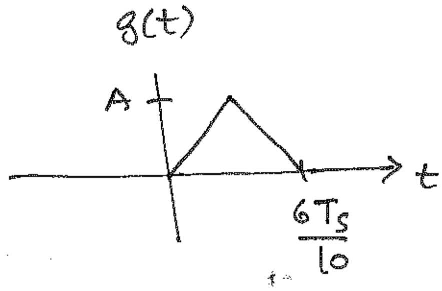
Prob 3-1

4-ary QAM , $M = 4$

$g(t) = g_{tri}(t)$

$\bar{P}_s = ?$ (Average Signal power)



Sol

$$\bar{P}_s = \bar{E}_s R_s.$$

where $\bar{E}_s = \dfrac{2(M-1)}{3} \cdot \dfrac{E_g}{2}$. [Eq. 2.98].

and E_g of $g_{tri}(t) = g(t)$ given above is [Eq D.10 at p # 618].

$$E_g = \frac{A^2 T}{3}$$

where $T = \dfrac{6T_s}{10}$

$$E_g = \frac{A^2}{3} \cdot \frac{6T_s}{10} = \frac{A^2 T_s}{5}$$

and $\bar{E}_s = \dfrac{2(\cancel{3})}{\cancel{3}} \cdot \dfrac{1}{2} \cdot \dfrac{A^2 T_s}{5} = \dfrac{2A^2 T_s}{10}$.

and $\bar{P}_s = \dfrac{\bar{E}_s}{T_s}$ [since $R_s = \dfrac{1}{T_s}$].

$$\boxed{\bar{P}_s = \frac{2A^2}{10}}$$ True.

P#3.2 $T_s = 2\,\mu s$

(a) signaling method = ?

Since amplitude of each signal alternative in s(t) is ~~constant~~ same and only phase of each signal alternative is varying. Therefore, signaling method can be either 8-PSK or 8-QAM.

message sequence $m = ?$

for PSK $s_\ell(t) = g(t)\cos(2\pi f_c t + v_\ell)$

$$v_\ell = \frac{2\pi \ell}{M}$$

from s(t) we can infer that $g(t) = \begin{cases} 1 & 0 \le t \le T_s \\ 0 & \text{o.w} \end{cases}$

pulse shape is used. Signal alternatives with corresponding phase are

$s_0(t)$	has	phase	$v_0 = 0$.
$s_1(t)$	has	phase	$v_1 = \frac{\pi}{4}$
$s_2(t)$	"	"	$v_2 = \frac{\pi}{2}$
$s_3(t)$	"	"	$v_3 = \frac{3\pi}{4}$
$s_4(t)$	"	"	$v_4 = \pi$
$s_5(t)$			$v_5 = \frac{5\pi}{4}$
$s_6(t)$			$v_6 = \frac{3\pi}{2}$
$s_7(t)$			$v_7 = \frac{7\pi}{4}$

From give s(t), it can be seen that

i	s(t)	Phase	$s_\ell(t)$	m[i]
0	$0 - T_s$	v_2	$s_2(t)$	2
1	$T_s - 2T_s$	v_5	$s_5(t)$	5
2	$2T_s - 3T_s$	v_0	$s_0(t)$	0
3	$3T_s - 4T_s$	v_4	$s_4(t)$	4
4	$4T_s - 5T_s$	v_3	$s_3(t)$	3

(b) Draw amplitudes A_ℓ and B_ℓ in a diagram
(Similar to fig 2.10)

Trigonometric identity
$$\boxed{\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta}$$

$$s_\ell(t) = \cos(\underbrace{2\pi f_c t}_{\alpha} + \underbrace{v_\ell}_{\beta})$$

$$s_\ell(t) = \cos(2\pi f_c t)\cos(v_\ell) - \sin(2\pi f_c t)\sin(v_\ell)$$

$$\ell=0 \Rightarrow s_0(t) = \underbrace{\cos(v_0)}_{A_0}\cos(2\pi f_c t) - \underbrace{\sin(v_0)}_{B_0}\sin(2\pi f_c t) \quad \text{--- (1)}$$

$$\ell=1 \Rightarrow s_1(t) = \underbrace{\cos(v_1)}_{A_1}\cos(2\pi f_c t) - \underbrace{\sin(v_1)}_{B_1}\sin(2\pi f_c t) \quad \text{--- (2)}$$

$$\vdots$$

$$\ell=7 \Rightarrow s_7(t) = \underbrace{\cos(v_7)}_{A_7}\cos(2\pi f_c t) - \underbrace{\sin(v_7)}_{B_7}\sin(2\pi f_c t) \quad \text{--- (7)}$$

For QAM signal $\left[s_\ell(t) = A_\ell \overset{g(t)}{\cos}(2\pi f_c t) - B_\ell \overset{g(t)}{\sin}(2\pi f_c t) \right]$ (Lec 2: Slide 19)

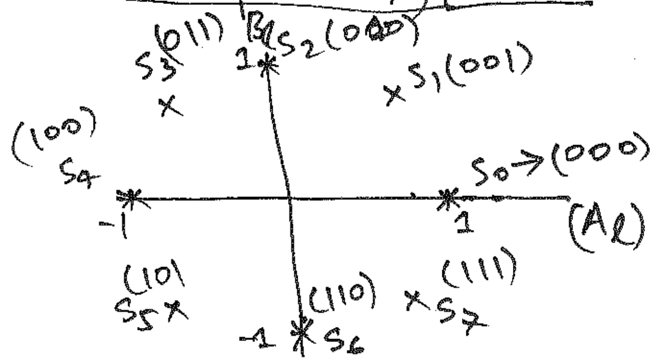
since $g(t) = 1$ during a symbol time

$$s_\ell(t) = A_\ell\cos(2\pi f_c t) - B_\ell\sin(2\pi f_c t) \quad \text{--- (8)}$$

by comparing eq. 8 to equations 1 to 7 we can identify amplitudes

A_0	$\cos(v_0)$	1
A_1	$\cos(v_1)$	.707
A_2	$\cos(v_2)$	0
A_3	$\cos(v_3)$	-.707
A_4	$\cos(v_4)$	-1
A_5	$\cos(v_5)$	-.707
A_6	$\cos(v_6)$	0
A_7	$\cos(v_7)$	.707

B_0	$\sin(v_0)$	0
B_1	$\sin(v_1)$	.707
B_2	$\sin(v_2)$	1
B_3	$\sin(v_3)$	.707
B_4	$\sin(v_4)$	0
B_5	$\sin(v_5)$	-.707
B_6	$\sin(v_6)$	-1
B_7	$\sin(v_7)$	-.707

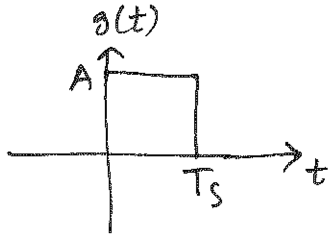


transmitted bits are

$$\underbrace{010}_{s_2(t)}\ \underbrace{101}_{s_5(t)}\ \underbrace{000}_{s_0(t)}\ \underbrace{100}_{s_4(t)}\ \underbrace{011}_{s_3(t)}$$

Prob 3.3

$g(t) = g_{rec}(t)$

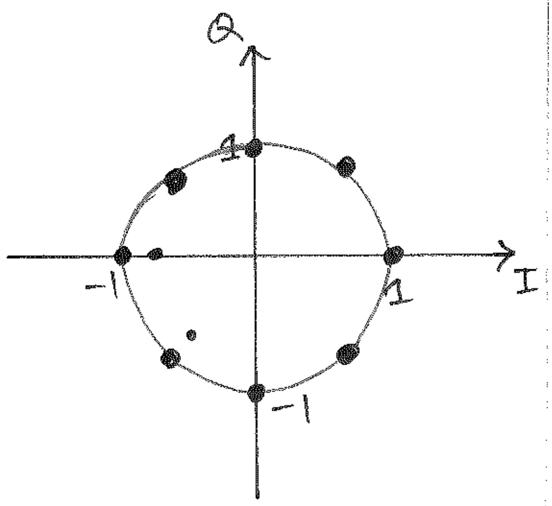


(a) Draw constellation diagram for 8 PSK and QAM

For PSK we have

$v_0 = 0,\ v_1 = \frac{\pi}{4},\ v_2 = \frac{\pi}{2},\ v_3 = \frac{3\pi}{4},\ v_4 = \pi,\ v_5 = \frac{5\pi}{4}$

$v_6 = \frac{3\pi}{2},\ v_7 = \frac{7\pi}{4}$



normalized Energy on I and Q axis.

Fig. 8-PSK constellation

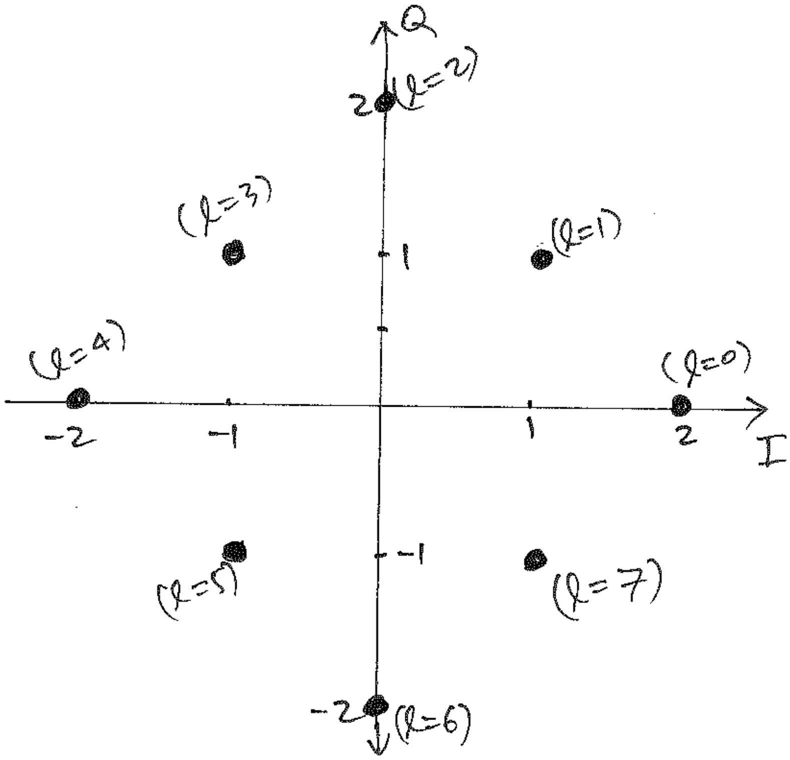


Fig. QAM constellation.

(b) Avg. Energy of PSK

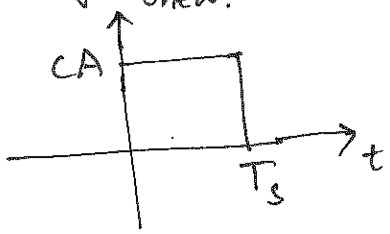
$$\bar{E}_s = \frac{E_g}{2} \quad [\text{Eq. } 2.58 \ \text{P\# } 37].$$

where $E_g = A^2 T_s$ when $g(t) = g_{rec}(t)$ is used.

Since we want to scale $g(t)$ with c the resultant $E_{g\,(new)}$ is

$$E_{g(new)} = c^2 A^2 T_s = c^2 E_g$$

when g_{new} is



[since $\bar{E}_b = \frac{\bar{E}_s}{k}$].

hence $\bar{E}_s = \frac{c^2 E_g}{2}$

and $\boxed{\underset{(PSK)}{\bar{E}_b} = \frac{c^2 E_g}{6}}$ —— (1)

Avg. Energy per bit of QAM constellation.

$$\bar{E}_s = \sum_{l=0}^{7} P_l E_l \qquad P_l = \frac{1}{8} \text{ for } l=0.$$

$$\bar{E}_s = \frac{1}{8}(4E_0 + 4E_1)$$

$E_0 = ?$ For QAM $E_l = (A_l^2 + B_l^2)\, E_g/2$ [Eq. 2.94 P# 47].

hence $E_0 = 2E_g$ and $E_1 = E_g$.

$$\bar{E}_s = \frac{1}{8}(8E_g + 4E_g) = \frac{3}{2}E_g.$$

$\boxed{\underset{(QAM)}{\bar{E}_b} = \frac{E_g}{2}}$ —— (2)

In order for $\bar{E}_b$ of 8-PSK and QAM to be equal c should be

From ① and ②

$$\bar{E}_{b(PSK)} = \bar{E}_{b(QAM)}$$

$$\frac{c^2 E_g}{6} = \frac{E_g}{2} \Rightarrow \boxed{c = \sqrt{3}.}$$

which constellation have larger min. squared Euclidean distance $\min_{ij} D_{ij}^2$?

PSK $\quad D_{ij}^2 = E_g(1 - \cos(\theta_i - \theta_j))$ [Eq. 2.60 p# 38].

Since distance between neighboring signal points is same, $\min_{ij} D_{ij}^2$ in this case is the distance between any two nearest neighbors on the constellation diagram

$$D_{ij}^2 = 3E_g\left(1 - \cos\left(0 - \frac{\pi}{4}\right)\right) = D_{01}^2$$

$$\boxed{\min_{ij} D_{ij}^2 = D_{01}^2 = .8787\, E_g.}$$

QAM $\quad D_{ij}^2 = \frac{E_g}{2}\left((A_i - A_j)^2 + (B_i - B_j)^2\right)$ [Eq. 2.95 on P#47

$\min D_{ij}^2$ is the distance between two closest points in the constellation diagram, e.g., between $l=0$ and $l=1$

$$\min \boxed{D_{ij}^2 = E_g}$$

hence QAM has largest $\min D_{ij}^2$