/5$ (larger=better)

(10p)

Problem 3

(a)



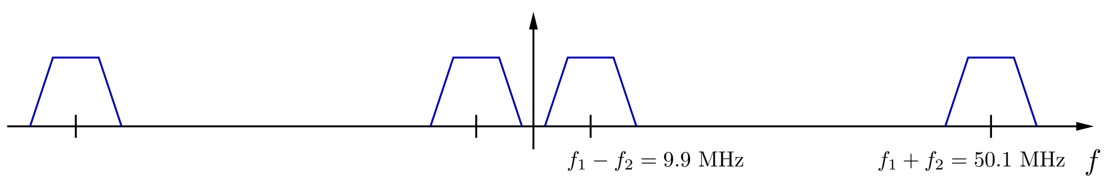
(b) $x_i(t) = s_i(t)\ \cos(2\pi\, f_{c,i}\, t)$ (assuming real valued signal $s(t)$)

or $x_i(t) = s_{I,i}(t)\ \cos(2\pi\, f_{c,i}\, t) - s_{Q,i}(t)\ \sin(2\pi\, f_{c,i}\, t)$ if $s_i(t) = s_{I,i}(t) + j\, s_{Q,i}(t)$

(each of the two solutions is fine)

(c) We can choose a carrier wave with frequency $f_2 = f_{c,1} - f_1 = 20.1\,\mathrm{MHz}$ to move the signal $\tilde{x}(t)$ to the frequency location $f_{c,1}$

$$x_1(t) = [\tilde{x}(t)\, 2\ \cos(2\pi\, f_2\, t)]_{BP}$$



A bandpass filter with center frequency $f_{c,1} = 50.1$ MHz and bandwidth 200 kHz is then applied to remove the other copies of the depicted signal.
Remark: we could alternatively use $f_2 = f_{c,1} + f_1$ to obtain the same result

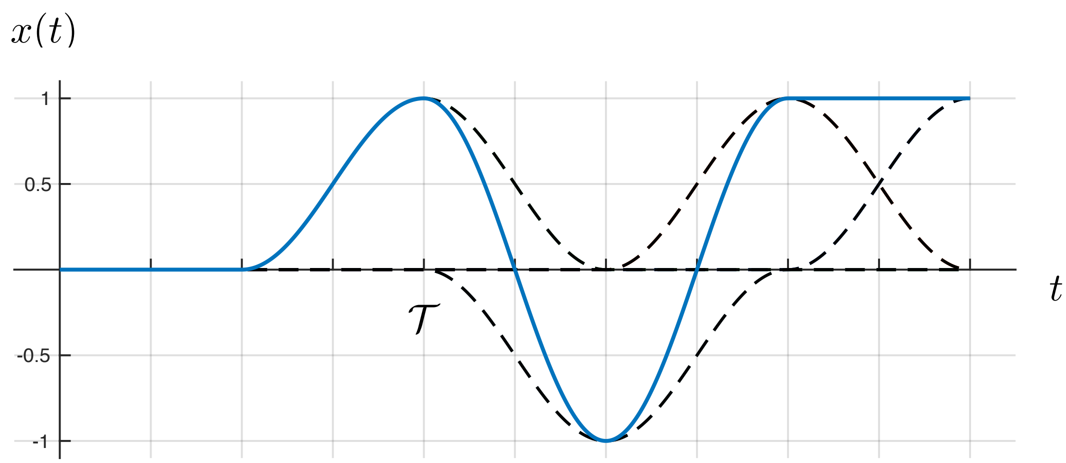
(d)
- with complex signal notation we can represent bandpass signals whose spectrum is not symmetric around f_c in the baseband domain
- the equivalent baseband model with complex notation is more compact than if we always have to compute the inphase and quadrature components explicitly
- the effect of noise and of the channel filter can be described within the equivalent baseband model

(10p)

Problem 4

(a) $T_s = T = 0.2\,\mu\text{s} \;\Rightarrow\; R_b = \frac{2}{T_s} = 10\,\text{Mbps}$

(b)



(c) 20 Mpbs $\Rightarrow T_s = 0.1\,\mu\text{s}$

$x[i] = 0.5\,\delta[i-1] + \delta[i] + 0.5\,\delta[i+1]$

(d)

$$\xi[i] = \sum_{k=-\infty}^{\infty} A[n]\,x[i-n] \overset{(i=100)}{=} A[101]\,x[-1] + A[100]\,x[0] + A[99]\,x[1] = 2$$

The value $\xi[100] = 2$ lies directly on the decision boundary between the ideal values $+1$ and $+3$. For this reason a correct decision is not guaranteed.

(e)

$$\sum_{n=-\infty}^{\infty} X_{nc}(f - n\,R_s) = \frac{x_0}{R_s}$$

- the shifted and repeated spectrum X_{nc} has to add up to a constant
- for the spectral raised cosine pulse, the overlapping roll-off regions satisfy this condition
- ISI-free transmission is possible although $T \nleq T_s$. The pulse is actually not limited at all in time domain, but it is equal to zero at the other time instants at which the signal is sampled.

(10p)

Problem 5

(a) $W_{lobe} = \frac{2}{T} = \frac{4}{T_s} \Rightarrow R_b = \frac{2}{T_s} = \frac{W_{lobe}}{2} = 50\,\text{kbps}$

(b) With $c\,Q\left(\sqrt{\mathcal{X}}\right) \leqslant k\,P_b$, where $c = 2$ and $k = 2$, we get $\mathcal{X} \geqslant (7.0345)^2 = 49.4209$. Furthermore, we have $\mathcal{X} = \frac{d_{min}^2\,\alpha^2\,P_{sent}}{R_b\,N_0}$ with $\alpha^2 = (0.01 \cdot 1/100)^2 = 10^{-8}$, $d_{min}^2 = 2$.

$$\Rightarrow \quad P_{sent} \geqslant 5.115 \cdot 10^{-6}\,\text{W}$$

(c) Assume a time raised cosine pulse, for which the mainlobe ends at $f_c + \frac{2}{T}$. Under the given conditions, $R(f)$ needs to be at least 45 dB below the peak of its mainlobe at $f_c + 100\,\text{kHz} = f_c + 4/T$ (smallest frequency of the other system). In Appendix D we can see that this pulse satisfies this condition.

Now $W_{lobe} = \frac{4}{T} = \frac{8}{T_s} \Rightarrow R_b = \frac{1}{T_b} = \frac{2}{T_s} = \frac{W_{lobe}}{4} = 25\,\text{kbps}$

In the calculation of P_{sent}, the bitrate changes by a factor $1/2$ while the rest stays the same $\Rightarrow$ $P_{sent} \geqslant 2.5575 \cdot 10^{-6}\,\text{W}$
(since we need to lower R_b, we need less power)

On the other hand, for the triangular pulse or the half-cycle sinusoidal pulse, we can see in Appendix D that the condition is not satisfied if we choose $W_{lobe} = 100\,\text{kHz}$. In these cases we would have to reduce the bandwidth dramatically to satisfy the requirements, and the resulting information bit rates would have to be much smaller than for the time raised cosine pulse.

(10p)