

# Exam in Digital Communications, EITG05

## October 26, 2017

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- During this exam you are allowed to use a calculator, the compendium, a printout of the lecture slides, and Tefyma (or equivalent).
- Please use a new sheet of paper for each solution. Write your anonymized assessment code + a personal identifier on each paper.
- Solutions should clearly show the line of reasoning and follow the methods presented in the course. If you use results from the compendium or lecture slides, please add a reference in your solution.
- ▶ If any data is lacking, make reasonable assumptions.

## Good luck!

Determine for each of the five statements below if it is true or false. Give a motivation for each of your answers.

- (a) *"For any type of M-ary signal constellation, increasing M reduces energy efficiency."*
- (b) Consider a conventional *M*-ary QAM system with a time raised cosine pulse of duration  $T = 0.25 \,\mu s$  and a symbol rate  $T_s = T$ .

"If the value of M is increased from M = 16 to M = 64, then both the bandwidth efficiency and the information bit rate are increased by a factor 1.5."

(c) Assume a conventional 16-ary PSK system that uses  $g(t) = g_{rc}(t)$  with duration  $T = T_s/2$ , a conventional AWGN channel, and an ML receiver. "If  $E_b/N_0$  is 19.65 dB, then  $P_s \approx 1.158 \cdot 10^{-7}$ ."

(d) *"For any QAM constellation, the spectrum* |X(f)| *of the transmitted signal* x(t) *is symmetric around* f = 0*."* 

(e) "An OFDM system with N = 16 carriers and 16 QAM modulation per carrier can achieve a higher information bit rate than a conventional 64 QAM single-carrier system of the same bandwidth."

The following four transmit signals are created using different signal constellations:



The information sequences are unknown and may be different for each signal.

(a) Match each of the above signals (A,B,C,D) to one of the following constellation diagrams (I, II, III, IV). Motivate each of your choices.



- (b) Consider now the signal corresponding to the constellation depicted in III. What is the carrier frequency  $f_c$ ? Using the same constellation, draw the transmit signal s(t) corresponding to the information sequence  $\mathbf{b} = 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0$ . For assigning bits to signal alternatives use some Gray mapping of your choice.
- (c) Another signal constellation is given as follows:

$$\sqrt{\frac{5E_g}{21}} \quad 3\sqrt{\frac{5E_g}{21}} \quad 5\sqrt{\frac{5E_g}{21}} \quad 7\sqrt{\frac{5E_g}{21}} \qquad \phi_1(t) = \frac{g(t)}{\sqrt{E_g}}$$

$$\mathbf{z}_0 \quad \mathbf{z}_1 \quad \mathbf{z}_2 \quad \mathbf{z}_3 \qquad \phi_1 \qquad \int_0^{T_s} \phi_1^2(t) \ dt = 1$$

Assume equally likely signal alternatives and determine the average energy per bit  $\mathcal{E}_b$  and the normalized squared minimum Euclidean distance  $d_{min}^2$ .

Compare the values with conventional PAM as given in (2.45) on page 33. What can we conclude from this result?

A communication system can serve five users simultaneously within the frequency band 50 MHz  $\leq f \leq 52$  MHz. The last two users have access to a larger bandwidth than the others. As a result, the baseband signals  $s_i(t)$ , i = 1, ..., 5 can be divided into two categories, having spectrum  $R_A(f)$  and  $R_B(f)$ , respectively.



Between two users an empty frequency range of 100 kHz (guard interval) is required.

- (a) Determine a suitable set of carrier frequencies  $f_{c,i}$ , i = 1, ...5 for the different users. Sketch the frequency content of the bandpass signals within the range  $50 \text{ MHz} \le f \le 52 \text{ MHz}$ .
- (b) Give an expression for the bandpass signal  $x_i(t)$  of user *i* in terms of the baseband signal  $s_i(t)$ . (Remark: you do not have to determine  $s_i(t)$  explicitly)
- (c) Consider now user 1 and assume that the transmitter has access to the signal  $\tilde{x}(t)$  only, which is a bandpass version of  $s_1(t)$  with carrier frequency  $f_1 = 30$  Mhz.



Describe how the transmitter can create the final bandpass signal  $x_1(t)$  of user 1 from  $\tilde{x}(t)$  using multiplier and a bandpass filter. Draw the spectrum of the converted signal before the filtering.

(d) List some reasons why a complex representation of baseband signals is useful (the more good reasons you find the better).

Assume a communication system employing 4 PAM modulation with equally likely signal alternatives. The combination of the transmit pulse g(t), channel filter h(t), and receiver filter v(t) can be written as x(t) = g(t) \* h(t) \* v(t) and is given as follows:



In the noise-free case, the signal y(t) at the output of the receiver filter is therefore (compare with Figure 6.2 in the compendium)

$$y(t) = \sum_{n=0}^{\infty} A[n] x(t - n T_s)$$

The signal is sampled in the receiver at time instants  $T + i T_s$ , i = 0, 1, 2, ...

- (a) What is the maximum possible bitrate that can be achieved so that there is no intersymbol interference (overlap of symbols at the sampling instances in the receiver)?
- (b) Sketch the corresponding signal y(t) in the interval  $0 < t < T + 0.6 \,\mu s$  for the amplitude sequence A[0] = 1, A[1] = -1, A[2] = 1, A[3] = 1.
- (C) Assume now that we want to achieve a bitrate of 20 Mbps and that we can tolerate ISI. Determine the discrete impulse response of the system

$$x[i] = x(\mathcal{T} + i\,T_s)$$

for this particular case.

(d) For the case considered in (c), determine the value of the decision variable  $\xi[i]$  at i = 100 given the sequence

A[98] = -3, A[99] = 1, A[100] = 3, A[101] = -3, A[102] = -1.

Assume that the same threshold detector is used in the receiver as for the ISI-free case. Will the message  $\hat{m}[i]$  at i = 100 be detected correctly?

(e) Use the spectral raised cosine pulse  $x_{nc}(t)$ , defined on page 452, as an example to explain the ISI-free condition in frequency domain.

As the chief technology officer (CTO) of a new and hot startup DigComm AB, you are given the task of designing a communication system for high reliability communications between sensor nodes. Through talks with your boss and hardware designers in your company, you learn of some practical constraints for your system:

- The system will operate at a carrier frequency  $f_c$  and use a very small bandwidth of W = 100 kHz (W is measured as the width of the mainlobe)
- You need to use QPSK as the modulation method
- The bit error probability  $P_b$  must be kept below  $10^{-12}$  (at such high SNR you can approximate  $P_b$  by  $P_s/k$  assuming Gray mapping)
- (a) At first, you choose a rectangular pulse. To combat possible ISI at the receiver, you make the pulse shorter than the total symbol duration, in particular,  $T = T_s/2$ . What is the bitrate that can be achieved?
- (b) Another constraint that the system has to fulfill is that the distance *d* between transmitter and receiver should be at most 100 meters. Your colleagues report that the propagation attenuation  $\alpha$  and the parameter  $N_0$  of the additive white Gaussian noise are given by

$$\alpha = 0.01 \cdot d^{-1}$$
,  $N_0 = 4.14 \cdot 10^{-20} \, [W/Hz]$ .

The ML/matched filter receiver is used for detection. The hardware designers would like to know from you the required transmit power for worst case coverage, i.e., when d = 100 m. Which value do you report to them?

(c) After you have done the design and written the technical report, your boss calls you. He tells you that he was informed by the Swedish frequency regulation agency that there is another system in the frequency band right next to your system. The carrier frequency of that system is equal to  $f_c + 150$  kHz and its bandwidth is 100 kHz.

The problem is that for your current system design, the side-lobes are causing too strong out-of-band leakage, which causes interference to the neighboring system. The regulation agency requires that, within the frequency band of the other system, the power spectral density R(f) of your signal has to be at least 45 dB below the peak of its main-lobe. If this requirement is violated you will be charged huge fines from the agency.

You quickly realize that you need to change the pulse shape. The hardware designers tell you that they are able to implement triangular, half-cycle sinusoidal or time raised cosine pulses.

Would choosing any of these pulses help achieve the goals set by the regulation agency? If yes, what would be the bitrate after changing the pulse, and the required transmit power (calculated as in part b))?