LUND UNIVERSITY

EITG05 – Digital Communications

Lecture 10

Equivalent baseband model, Compact description

Chapter 6: Intersymbol interference
ISI, Increasing the signaling rate

Michael Lentmaier
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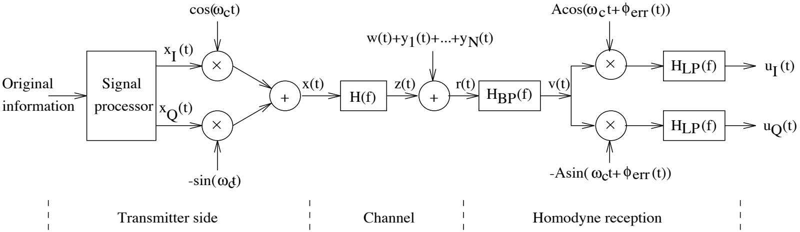
Overall transmission model



- The signal $y(t)$ is given by

$$y(t) = z(t) + w(t) = x(t) \ * \ h(t) + w(t)$$

- It can be written as

$$y(t) = y_I(t)\cos(2\pi f_c t) - y_Q(t)\sin(2\pi f_c t)$$

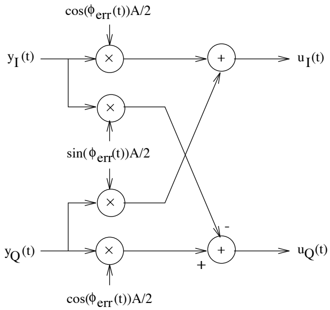
Can we express $u_I(t)$ and $u_Q(t)$ in terms of $x_I(t)$ and $x_Q(t)$?

Inphase and quadrature relationship

- With the complete signal $r(t)$ entering the receiver the output signals become

$$
\begin{aligned}
u_I(t) &= \Big[y(t) A \cos\left(2\pi f_c t + \phi_{err}(t)\right)\Big]_{LP} \\
&= \frac{y_I(t)}{2} A \cos(\phi_{err}(t)) \\
&\quad + \frac{y_Q(t)}{2} A \sin(\phi_{err}(t))
\end{aligned}
$$

$$
\begin{aligned}
u_Q(t) &= \Big[-y(t) A \sin\left(2\pi f_c t + \phi_{err}(t)\right)\Big]_{LP} \\
&= \frac{y_Q(t)}{2} A \cos(\phi_{err}(t)) \\
&\quad - \frac{y_I(t)}{2} A \sin(\phi_{err}(t))
\end{aligned}
$$



Including the channel filter

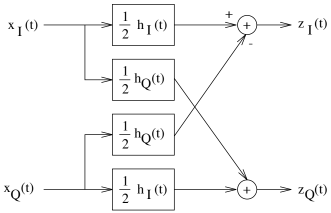
- Before we can relate $y(t) = z(t) + w(t)$ to $x(t)$ we need to consider the effect of the channel

$$z(t) = x(t) \ * \ h(t) \qquad\qquad x(t) \longrightarrow \boxed{h(t)} \longrightarrow z(t)$$

- We assume that the impulse response $h(t)$ can be represented as a bandpass signal

$$h(t) = h_I(t)\cos(2\pi f_c t) - h_Q(t)\sin(2\pi f_c t)$$

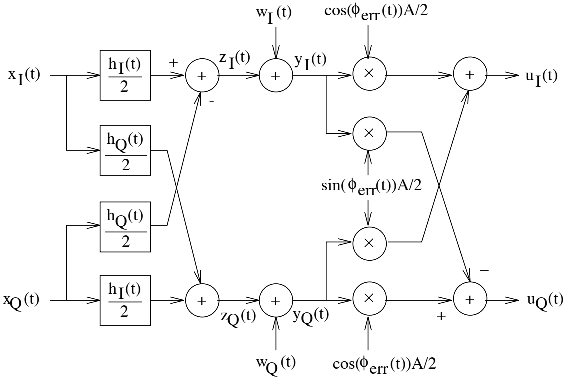
- With some calculations the signals can be written as (p. 159-160)



$$z_I(t) = \frac{1}{2}\big(x_I(t) \ * \ h_I(t) - x_Q(t) \ * \ h_Q(t)\big)$$
$$z_Q(t) = \frac{1}{2}\big(x_I(t) \ * \ h_Q(t) + x_Q(t) \ * \ h_I(t)\big)$$

Equivalent baseband model

- Combining the channel with the receiver frontend we obtain



- Observe that all the involved signals are in the baseband
- The same is true for channel filter, noise and phase error

Digital signal processing can be applied easily in baseband

What happened with the carrier waveforms?

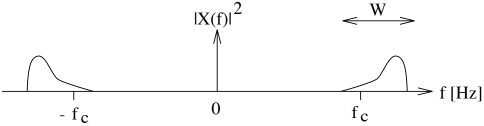
A compact description

- A more compact description is possible by combining $x_I(t)$ and $x_Q(t)$ to an equivalent baseband signal

$$\tilde{x}(t) = x_I(t) + j\,x_Q(t)$$

- The transmitted signal can then be described as

$$x(t) = Re\left\{\left(x_I(t) + j\,x_Q(t)\right) e^{+j2\pi f_c t}\right\} = Re\left\{\tilde{x}(t)\, e^{+j2\pi f_c t}\right\}$$



- With $Re\{a\} = (a + a^*)/2$ we can write

$$x(t) = \frac{\tilde{x}(t)}{2} \cdot e^{+j2\pi f_c t} + \frac{\tilde{x}^*(t)}{2} \cdot e^{-j2\pi f_c t}$$

A compact description

- Let us first ignore the effect of the channel: $w(t) = 0$, $h(t) = \delta(t)$
- The receiver can invert the frequency shift operation by

$$\tilde{u}(t) = \Big[x(t) \cdot A\, e^{-j(2\pi f_c\, t + \phi_{err}(t))} \Big]_{LP}$$

- Using the expression for $x(t)$ from the previous slide we get

$$\begin{aligned} \tilde{u}(t) &= \left[\frac{A}{2} \left(\tilde{x}(t) e^{+j2\pi f_c\, t} + \tilde{x}^*(t) e^{-j2\pi f_c\, t} \right) \cdot e^{-j(2\pi f_c\, t + \phi_{err}(t))} \right]_{LP} \\ &= \frac{\tilde{x}(t)}{2} A \cdot e^{-j\phi_{err}(t)} = u_I(t) + j\, u_Q(t) \end{aligned}$$

- Observe that this expression is equivalent to our earlier result

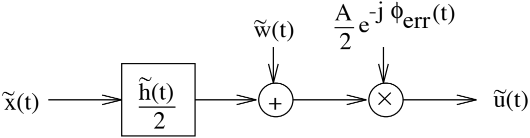
$$\begin{aligned} \tilde{u}(t) = & \left(\frac{x_I(t)}{2} A \cos(\phi_{err}(t)) + \frac{x_Q(t)}{2} A \sin(\phi_{err}(t)) \right) \\ & + j \left(\frac{x_Q(t)}{2} A \cos(\phi_{err}(t)) - \frac{x_I(t)}{2} A \sin(\phi_{err}(t)) \right) \end{aligned}$$

Compact equivalent baseband model

- The effect of the channel filter becomes

$$\tilde{z}(t) = z_I(t) + j z_Q(t) = \tilde{x}(t) \ * \ \frac{\tilde{h}(t)}{2}$$

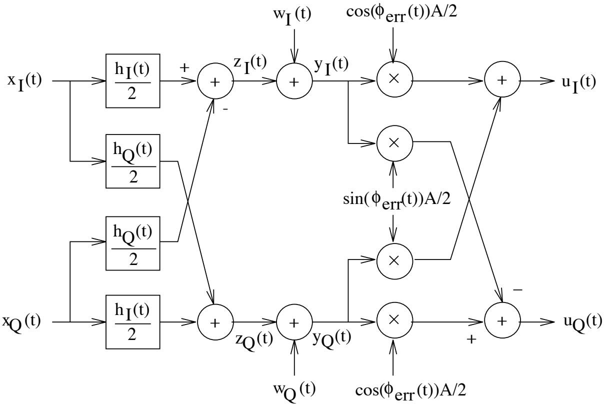
- Combining these parts and the noise we obtain the simple model

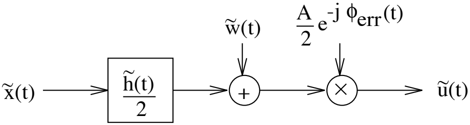


$$\tilde{u}(t) = \left[\left(\tilde{x}(t) \ * \frac{\tilde{h}(t)}{2} \right) + \tilde{w}(t) \right] \cdot e^{-j\phi_{err}(t)} \cdot \frac{A}{2} \ , \quad \tilde{w}(t) = w_I(t) + j w_Q(t)$$

- Complex signal notation simplifies expressions significantly

The two equivalent baseband models





M-ary QAM signaling

- Considering M-ary QAM signals we get

$$x_I(t) = \sum_{n=-\infty}^{\infty} A_{m[n]}\, g(t - nT_s)\ , \quad x_Q(t) = \sum_{n=-\infty}^{\infty} B_{m[n]}\, g(t - nT_s)$$

- Let us now introduce

$$\tilde{A}_m[n] = A_{m[n]} + j B_{m[n]}$$

- Then our complex baseband signal $\tilde{x}(t)$ can be written as

$$\tilde{x}(t) = x_I(t) + j\, x_Q(t) = \sum_{n=-\infty}^{\infty} \tilde{A}_{m[n]}\, g(t - nT_s)$$

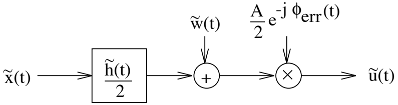
- **Example:** (on the board)

  Consider 4-QAM transmission of $\mathbf{b} = 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1$
  Determine $A_{m[n]}$, $B_{m[n]}$ and $\tilde{A}_{m[n]}$

How can we design the receiver for QAM signals?

Matched filter receiver

- At the receiver we see the complex baseband signal $\tilde{u}(t)$

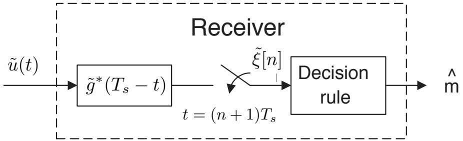


- If we know the channel we can design a matched filter for

$$\tilde{z}(t) = \tilde{x}(t) * \frac{\tilde{h}(t)}{2} \quad \Rightarrow \quad \tilde{v}(t) = \tilde{z}^*(T_s - t)$$

- It is often convenient to match $\tilde{v}(t)$ to the pulse $g(t)$ instead

$$\tilde{v}(t) = g^*(T_s - t) \quad \Rightarrow \quad \tilde{\xi}[n] = \left[\tilde{u}(t) * g^*(T_s - t)\right]_{t=(n+1)T_s}$$



Decision rule

- Consider now $\tilde{h}(t) = \delta(t)$ and $\tilde{w}(t) = 0$
- The ideal values of the decision variable are then given by

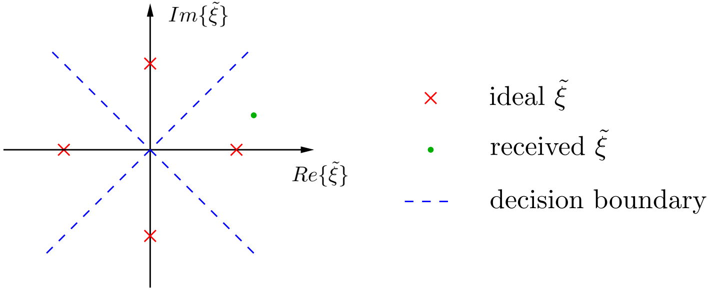
$$\begin{aligned}
\tilde{\xi}_{m[n]} &= \left[\tilde{u}(t) \, * \, g^*(T_s - t)\right]_{t=(n+1)T_s} \\
&= \left[\left(\tilde{A}_{m[n]} g(t - nT_s) \cdot e^{-j\phi_{err}(t)} \cdot \frac{A}{2}\right) * \, g^*(T_s - t)\right]_{t=(n+1)T_s} \\
&= \tilde{A}_{m[n]} e^{-j\phi_{err}(t)} \cdot \frac{A}{2}\left[g(t - nT_s) \, * \, g^*(T_s - t)\right]_{t=(n+1)T_s} \\
&= \tilde{A}_{m[n]} e^{-j\phi_{err}((n+1)T_s)} \cdot \frac{A}{2} \, E_g
\end{aligned}$$

- Due to noise $w(t) \neq 0$ and non-ideal channel $\tilde{h}(t)$ the decision variables at the receiver will differ from these ideal values
- The Euclidean distance receiver will base its decision on the ideal value $\tilde{\xi}_{m[n]}$ which is closest to the received value $\tilde{\xi}[i]$

Example: 4-PSK

- Assuming $\phi_{err}(t) = 0$ we obtain the ideal decision variables

$$\tilde{\xi}_{m[n]} = \tilde{A}_{m[n]} \cdot \frac{A}{2}\, E_g = (A_{m[n]} + jB_{m[n]}) \cdot \frac{A}{2}\, E_g$$

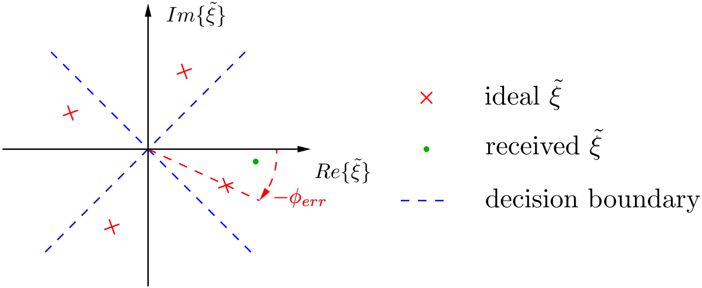


- Based on the received value $\tilde{\xi}[n]$ we decide for

$$\hat{m}[n]: \quad \tilde{A}_{\hat{m}[n]} = (1 + j \cdot 0)$$

Example: 4-PSK with phase offset

- Consider now a constant phase offset of $\phi_{err}(t) = \phi_{err} = 25^\circ$
- As a result the values $\tilde{\xi}_{m[n]}$ and $\tilde{\xi}[n]$ are rotated accordingly



How can we compensate for ϕ_{err}?

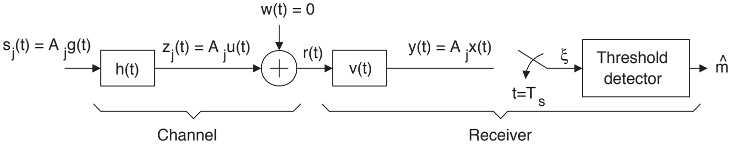
1. we can rotate the decision boundaries by the same amount
2. or we can rotate back $\tilde{\xi}[n]$ by multiplying with $e^{+j\phi_{err}}$

Summary: M-ary QAM transmission

- We can describe the transmitted messages $\tilde{A}_{\hat{m}[n]}$ and the decision variables $\tilde{\xi}[n]$ at the receiver as complex variables
- The effect of the noise $\tilde{w}(t)$ and the channel filter $\tilde{h}(t)$ on $\tilde{\xi}[n]$ can be described by the equivalent baseband model
- The transmitter and receiver frontends can be separated from the (digital) baseband processing
- Assumptions:
  - the pulse shape $g(t)$ satisfies the ISI-free condition
  - the carrier frequency f_c is much larger than the bandwidth of $g(t)$
- Under these conditions the design of the baseband receiver and its error probability analysis can be applied as in Chapter 4

Intersymbol Interference (ISI)

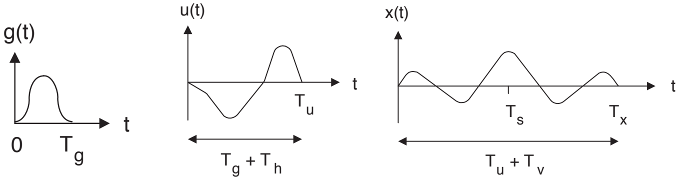
- Consider transmission of a single M-ary PAM signal alternative



- In the noise-free case ($w(t) = 0$) the signal $x(t)$ can be written as

$$x(t) = u(t) * v(t) = g(t) * h(t) * v(t)$$

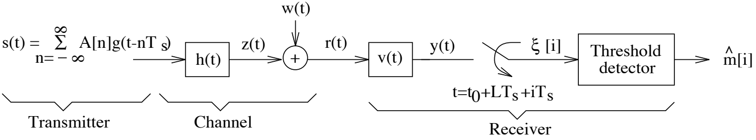
Example:



What happens if $T_u = T_g + T_h \geq T_s$? $\Rightarrow$ ISI occurs

Intersymbol Interference (ISI)

- For $R_s = 1/T_s < 1/T_u$ we can use the ML receiver from Chapter 4
- **Question:** can we use such a receiver for larger rates $R_s \geq 1/T_u$?
- Consider the following receiver structure (compare to last slide)



- Note that $z(t)$ now is a superposition of overlapping pulses $u(t)$
- The signal $y(t)$ after the receiver filter $v(t)$ is

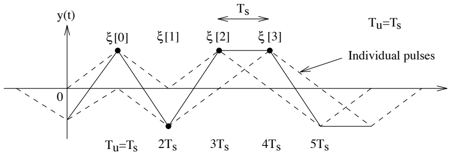
$$y(t) = \sum_{n=-\infty}^{\infty} A[n]\, x(t - nT_s) + w_c(t) \ ,$$

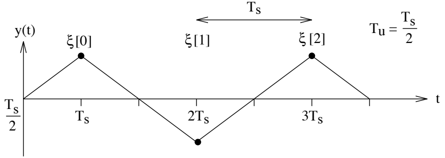
where $w_c(t)$ is a filtered Gaussian process

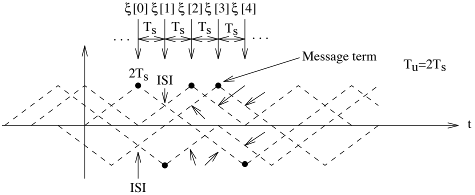
- The decision variable is obtained after sampling

$$\xi[i] = y(\mathcal{T} + iT_s) \ , \quad \mathcal{T} = t_0 + LT_s \ , \text{ where } LT_s \geq T_u$$

Illustration of ISI in the receiver







Discrete time model for ISI

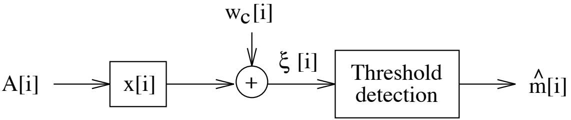
- According to our model the decision variable can be written as

$$\xi[i] = y(\mathcal{T} + i T_s) = \sum_{n=-\infty}^{\infty} A[n]\, x(\mathcal{T} + i T_s - n T_s) + w_c(\mathcal{T} + i T_s)$$

- Let us introduce the discrete sequences

$$x[i] = x(\mathcal{T} + i T_s)\ , \quad w_c[i] = w_c(\mathcal{T} + i T_s)$$

- This leads to the following discrete-time model of our system

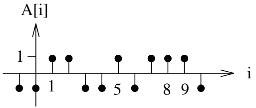


$$\xi[i] = \sum_{n=-\infty}^{\infty} A[n]\, x[i-n] + w_c[i] = A[i] * x[i] + w_c[i]$$

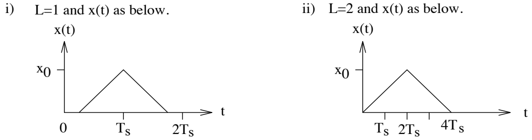
Remark: the discrete-time impulse response $x[i]$ represents pulse shape $g(t)$, channel filter $h(t)$, and receiver filter $v(t)$

Example 6.1

The transmitted sequence of amplitudes $A[i]$ is given as,



Calculate, and plot, the sequence of decision variables $\xi[i]$ in Figure 6.2, for $0 \leq i \leq 8$, in the noiseless case (i.e. $w(t) = 0$) if $t_0 = 0$ and if the output pulse $x(t)$ is:



- i) $\xi[i] = x_0 A[i]$ ii) $\xi[i] = \frac{x_0}{2} A[i+1] + x_0 A[i] + \frac{x_0}{2} A[i-1]$