LUND UNIVERSITY

EITG05 – Digital Communications

Lecture 9

Chap. 3: N-ray channel model, noise,
Receivers for bandpass signals

Chap. 4: Filtered channel receiver

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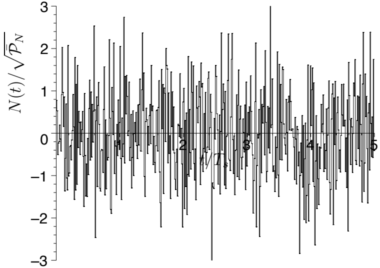
Thursday, October 3, 2019

Channel Noise

- In almost all applications the received signal $r(t)$ is disturbed by some additive noise $N(t)$:

$$r(t) = z(t) + N(t)$$



- Since the received noise disturbs that transmitted signal, we need to characterize its influence on the performance in terms of bit error rate or achievable information bit rate

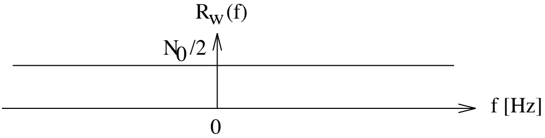
White Gaussian Noise

- White Gaussian noise $w(t)$ is a common model for background noise, such as created by electronic equipment
- The samples of $w(t)$ have a zero-mean Gaussian distribution
- Any two distinct samples of $w(t)$ are uncorrelated

$$r_w(\tau) = E\{w(t+\tau)\,w(t)\} = \frac{N_0}{2}\,\delta(\tau)$$

- This leads to a constant power spectral density

$$R_w(f) = \int_{-\infty}^{\infty} r_w(\tau)\,e^{-j2\pi f\,\tau}\,d\tau = \frac{N_0}{2}\;, \quad -\infty \le f \le \infty$$



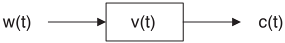
All frequencies are disturbed equally strongly

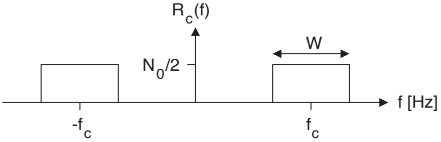
Filtered Gaussian Noise

- In reality we usually deal with filtered noise of limited bandwidth, so-called colored noise
- Assuming that white Gaussian noise $w(t)$ passes a filter $v(t)$ we obtain colored noise $c(t)$ with power spectral density

$$R_c(f) = R_w(f)\,|V(f)|^2 = \frac{N_0}{2}\,|V(f)|^2$$

- For an ideal bandpass filter $v(t)$ with bandwidth W the spectrum is shown below:





Filtered Gaussian Noise

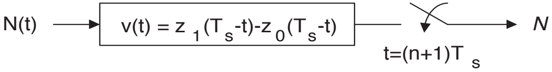
- Since $R(f)$ is constant within the bandwidth W, such a process $c(t)$ is usually referred to as "white" bandpass process
- Let the noise process $c(t)$ be sampled at some time $t = t_0$. Then the sample value $c(t_0)$ is a Gaussian random variable with

$$p(c) = \frac{1}{\sqrt{2\pi\,\sigma^2}}\, e^{-(c-m)^2/2\sigma^2}$$

with mean $m = 0$ and variance $\sigma^2 = N_0/2\, E_v = N_0\, W = \mathcal{P}_c$

Example: matched filter output (recall Chapter 4)

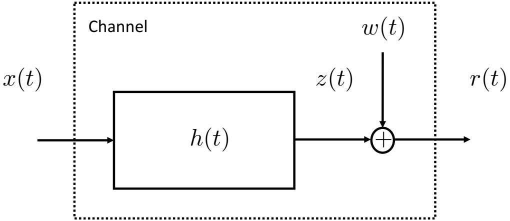
The additive noise $\mathcal{N}$ is sampled from a filtered noise process



$$\sigma^2 = N_0/2 \cdot E_v = N_0/2 \int_0^{T_s} \left(z_1(t) - z_0(t)\right)^2\, dt$$

Linear-Filter Channels

- The channel is often modeled as time-invariant filter with noise



- $h(t)$ is the channel impulse response and $w(t)$ the additive noise
- The received signal becomes

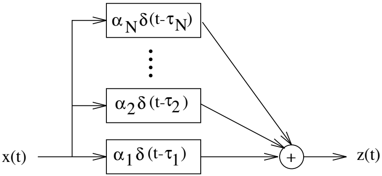
$$r(t) = x(t) * h(t) + w(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\; d\tau + w(t)$$

- The simplest case is an attenuated noisy channel:

$$h(t) = \alpha\, \delta(t) \qquad \Rightarrow r(t) = \alpha\, s(t) + w(t)$$

N-ray Channel Model

- In many applications (wired and wireless) the transmitted signal $x(t)$ reaches the receiver along several different paths
- Such multi-path propagation motivates the N-ray channel model



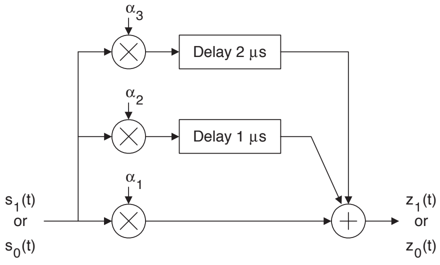
- The output signal becomes

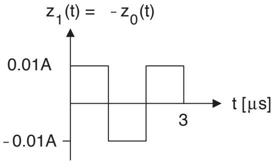
$$z(t) = \sum_{i=1}^{N} \alpha_i \, x(t - \tau_i) = x(t) * h(t)$$

- The impulse response $h(t)$ and its Fourier transform are given by

$$h(t) = \sum_{i=1}^{N} \alpha_i \, \delta(t - \tau_i) \; , \quad H(f) = \sum_{i=1}^{N} \alpha_i \, e^{-j 2\pi f \tau_i}$$

Example 3.19: multipath propagation





$$s_1(t) = -s_0(t) = \begin{cases} A & , \quad 0 \leq t \leq 10^{-6} \\ 0 & , \quad otherwise \end{cases}$$

$$\alpha_1 = 0.01, \alpha_2 = -0.01, \alpha_3 = 0.01$$

- The channel (= filter) increases the length of the signals
- Signals exceed their time interval and will overlap if T_s is not increased accordingly ⇒ inter-symbol interference (ISI)

Example 3.20

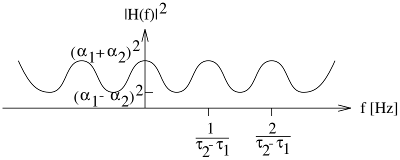
EXAMPLE 3.20

Calculate and sketch $|H(f)|^2$ for the 2-ray channel model.

Solution:

From (3.128) we obtain,

$$\begin{aligned}
H(f) &= \alpha_1 e^{-j2\pi f\tau_1} + \alpha_2 e^{-j2\pi f\tau_2} = \\
&= e^{-j2\pi f\tau_1}\left(\alpha_1 + \alpha_2 e^{-j2\pi f(\tau_2-\tau_1)}\right) \\
|H(f)|^2 &= \left(\alpha_1 + \alpha_2 e^{-j2\pi f(\tau_2-\tau_1)}\right)\left(\alpha_1 + \alpha_2 e^{+j2\pi f(\tau_2-\tau_1)}\right) = \\
&= \alpha_1^2 + \alpha_2^2 + \alpha_1\alpha_2\left(e^{j2\pi f(\tau_2-\tau_1)} + e^{-j2\pi f(\tau_2-\tau_1)}\right) = \\
&= \alpha_1^2 + \alpha_2^2 + 2\alpha_1\alpha_2\cos(2\pi f(\tau_2-\tau_1))
\end{aligned}$$



Channel fading: some frequencies are attenuated strongly

Features of Multipath Channels

Challenges:

- the receiver needs to know the channel
- training sequences need be transmitted for channel estimation
- the impulse response can change over time
- the line-of-sight (LOS) component is sometimes not received

Opportunities:

- with multiple paths we can collect more signal energy
- receiver can work without direct LOS component
- channel knowledge, once we have it, can give useful information:
  Examples: distance, angle of arrival, speed (Doppler)
- positioning/navigation is often based on channel estimation

If you want to know more:

EITN85: Wireless Communication Channels, VT 1

Receiver for linear filter channel model

- For a simple channel with a direct transmission path only

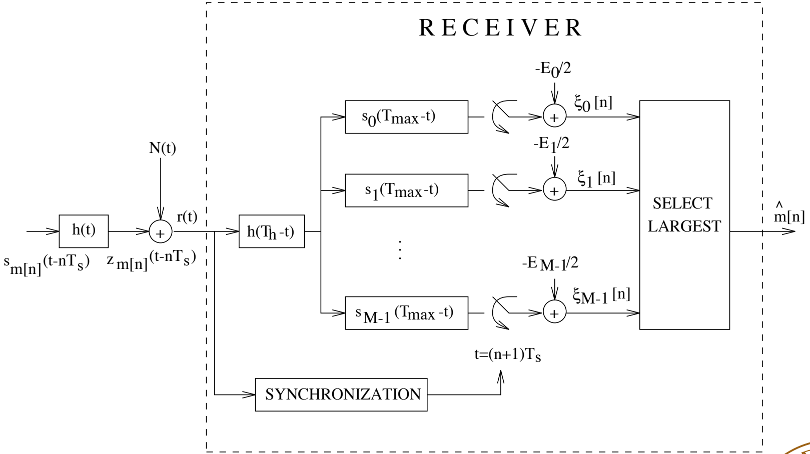
$$h(t) = \alpha\,\delta(t) \quad \Rightarrow \; z_\ell(t) = \alpha\, s_\ell(t)$$

- In case of multipath propagation the channel filter can change the shape and duration of the signals $z_\ell(t)$
- It can be shown that the matched filter of the overall system can be replaced with a cascade of two separate matched filters

$$z_\ell(T_s - t) \quad \Leftrightarrow \quad h(T_h - t)\ ,\ s_\ell(T_{max} - t)\ , \quad T_s = T_{max} + T_h$$

- The channel matching filter $h(T_h - t)$ simplifies the implementation of the receiver

ML receiver with channel matching filter



Example: three-ray channel

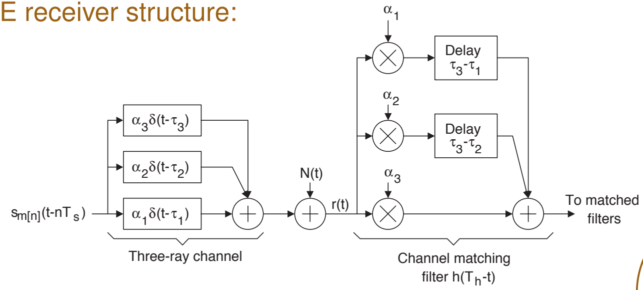
- Consider a channel with three signal paths

$$h(t) = \alpha_1\,\delta(t-\tau_1) + \alpha_2\,\delta(t-\tau_2) + \alpha_3\,\delta(t-\tau_3)$$

- Assuming $\tau_1 < \tau_2 < \tau_3$ we have $T_h = \tau_3$
- The channel matching filter becomes

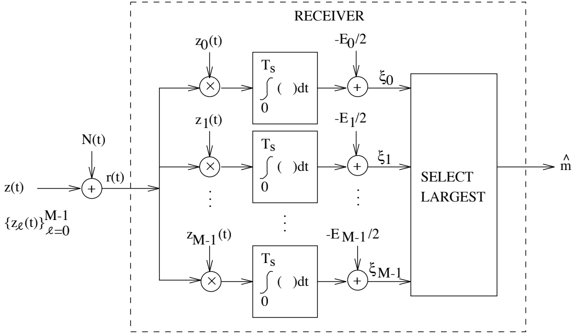
$$\begin{aligned} h(T_h - t) &= h(\tau_3 - t) \\ &= \alpha_3\;\delta(t) + \alpha_2\,\delta(t-(\tau_3-\tau_2)) + \alpha_1\,\delta(t-(\tau_3-\tau_1)) \end{aligned}$$

RAKE receiver structure:



Recall: receiver for M-ary signaling

- Consider the general receiver structure from Chapter 4:

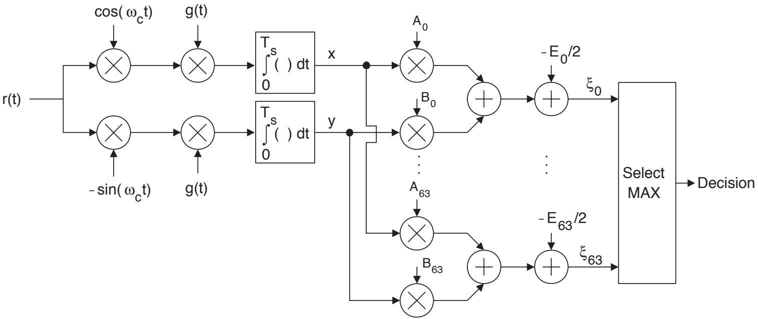


- Decision variables are computed by correlators or matched filters
- Each possible signal alternative is recreated in the receiver
- **Question:** can we apply this to bandpass signals? Yes!

But: recreating signals at large frequencies f_c is a challenge

Example: QAM Signaling

- Recall the simplified receiver considered in Example 4.4:

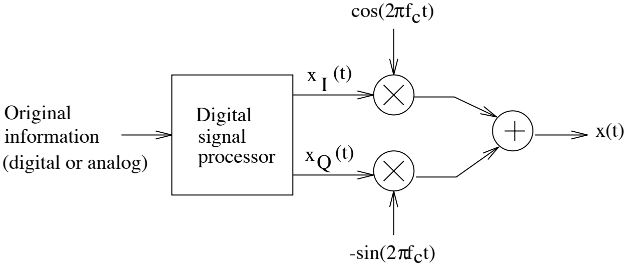


- Only two correlator branches are required instead of M
- Separation of carrier waveforms from baseband pulse possible

Our aim: a general baseband representation of the receiver

Transmission of bandpass signals

- Recall from last lecture:



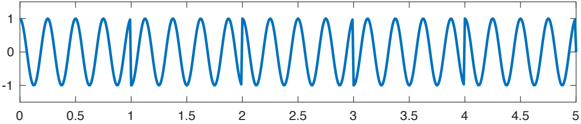
- A general bandpass signal can always be written as

$$x(t) = x_I(t)\ \cos(2\pi f_c t)\ -\ x_Q(t)\ \sin(2\pi f_c t)\ , \quad -\infty \leq t \leq \infty$$

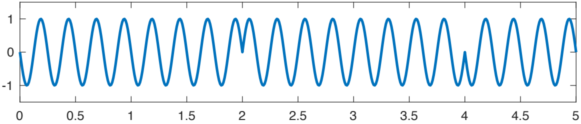
- $x_I(t)$: inphase component $\qquad x_Q(t)$: quadrature component

QPSK Example

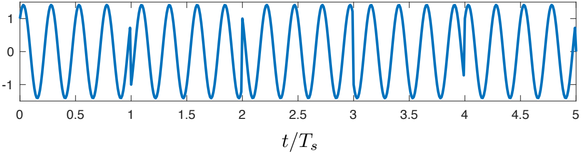
$x_I(t)\ \cos(2\pi\, f_c\, t)$



$x_Q(t)\ \sin(2\pi\, f_c\, t)$



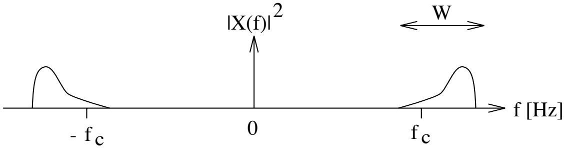
$x(t) = x_I(t)\ \cos(2\pi\, f_c\, t) - x_Q(t)\ \sin(2\pi\, f_c\, t)$



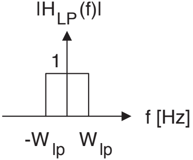
What are $x_I(t)$ and $x_Q(t)$ in this case?

Receivers for bandpass signals

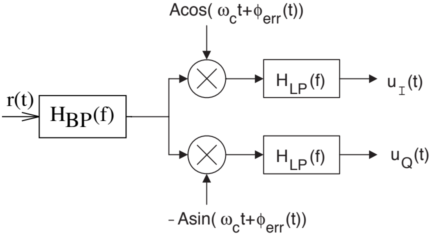
- **Our goal:** reproduce components $x_I(t)$ and $x_Q(t)$ at the receiver
- In the transmitted bandpass signal $x(t)$ these components were shifted to the carrier frequency f_c



- **Idea:** shifting the signal back to the baseband by multiplying with the carrier waveform again (see Ex. 2.19 and Problem 3.9)
- A lowpass filter $H_{LP}(f)$ is then applied in the baseband to remove undesired other signals or copies from the carrier multiplication



Homodyne receiver frontend



- Receiver is not synchronized to transmitter: phase errors $\phi_{err}(t)$
- Assume first $r(t) = x_I(t)\cos(2\pi f_c t)$ $\quad (x_Q(t) = 0$ and no noise)

$$u_I(t) = \left[x_I(t)\cos(2\pi f_c t)\cdot A\cos\left(2\pi f_c t + \phi_{err}(t)\right)\right]_{LP}$$

$$= \left[\frac{x_I(t)}{2} A\left(\cos(\phi_{err}(t)) + \cos(2\pi\, 2 f_c t + \phi_{err}(t))\right)\right]_{LP}$$

$$= \frac{x_I(t)}{2} A\cos(\phi_{err}(t))$$

- Likewise $\quad u_Q(t) = -\frac{x_I(t)}{2} A\sin(\phi_{err}(t))$

The impact of phase errors

- Assuming $r(t) = x_I(t)\cos(2\pi f_c t)$ we have found that

$$u_I(t) = \frac{x_I(t)}{2} A \cos(\phi_{err}(t)) \,, \quad u_Q(t) = -\frac{x_I(t)}{2} A \sin(\phi_{err}(t))$$

- **Ideal case:** $\phi_{err}(t) = 0$

$$u_I(t) = x_I(t)/2 \cdot A \quad \text{and} \quad u_Q(t) = 0$$

$\Rightarrow$ the inphase branch is independent of the quadrature branch

- **Phase errors:** $\phi_{err}(t) \neq 0$

$$u_I(t) < x_I(t)/2 \cdot A \quad \text{and} \quad u_Q(t) \neq 0 \;\; \text{(crosstalk)}$$

- If $\phi_{err}(t)$ changes randomly (jitter) the average $u_I(t)$ can vanish
- Ignoring the effect of phase errors can lead to bad performance

Question: what can we then do about phase errors?

Coherent receivers

- Assume now that we can estimate $\phi_{err}(t)$
- The signal $x_I(t)$ is contained in both $u_I(t)$ and $u_Q(t)$

$$u_I(t) = \frac{x_I(t)}{2} A \cos(\phi_{err}(t)) \ , \quad u_Q(t) = -\frac{x_I(t)}{2} A \sin(\phi_{err}(t))$$

- **Coherent reception:**

  by combining both components the signal can be recovered by

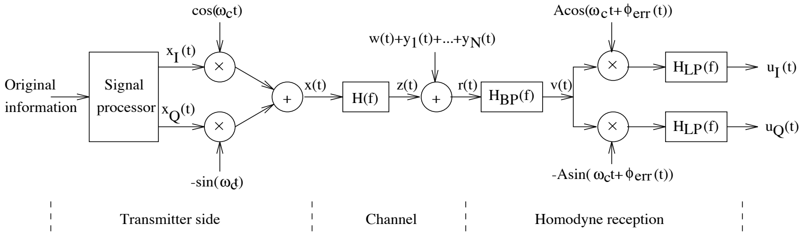
$$\hat{u}_I(t) = u_I(t) \cdot \cos(\phi_{err}(t)) - u_Q(t) \cdot \sin(\phi_{err}(t))$$

$$= \frac{x_I(t)}{2} A \cos^2(\phi_{err}(t)) + \frac{x_I(t)}{2} A \sin^2(\phi_{err}(t)) = \frac{x_I(t)}{2} A$$

- Observe: same result as in the ideal case $\phi_{err}(t) = 0$

Compare: non-coherent DPSK receiver (last lecture, p. 400-403) can be used if phase estimation is not possible

Overall transmission model



- The signal $y(t)$ is given by

$$y(t) = z(t) + w(t) = x(t) \ * \ h(t) + w(t)$$

- It can be written as

$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

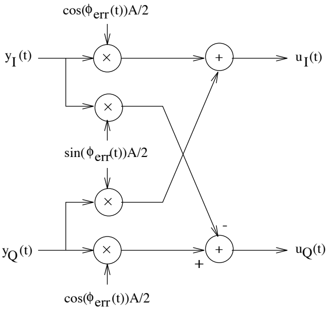
Can we express $u_I(t)$ and $u_Q(t)$ in terms of $x_I(t)$ and $x_Q(t)$?

Inphase and quadrature relationship

- With the complete signal $r(t)$ entering the receiver the output signals become

$$
\begin{aligned}
u_I(t) &= \big[y(t)\, A \cos(2\pi f_c t + \phi_{err}(t)) \big]_{LP} \\
&= \frac{y_I(t)}{2} A \cos(\phi_{err}(t)) \\
&\quad + \frac{y_Q(t)}{2} A \sin(\phi_{err}(t))
\end{aligned}
$$

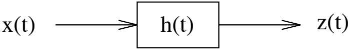
$$
\begin{aligned}
u_Q(t) &= \big[-y(t)\, A \sin(2\pi f_c t + \phi_{err}(t)) \big]_{LP} \\
&= \frac{y_Q(t)}{2} A \cos(\phi_{err}(t)) \\
&\quad - \frac{y_I(t)}{2} A \sin(\phi_{err}(t))
\end{aligned}
$$



Including the channel filter

- Before we can relate $y(t) = z(t) + w(t)$ to $x(t)$ we need to consider the effect of the channel

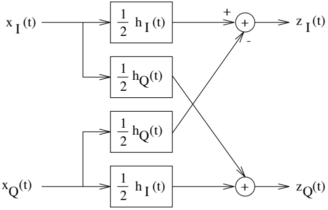
$$z(t) = x(t) \; * \; h(t)$$



- We assume that the impulse response $h(t)$ can be represented as a bandpass signal

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t)$$

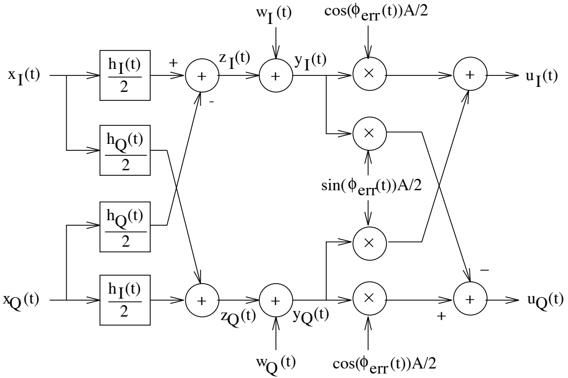
- With some calculations the signals can be written as (p. 159-160)



$$z_I(t) = \frac{1}{2}\big(x_I(t) \; * \; h_I(t) - x_Q(t) \; * \; h_Q(t)\big)$$
$$z_Q(t) = \frac{1}{2}\big(x_I(t) \; * \; h_Q(t) + x_Q(t) \; * \; h_I(t)\big)$$

Equivalent baseband model

- Combining the channel with the receiver frontend we obtain



- Observe that all the involved signals are in the baseband
- The same is true for channel filter, noise and phase error

Digital signal processing can be applied easily in baseband

What happened with the carrier waveforms?