LUND UNIVERSITY

EITG05 – Digital Communications

Lecture 8

Chapter 3: Carrier modulation techniques

Bandpass signals, digital and analog modulation

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Monday, September 30, 2019

Course Outline

39	Mon, Sep 23	Receivers continued: System design criteria, Performance for M-ary signaling	p. 254-286	Lec6
	Thu, Sep 26	Receivers continued: Geometric representation, Capacity, Multiuser receiver, Non-coherent receiver	p. 329-331, 360-366, 369, 395-396, 400-403	Lec7
40	Mon, Sep 30	Carrier modulation techniques, Bandpass signals, digital and analog modulation	p. 117-136, 139-152	
	Thu, Oct 3	Carrier modulation techniques continued, N-ray channel model, noise, Receivers for bandpass signals, Filtered channel receiver	p. 167-189, 160, 287-293 (not 3.5.1-3.5.2)	
41	Mon, Oct 7	Carrier modulation techniques continued, Equivalent baseband model, Compact description Intersymbol interference, ISI, Increasing the signaling rate	p. 201-205 p. 435-446	
	Thu, Oct 10	Intersymbol interference, Nyquist condition, Spectral raised cosine, Equalizers	p. 446-459	
42	Mon, Oct 14	Course summary and outlook		

From Section 5.4.5: Non-coherent receivers

- With phase-shift keying (PSK) the message $m[n]$ at time nT_s is put into the phase θ_n of the transmit signal

$$s(t) = g(t)\,\sqrt{2E}\,\cos(2\pi f_c t + \theta_n)\;, \quad nT_s \leq t \leq (n+1)T_s$$

- The channel introduces some attenuation α, some additive noise $N(t)$ and also some phase offset ν into the received signal

$$r(t) = \alpha\; g(t)\,\sqrt{2E}\,\cos(2\pi f_c t + \theta_n + \nu) + N(t)$$

- **Challenge:** the optimal receiver needs to know α and ν
- In some applications an accurate estimation of ν is infeasible (cost, complexity, size)
- **Non-coherent receivers:**
  receiver structures that can work well without knowledge of the exact phase offset

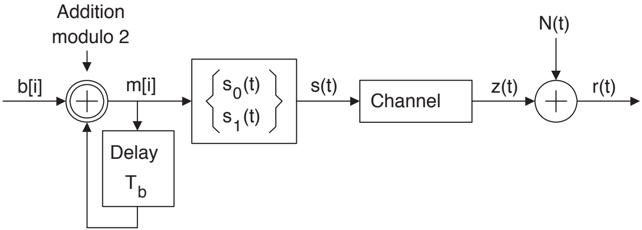
How can we modify our PSK transmission accordingly?

Differential Phase Shift Keying

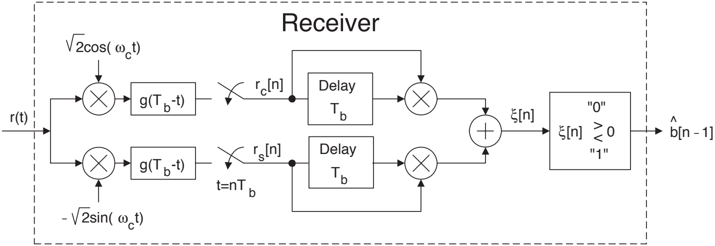
- With differential PSK, the message $m[n] = m_\ell$ is mapped to the phase according to

$$\theta_n = \theta_{n-1} + \frac{2\pi\ell}{M} \qquad \ell = 0, \dots, M-1$$

- The transmitted phase θ_n depends on both θ_{n-1} and $m[n]$
- This differential encoding introduces memory and the transmitted signal alternatives become dependent
- **Example 5.25:** binary DPSK



Differential Phase Shift Keying ($M = 2$)



- The receiver uses no phase offset ν in the carrier waveforms
- Without noise, the decision variable is

$$
\begin{aligned}
\xi[n] &= r_c[n]\, r_c[n-1] + r_s[n]\, r_s[n-1] \\
&= A\cos(\theta_{n-1}+\nu)\; A\cos(\theta_{n-2}+\nu) + A\sin(\theta_{n-1}+\nu)\; A\sin(\theta_{n-2}+\nu) \\
&= A^2\cos(\theta_{n-1}-\theta_{n-2}) \quad \Rightarrow \text{ independent of } \nu
\end{aligned}
$$

- Note: non-coherent reception increases variance of noise

Chapter 3: Carrier modulation techniques

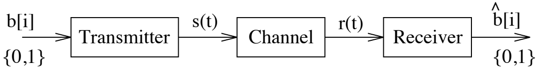
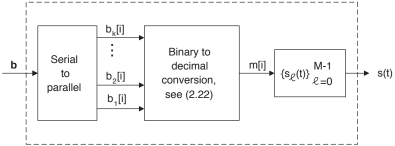


Figure 4.1: A digital communication system.

What we have done so far:

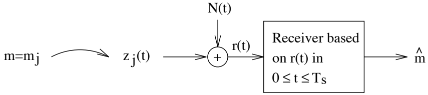
Chapter 2:

From $b[i]$ and $m[i]$ to signals $s_\ell(t)$



Chapter 4:

From signals $z_j(t) + N(t)$ to $\hat{m}[i]$ and $\hat{b}[i]$



Now more on:

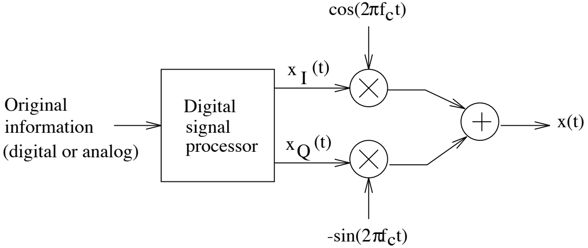
- properties of bandpass signals
- the channel: from $s(t)$ over $z(t)$ to $r(t)$
- efficient receivers for bandpass signals

Bandpass Signals

- A general bandpass signal can always be written as

$$x(t) = x_I(t)\ \cos(2\pi f_c t)\ -\ x_Q(t)\ \sin(2\pi f_c t)\ , \quad -\infty \le t \le \infty$$

- $x_I(t)$: inphase component $\qquad x_Q(t)$: quadrature component
- Corresponding transmitter structure:



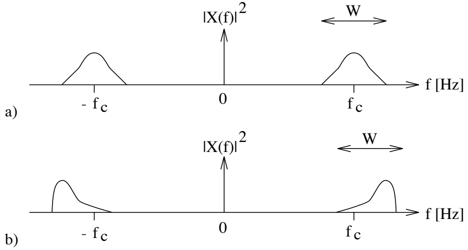
- The information is contained in the signals $x_I(t)$ and $x_Q(t)$ (for both analog or digital modulation)
- Not only wireless systems use carrier modulation

Spectrum of bandpass signals

- Computing the Fourier transform of $x(t)$ we get

$$X(f) = \frac{X_I(f+f_c) - j\,X_Q(f+f_c)}{2} + \frac{X_I(f-f_c) + j\,X_Q(f-f_c)}{2}$$

- Normally, $X_I(t)$ and $X_Q(t)$ have baseband characteristic, and f_c is much larger than their bandwidth
- The spectrum can be symmetric or non-symmetric around f_c



- **Remember:** real signals $x(t)$ always have even $|X(f)|$

DSB-SC Carrier Modulation

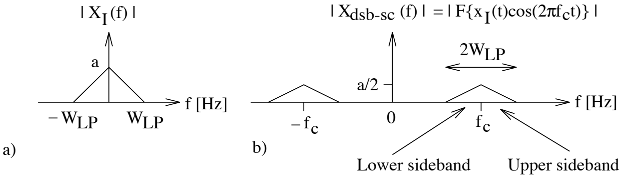
- Double sideband-suppressed (DSB-SC) carrier modulation is a special case of our general model
- In this case only $x_I(t)$ contains information and $x_Q(t) = 0$, i.e.,

$$x_{dsb-sc}(t) = x_I(t)\cos(2\pi f_c t)$$

- The Fourier transform then simplifies to

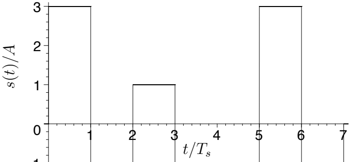
$$X(f) = \frac{X_I(f+f_c)}{2} + \frac{X_I(f-f_c)}{2}$$

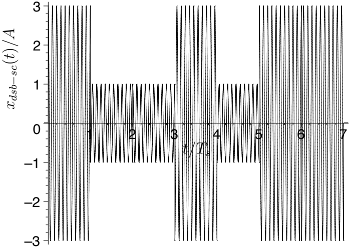
- $X_I(f)$ is symmetric around $f = 0$ $\Rightarrow$ $X(f)$ is symmetric around f_c



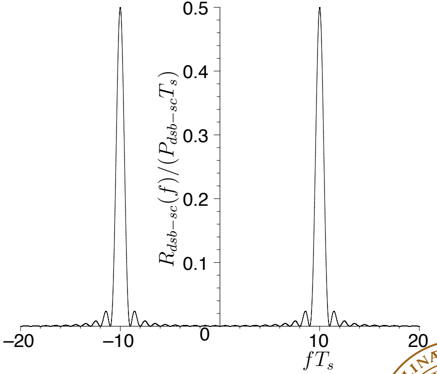
Where does the name come from?

Example 3.1: 4-ary PAM



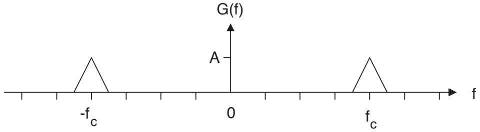


$$x_I(t) = s(t) = \sum_{n=-\infty}^{\infty} A_{m[n]}\, g_{rec}(t - nT_s)$$



How can we revert the frequency shift to f_c?

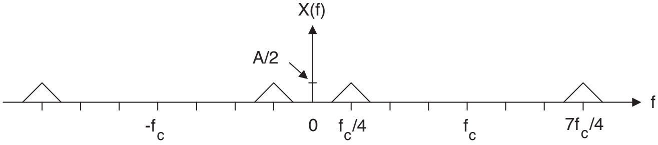
Hint: check Example 2.19 (p. 68)



Find the frequency content of

$$x(t) = g(t)\cos(2\pi f_0 t)\ , \quad f_0 = 3f_c/4$$

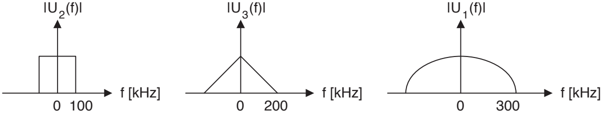
Solution:
If we apply (2.157) using $G(f)$ above, we obtain the frequency content in $x(t)$ as

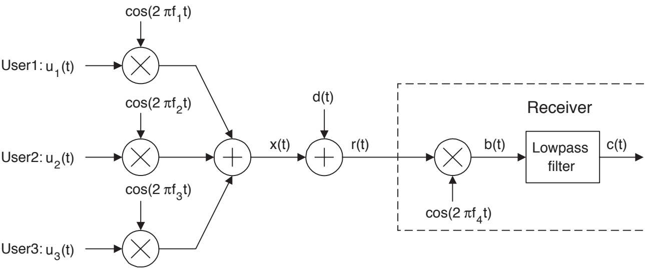


How should we choose f_0 to get the baseband signal back?

Problem 3.9

In the three-user (digital) communication system below, the frequency content in the user information signals $u_1(t)$, $u_2(t)$ and $u_3(t)$ are,





It is known that the individual carrier frequencies are: $f_1 = 3.5$ MHz, $f_2 = 4.0$ MHz, $f_3 = 3$ MHz. The disturbance $d(t)$ is $d(t) = \cos(2\pi 2 f_d t)$ where $f_d = 1.7$ MHz.

Only frequencies up to 100 kHz pass the lowpass filter.

Envelope and Phase

- A frequency shift corresponds to a multiplication with $e^{j2\pi f_c t}$
- For connecting this to the cosine and sine function we use

$$e^{j2\pi f_c t} = \cos(2\pi f_c t) \; + \; j \sin(2\pi f_c t)$$

- The general bandpass signal can then be written in terms of a frequency shifted version of a complex signal $x_I(t) + j x_Q(t)$

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) \; - \; x_Q(t)\sin(2\pi f_c t) \\ &= Re\left\{ \big(x_I(t) + j x_Q(t)\big) e^{j2\pi f_c t} \right\} \end{aligned}$$

- Expressing $x_I(t) + j x_Q(t)$ in terms of magnitude and phase we get

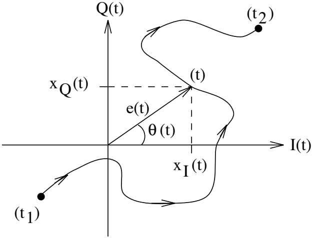
$$x(t) = e(t)\cos(2\pi f_c t + \theta(t)) \; , \quad -\infty \le t \le \infty$$

with

$$\begin{aligned} e(t) &= \sqrt{x_I^2(t) + x_Q^2(t)} \ge 0 \\ x_I(t) &= e(t)\cos(\theta(t)) \\ x_Q(t) &= e(t)\sin(\theta(t)) \end{aligned}$$

I-Q Diagram

- In the representation
$$x(t) = x_I(t)\cos(2\pi f_c t) \;-\; x_Q(t)\sin(2\pi f_c t)$$
the information is contained in the inphase component $x_I(t)$ and quadrature component $x_Q(t)$
- In the representation
$$x(t) = e(t)\cos(2\pi f_c t + \theta(t))\;, \quad -\infty \le t \le \infty$$
the information is contained in the envelope $e(t)$ and instantaneous phase $\theta(t)$



connection: I-Q diagram

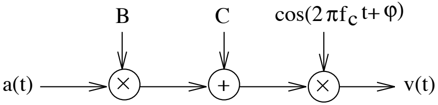
Analog Information Transmission

- Suppose that the information signal is an analog waveform $a(t)$
  Examples: music, speech, video
- If we use digital modulation, the waveform $a(t)$ is first converted to a binary sequence $b[i]$, which then is mapped to signals $s_\ell(t)$
- In case of analog modulation, the waveform $a(t)$ is used directly to modulate the carrier signal
- Let $v(t)$ denote the bandpass signal of an analog transmitter

$$\begin{aligned} v(t) &= v_I(t)\cos(2\pi f_c t) \; - \; v_Q(t)\sin(2\pi f_c t) \; , \quad -\infty \leq t \leq \infty \\ &= e(t)\cos\big(2\pi f_c t + \theta(t)\big) \end{aligned}$$

- **Amplitude modulation (AM):**
  the waveform $a(t)$ modulates the envelope $e(t)$ only
- **Frequency modulation (FM):**
  here $a(t)$ modulates the instantaneous phase $\theta(t)$ only

Amplitude Modulation (AM)



- The AM signal is the sum of a DSB-SC signal and carrier wave

$$\begin{aligned} v(t) &= \big(a(t)\,B + C\big)\,\cos(2\pi f_c\,t + \varphi) \\ &= a(t)\,B\cos(2\pi f_c\,t + \varphi) \;+\; C\cos(2\pi f_c\,t + \varphi) \end{aligned}$$

- Let us introduce the modulation index

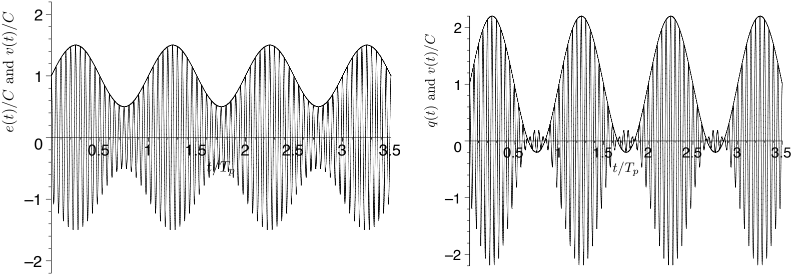
$$m = \frac{B\,a_{max}}{C} \leq 1\ , \quad \text{where}\ \ a_{max} = \max|a(t)|$$

- Using the normalized signal $a_n(t) = a(t)/a_{max}$ we can write

$$v(t) = \big(1 + m\,a_n(t)\big)\,C\cos(2\pi f_c\,t + \varphi)$$

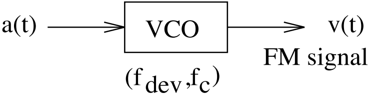
Example: AM signal

$$e(t)/C = 1 + m\,a(t)\ ,\ a_n(t) = \sin(2\pi f_p t)\ ,\ f_p = 1/Tp$$



- $m = 0.5 < 1$**:**
  the information signal $a_n(t)$ is contained in the envelope $e(t)$
- $m = 1.2 > 1$**:** (right picture)
  overmodulation: the baseband signal $q(t) = (1 + 1.2\,a_n(t))$
  is no longer equal to $e(t)$

Frequency Modulation (FM)



- With FM modulation, the transmitted signal

$$v(t)=\sqrt{2P}\cos(2\pi f_c t+\theta(t))$$

  is generated by a voltage controlled oscillator (VCO)
- The information carrying signal $a(t)$ is related to the phase $\theta(t)$ by

$$\frac{1}{2\pi}\,\frac{d\theta(t)}{dt}=f_{dev}\cdot a(t)$$

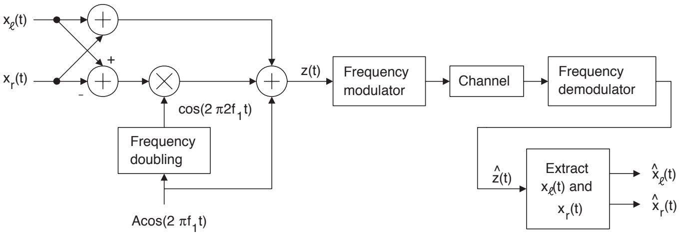
- The signal $a(t)$ hence modulates the instantaneous frequency

$$f_{ins}(t)=f_c+\frac{1}{2\pi}\,\frac{d\theta(t)}{dt}=f_c+f_{dev}\,a(t)$$

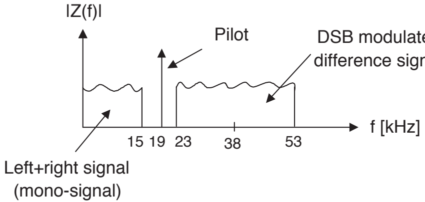
- FM modulation is a non-linear operation, hard to analyze

Example 3.13: FM stereo

A possible block-diagram of conventional analog FM stereo is shown below.



$x_\ell(t)$ and $x_r(t)$ denotes the left and the right audio-channel, respectively, and they are both bandlimited to 15 [kHz]*. The frequency $f_1 = 19$* [kHz] *(often referred to as a so-called pilot-tone).*



Digital Information Transmission

- In Chapter 2 the signal alternatives $s_\ell(t)$ could have arbitrary shape within the signaling interval $0 \leq t \leq T_s$
- The bandpass signal for digital modulation then has the form

$$\begin{aligned} x(t) &= x_I(t)\cos(2\pi f_c t) \; - \; x_Q(t)\sin(2\pi f_c t) \\ &= \left(\sum_{n=-\infty}^{\infty} s_{m[n],I}(t - nT_s)\right)\cos(2\pi f_c t) \\ &\quad - \left(\sum_{n=-\infty}^{\infty} s_{m[n],Q}(t - nT_s)\right)\sin(2\pi f_c t) \end{aligned}$$

- In case of M-ary QAM we have

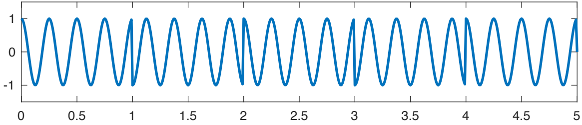
$$x_I(t) = \sum_{n=-\infty}^{\infty} A_{m[n]}\, g(t - nT_s)\ , \quad x_Q(t) = \sum_{n=-\infty}^{\infty} B_{m[n]}\, g(t - nT_s)$$

- Also M-ary FSK signals have bandpass characteristics

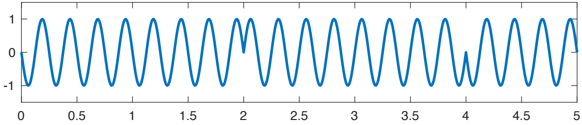
A simple Matlab example

How does a QPSK signal look like? Here is an example:

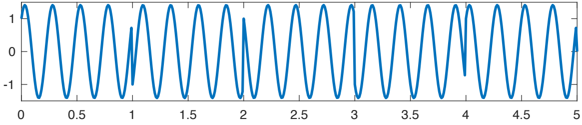
$x_I(t)\ \cos(2\pi\, f_c\, t)$



$x_Q(t)\ \sin(2\pi\, f_c\, t)$



$x(t) = x_I(t)\ \cos(2\pi\, f_c\, t) - x_Q(t)\ \sin(2\pi\, f_c\, t)$



t/T_s

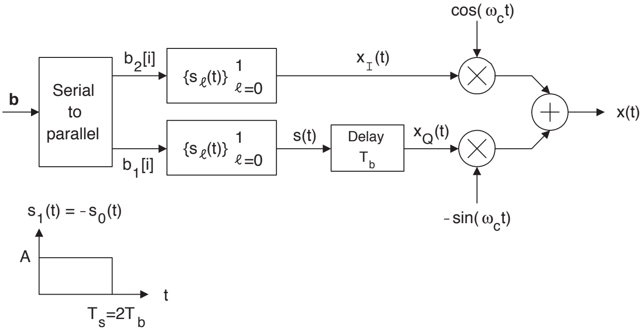
And how it was done:

```
% Example: QPSK signal

t=0:0.01:5;
fc=4;
pRec=ones(1,(length(t)-1)/5);
sI=zeros(1,length(t)); sQ=zeros(1,length(t));

dataI=[1 -1 1 -1 1];
indPulse=1:(length(t)-1)/5;
for i=1:length(dataI),
  sI(indPulse)=dataI(i)*pRec;
  indPulse=indPulse+length(indPulse);
end;

dataQ=[-1 -1 1 1 -1];
indPulse=1:(length(t)-1)/5;
for i=1:length(dataQ),
  sQ(indPulse)=dataQ(i)*pRec;
  indPulse=indPulse+length(indPulse);
end;

sCarI=cos(2*pi*t*fc); sCarQ=sin(2*pi*t*fc);

figure(1);
subplot(3,1,1); plot(t,sI.*sCarI);
set(gca,'YLim',[-1.5 1.5]); xlabel('fT_s');

subplot(3,1,2); plot(t,sQ.*sCarQ);
set(gca,'YLim',[-1.5 1.5]); xlabel('fT_s');

subplot(3,1,3); plot(t,sI.*sCarI - sQ.*sCarQ);
set(gca,'YLim',[-1.5 1.5]); xlabel('fT_s');
```

script | Ln 32 Col 30

Example 3.5: offset QPSK

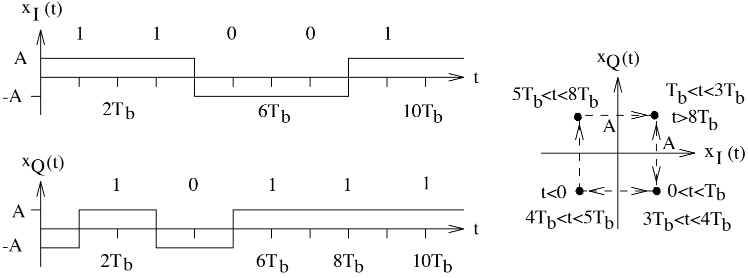
Below, two information carrying baseband signals $x_I(t)$ and $s(t)$ are first generated. Binary antipodal signaling with a rectangular pulse shape is used for both $x_I(t)$ and $s(t)$. The signal $x_Q(t)$ is a delayed version of $s(t)$, $x_Q(t) = s(t - T_b)$.



*The information bit rate (in **b**) is $R_b = 1/T_b$. Hence, the signaling rate in the quadrature components is $R_s = R_b/2$.*

QPSK signal with delayed transmission of $x_Q(t)$

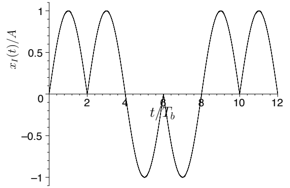
Example 3.5: offset QPSK

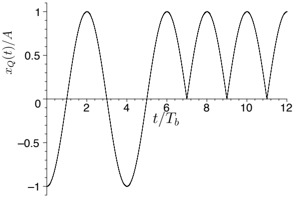


- **Special feature:**
  $x_I(t)$ and $x_Q(t)$ can never change at the same time
- it follows that the envelope does not pass the origin, i.e., $e(t) > 0$
- the variation of instantaneous power $\mathcal{P}(t) = e^2(t)/2$ is small, which allows more efficient power amplifiers

Example 3.6: constant envelope signaling

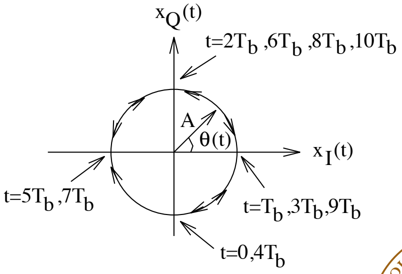
Change pulse shape:
half cycle sinusoidal $g_{hcs(t)}$ instead of $g_{rec}(t)$





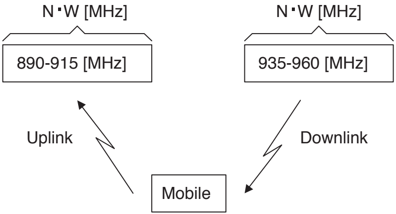
The squared envelope becomes

$$
\begin{aligned}
e^2(t) &= x_I^2(t) + x_Q^2(t) \\
&= A^2 \sin^2(\pi t/(2T_b)) + A^2 \cos^2(\pi t/(2T_b)) \\
&= A^2 \quad \Rightarrow \text{constant envelope } e(t) = A
\end{aligned}
$$



Continuous phase modulation (CPM) is used in GSM

Example 3.7: GSM



Each sub-band of W [Hz] carries information from X users, which are time-multiplexed using X time-slots. The total number of speech-channels (or data-channels) in the uplink (and in the downlink) is $N \cdot X$.

A specific user is allocated one of the N sub-bands, and one of the X time-slots. A time-slot has duration 576.92 [μs], and 148 binary symbols are transmitted within this time, see the figure below.

Start	Coded bits	Flag	Training sequence	Flag	Coded bits	Stop	ISI
3	57	1	26	1	57	3	margin

148

30.46 µs

576.92 µs

From 2G to 4G

- **GSM:** (Global System for Mobile Communications)
  based on combined time-division multiple access (TDMA) and frequency division multiple access (FDMA)
- **UMTS:** (Universal Mobile Telecommunications Service)
  based on wideband code division multiple access (W-CDMA)
  each user has an individual code, no TDMA or FDMA
- **LTE (advanced):** (Long Term Evolution)
  orthogonal frequency-division multiple access (OFDMA)

Multiple access:
refers to how different active users are separated