EITG05 – Digital Communications

Lecture 1

Introduction, Overview, Basic Concepts (p. 1–32)

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Digital Communications

We are in a global digital (r)evolution

- Mobile data and telephony (GSM, EDGE, 3G, 4G, 5G)
- Digital radio and television, Bluetooth, WLAN
- Data storage, CD, DVD, Flash, magnetic storage
- Optical fiber, DSL (long range, high rate)
- Cloud computing, big data, distributed storage
- Connected devices, Internet of things, machine-to-machine communication, distributed control, cyber physical systems

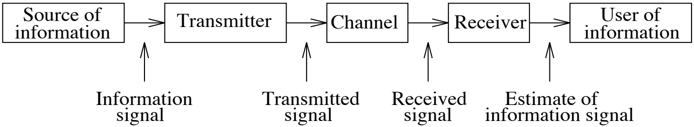
The large number of different application scenarios require flexible communication solutions (data rate / delay / reliability / complexity)

Remark storage of data falls also into the category of a communication system (why?)

What is communication?

- The purpose of a communication system is to transmit messages (information) from a source to a destination

  Examples: sound, picture, movie, text, etc.
- The messages are converted into signals that are suitable for transmission
- The physical medium for transmission is called the channel



- The received signal is used to estimate the messages

What are analog / digital signals?

Analog versus digital

- **Analog communication:**
  both source and processing are analog
- **Digital communication:**
  the source messages are digital, i.e., can be represented by discrete numbers (digits)

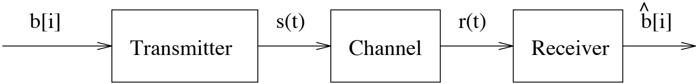
Example 1: I speak and you listen to the acoustic sound wave

Example 2: I record my speech to MP3 and send it to you, who plays it back on your computer or phone

Example 3: I use morse code and a flashlight to transmit a message to my neighbor

In all cases some analog medium has to be used during the transmission at some point

Scope of this course



- Transmitter principles: bits to analog signals (Chap. 2)
- Receiver principles: analog noisy signals to bits (Chap. 4,5,6)
- Characteristics of the communication link (Chap. 3,6)

Requirements:

- Data should arrive correctly at the receiver
- High bit rates are desirable
- Energy/power efficiency
- Bandwidth efficiency

What are the technical solutions and challenges?

Not in this course

- Analog to digital conversion, sampling theorem, quantization
  $\Rightarrow$ basic signals & systems or signal processing course
- Source coding (compression)
  $\Rightarrow$ covered in information theory course (elective)
- Channel coding (robust and reliable communication)
  $\Rightarrow$ covered in separate course (elective)
- Cryptography (secure communication)
  $\Rightarrow$ covered in separate course (elective))

There exist a large number of specialized courses that can be taken after this basic course.

There is also a project course in wireless communications.

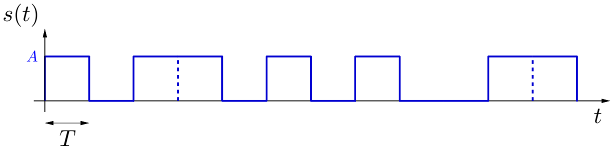
The Transmitter

How can we map digital data to analog signals?

$$b[i] = 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1$$

A simple approach:

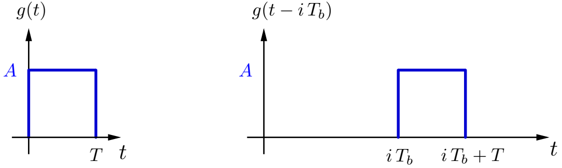
apply some voltage A during transmission of a 1



Basic operation: (more general)
represent the sequence of information bits $b[i]$ by a sequence of analog waveforms, resulting in the transmit signal $s(t)$

The Transmitter

- The analog waveform corresponding to the bit $b[i]$ can be written as a time-shifted version of an elementary pulse $g(t)$

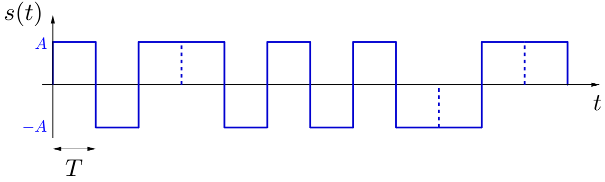


- T_b is the information bit interval, while T is the pulse duration
- For now we assume that $T \leq T_b$, i.e., the pulses do not overlap
- We can now represent the transmit sequence $s(t)$ as follows

$$s(t) = b[0]\,g(t) + b[1]\,g(t - T_b) + b[2]\,g(t - 2\,T_b) + \cdots$$

Variations of our signaling example

- In our example we only send a signal when $b[i] = 1$
  This modulation type is called on-off signaling
- Instead we could send a pulse with amplitude $-A$ for $b[i] = 0$:

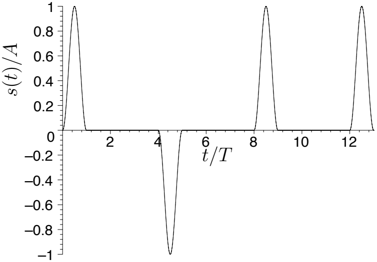


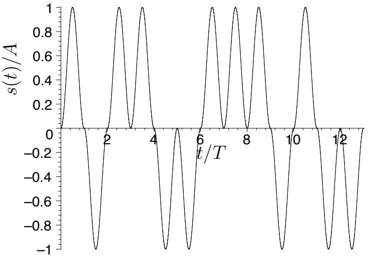
  This modulation type is called antipodal signaling

- We could also choose a different pulse shape $g(t)$

In this chapter: different modulation types and their properties

Another pulse example ($\rightarrow$ p. 10)

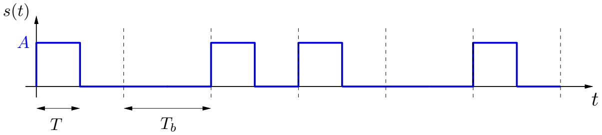




What are the input sequences $b[i]$ here?

What data rate can we achieve?

- We could also choose a shorter pulse, with $T < T_b$ (what for?)



- An important parameter is the information bit rate

$$R_b = \frac{B}{\tau} \text{ [bps] (bits per second) },$$

if the source produces B information bits during τ seconds

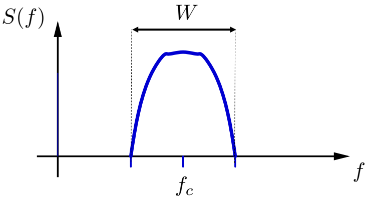
- If we avoid overlapping pulses we need $T \leq T_b$ and

$$R_b = \frac{1}{T_b} \leq \frac{1}{T}$$

Observe: T determines the pulse length and T_b the rate

What bandwidth is required?

- The bandwidth W of the transmit signal is a valuable resource



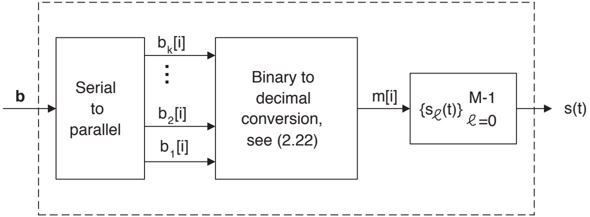
- For typical pulses $g(t)$ the bandwidth W is proportional to $\frac{1}{T}$
- More details about the bandwidth of $s(t)$ follow next week
- A challenging goal is to achieve a large bandwidth efficiency

$$\rho = \frac{R_b}{W} \quad \left[\frac{\mathrm{b/s}}{\mathrm{Hz}}\right]$$

Question: What happens when the pulse duration gets small?

Increasing the message alphabet

- Up to this point we have considered binary signaling only
- Each bit $b[i]$ was mapped to one of two signals $s_0(t)$ or $s_1(t)$
- More generally, we can combine k bits $b_1[i], b_2[i], \dots b_k[i]$ to a single message $m[i]$, which then is mapped to a signal $s_\ell(t)$



- In case of M-ary signaling, one of $M = 2^k$ messages $m[i]$ is transmitted by its corresponding signal alternative

$$s_\ell(t) \in \{s_0(t), s_1(t), \dots, s_{M-1}(t)\}$$

M-ary signaling

Example: $k = 2$, $M = 2^2 = 4$

The binary sequence

$$b_n[i] = 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1$$

is mapped by

$$m[i] = \sum_{n=1}^{k} b_n[i] \; 2^{n-1} = b_1[i] + b_2[i] \cdot 2$$

to $M = 4$ signal alternatives

$$b[i] = 00 \leftrightarrow m[i] = 0 \leftrightarrow s_0(t) \qquad b[i] = 10 \leftrightarrow m[i] = 1 \leftrightarrow s_1(t)$$
$$b[i] = 01 \leftrightarrow m[i] = 2 \leftrightarrow s_2(t) \qquad b[i] = 11 \leftrightarrow m[i] = 3 \leftrightarrow s_3(t)$$

The message sequence becomes

$$m[i] = 1 \quad 3 \quad 2 \quad 2 \quad 0 \quad 3$$

With $k = 14$ there are $M = 16\,384$ signal alternatives

Symbol rate versus bit rate

- When the message equals $m[i]=j$ then $s_j(t-iT_s)$ is sent

$$s(t)=s_{m[0]}(t)+s_{m[1]}(t-T_s)+s_{m[2]}(t-2T_s)+\cdots$$

How does k affect the bandwidth efficiency ρ?

- Since k information bits are transmitted with each symbol, the symbol interval (symbol time) becomes

$$T_s=kT_b$$

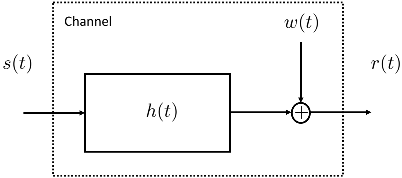
- Accordingly, the symbol rate (signaling rate) is given by

$$R_s=\frac{1}{T_s}\left[\frac{\text{symbols}}{\text{s}}\right]=\frac{R_b}{k}$$

Remark: Be careful with the different definitions of time:
t: time variable T: pulse duration T_b: bit time T_s: symbol time

The Channel

- The channel is often modeled as time-invariant filter with noise



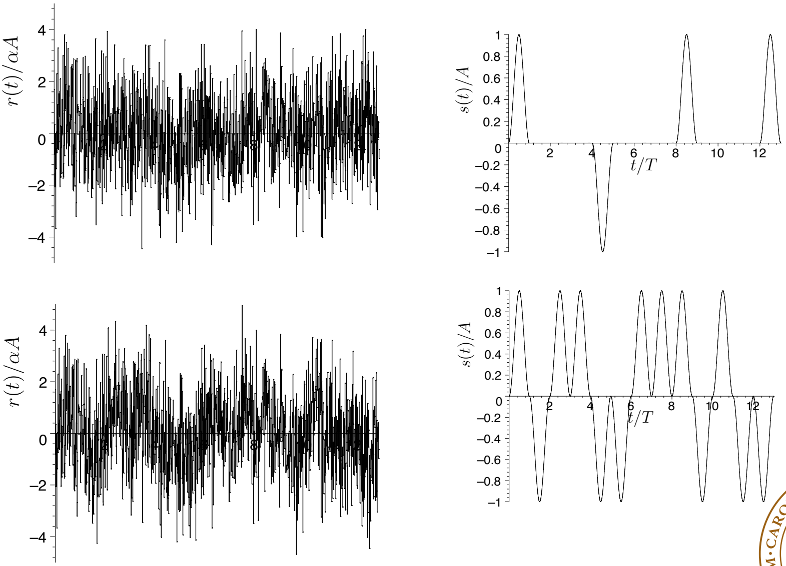
- $h(t)$ is the channel impulse response and $w(t)$ the additive noise
- The received signal becomes

$$r(t) = s(t) * h(t) + w(t) = \int_{-\infty}^{\infty} h(\tau)\, s(t-\tau)\; d\tau + w(t)$$

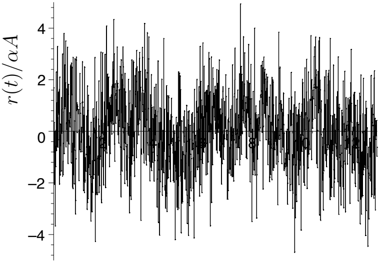
- For now we assume the simple case (α: attenuation)

$$h(t) = \alpha\, \delta(t) \qquad \Rightarrow r(t) = \alpha\, s(t) + w(t)$$

Example: noisy signal at the receiver (p. 13)



The Receiver



- Due to the attenuation α during transmission, the noise $w(t)$ has a strong impact on the received signal $r(t)$
- A well designed receiver can still detect the symbols correctly! In this example, only 1 of 10^5 bits will be wrong in average
- We will learn about the receiver and its performance later, in Chapters 4 and 5

Bit Errors

- The bit error probability is an important measure of communication performance
- It is defined as the average number of information bit errors per detected information bit

$$P_b = \frac{E\{B_{err}\}}{B}$$

Example:

- Assume a bit rate of 1 Mbps and that 10 bit errors occur per hour *on the average*. What is the bit error probability?
- The total number of bits in an hour is

$$B = 1\,000\,000 \cdot 60 \cdot 60 = 3.6 \cdot 10^9$$

This gives

$$P_b = \frac{10}{B} = 2.78 \cdot 10^{-9}$$

$\Rightarrow$ Computer simulations become very time consuming!

Signal energy and power

- The symbol energy E_ℓ of a signal alternative $s_\ell(t)$ is given by

$$E_\ell = \int_0^{T_s} s_\ell^2(t)\,dt < \infty\,, \quad \ell = 0, 1, \ldots, M-1$$

- An important system parameter is the average symbol energy

$$\overline{E}_s = \sum_{\ell=0}^{M-1} P_\ell \, E_\ell\,, \quad P_\ell = \Pr\{m[i] = \ell\}$$

  and the average signal energy per information bit

$$\overline{E}_b = \frac{\overline{E}_s}{k}$$

- The average signal power is then given by

$$\overline{P} = R_s \overline{E}_s = \frac{R_b}{k} \cdot k\overline{E}_b = R_b \overline{E}_b$$

Signal energy and power

- The attenuation α and the noise $w(t)$ determine the quality of a communication link

$$r(t) = \alpha\, s(t) + w(t)$$

Example:

If a transmitted signal $s(t)$ has energy $\overline{E}_b$, how much energy $\mathcal{E}_b$ is then in the received signal $z(t) = \alpha \cdot s(t)$ if $\alpha = 0.001$?

- Using $z^2(t) = \alpha^2 s^2(t)$ we obtain

$$\overline{P}_z = \alpha^2\, \overline{P} = \alpha^2 R_b \overline{E}_b$$

$$\text{and} \quad \mathcal{E}_b = \frac{\overline{P}_z}{R_b} = \alpha^2 \frac{\overline{P}}{R_b} = \alpha^2 \overline{E}_b$$

- If $\alpha = 0.001$ then the power is reduced by a factor 10^{-6}

This will increase the bit error probability!

How well can we distinguish two signals?

- The squared Euclidean distance between two signals $s_i(t)$ and $s_j(t)$ is defined as

$$\begin{aligned} D_{i,j}^2 &= \int_0^{T_s} \left(s_i(t) - s_j(t)\right)^2 \, dt \\ &= \int_0^{T_s} s_i^2(t) + s_j^2(t) - 2\, s_i(t) s_j(t) \, dt \\ &= E_i + E_j - 2 \int_0^{T_s} s_i(t) s_j(t) \, dt \end{aligned}$$

- Two signals are antipodal if

$$s_i(t) = -s_j(t) \; , \quad 0 \leq t \leq T_s$$

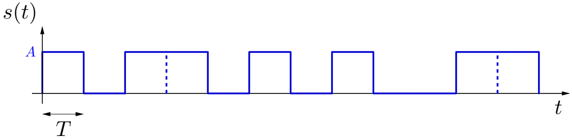
- Two signals are orthogonal if

$$\int_0^{T_s} s_i(t) s_j(t) \, dt = 0$$

Antipodal signals have larger Euclidean distance

Euclidean distance example $M = 2$

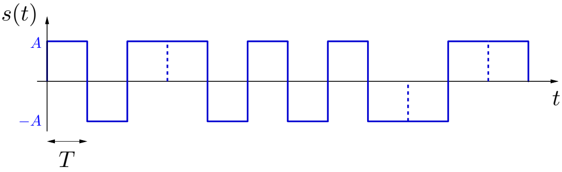
Case 1: on-off signaling



$s_0(t) = A$ and $s_1(t) = 0$ for $0 < t < T_s = T$, which gives $D^2_{0,1} = 2\overline{E}_b$

Observe: on-off signaling is orthogonal

Case 2: antipodal signaling



$s_0(t) = A$ and $s_1(t) = -A$ for $0 < t < T_s = T$, and $D^2_{0,1} = 4\overline{E}_b$

Examples of pulse shapes: Appendix D

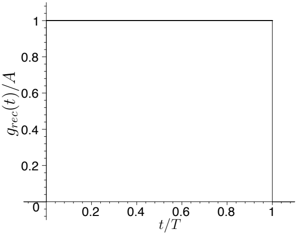


Figure D.1: $g_{rec}(t)/A$.

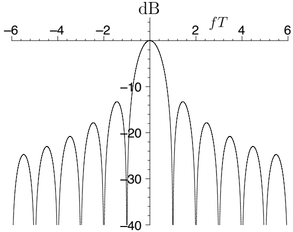


Figure D.2: $\frac{|G_{rec}(f)|^2}{E_g T}$ in dB.

1. The rectangular pulse:

$$g_{rec}(t) = \begin{cases} A & , \quad 0 \leq t \leq T \\ 0 & , \quad \text{otherwise} \end{cases} \tag{D.1}$$

$$E_g = \int_0^T g_{rec}^2(t)dt = \int_{-\infty}^{\infty} |G_{rec}(f)|^2 df = A^2 T \tag{D.2}$$

Examples of pulse shapes: Appendix D

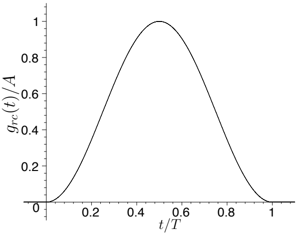


Figure D.9: $g_{rc}(t)/A$.

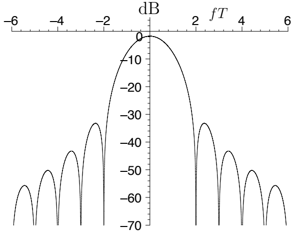


Figure D.10: $\frac{|G_{rc}(f)|^2}{E_g T}$ in dB.

5. The time raised cosine pulse:

$$g_{rc}(t) = \begin{cases} \frac{A}{2}\left(1-\cos(2\pi t/T)\right) & , \quad 0 \le t \le T \\ 0 & , \quad \text{otherwise} \end{cases} \tag{D.18}$$

$$E_g = 3A^2T/8 \tag{D.19}$$

Short summary

- We map digital data to analog transmit signals
- Elementary signal operations are important: get familiar with scaling, shifting, reflection
- M-ary signaling: more than one bit per signal alternative
- Study definitions of paramters: bit rate, bit time, pulse duration, symbol rate, symbol time, bandwidth efficiency, bit error probability
- Signal energy, power, and Euclidean distance: design criteria for efficient transmission