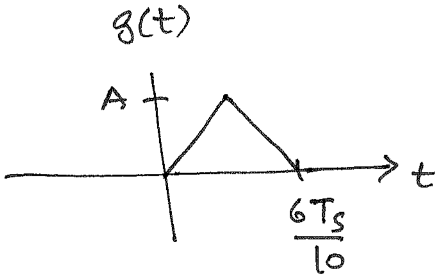
Prob 3-1

4-ary QAM, $M = 4$

$g(t) = g_{tri}(t)$

$\bar{P}_s = ?$ (Average Signal power)



Sol

$$\bar{P}_s = \bar{E}_s R_s.$$

where $\bar{E}_s = \frac{2(M-1)}{3} \cdot \frac{E_g}{2}$. [Eq. 2.98].

and E_g of $g_{tri}(t) = g(t)$ given above is

$$E_g = \frac{A^2 T}{3}$$ [Eq D.10 at P # 618].

where $T = \frac{6T_s}{10}$.

$$E_g = \frac{A^2}{3} \cdot \frac{6T_s}{10} = \frac{A^2 T_s}{5}$$

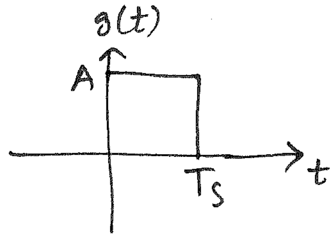
and $\bar{E}_s = \frac{2(\cancel{3})}{\cancel{3}} \cdot \frac{1}{2} \cdot \frac{A^2 T_s}{5} = \frac{2A^2 T_s}{10}$.

and $\bar{P}_s = \frac{\bar{E}_s}{T_s}$ [since $R_s = \frac{1}{T_s}$].

$$\boxed{\bar{P}_s = \frac{2A^2}{10}}$$ True.

Prob 3.2

$g(t) = g_{rec}(t)$

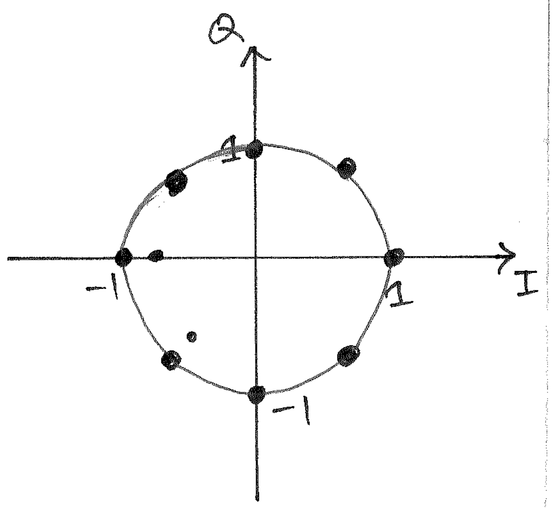


(a) Draw constellation diagram for 8 PSK and QAM

For PSK we have

$v_0 = 0,\ v_1 = \frac{\pi}{4},\ v_2 = \frac{\pi}{2},\ v_3 = \frac{3\pi}{4},\ v_4 = \pi,\ v_5 = \frac{5\pi}{4}$

$v_6 = \frac{3\pi}{2},\ v_7 = \frac{7\pi}{4}$



normalized Energy on I and Q axis.

Fig. 8-PSK constellation

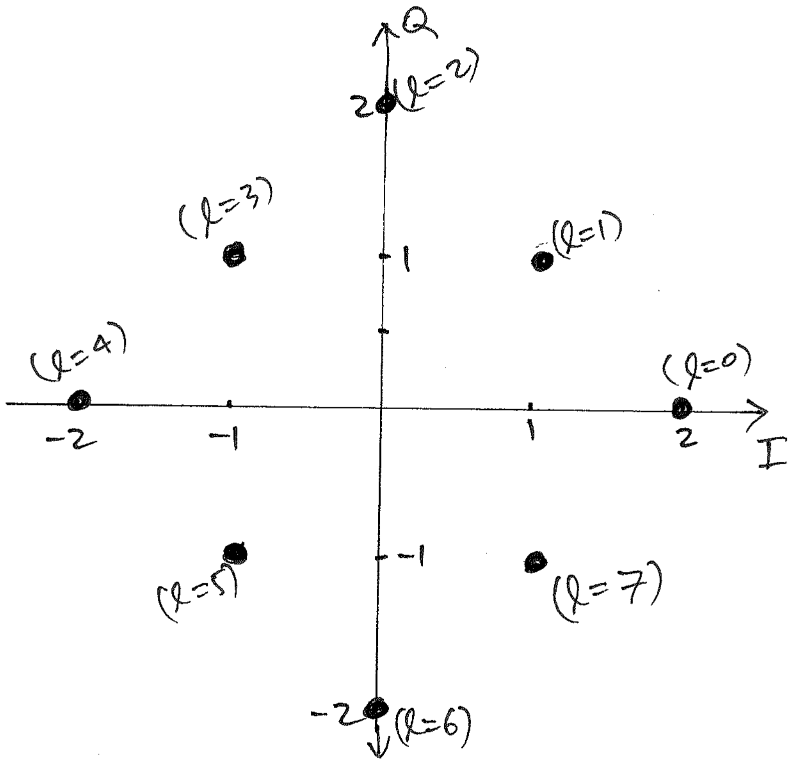


Fig. QAM constellation.

(b) Avg. Energy of PSK

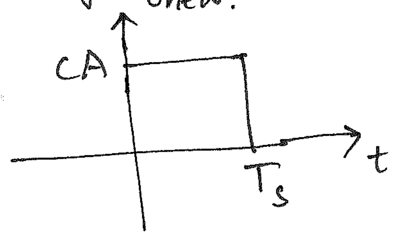
$$\bar{E}_s = \frac{E_g}{2} \quad [\text{Eq. } 2.58 \ \text{P\# } 37].$$

where $E_g = A^2 T_s$ when $g(t) = g_{rec}(t)$ is used.

Since we want to scale $g(t)$ with c the resultant E_g is (new)

$$E_{g(new)} = c^2 A^2 T_s = c^2 E_g$$

where g_{new} is



[since $\bar{E}_b = \frac{\bar{E}_s}{k}$].

hence $\bar{E}_s = \frac{c^2 E_g}{2}$

and $\boxed{\underset{(PSK)}{\bar{E}_b} = \frac{c^2 E_g}{6}}$ —— (1)

Avg. Energy per bit of QAM constellation.

$$\bar{E}_s = \sum_{l=0}^{7} P_l E_l \qquad P_l = \frac{1}{8} \text{ for } l=0.$$

$$\bar{E}_s = \frac{1}{8}(4E_0 + 4E_1)$$

$E_0 = ?$ For QAM $E_l = (A_l^2 + B_l^2) E_g/2$ [Eq. 2.94 P# 47].

hence $E_0 = 2E_g$ and $E_1 = E_g$.

$$\bar{E}_s = \frac{1}{8}(8E_g + 4E_g) = \frac{3}{2} E_g.$$

$\boxed{\underset{(QAM)}{\bar{E}_b} = \frac{E_g}{2}}$ —— (2)

In order for $\bar{E}_b$ of 8-PSK and QAM to be equal c should be

From ① and ②

$$\bar{E}_{b(PSK)} = \bar{E}_{b(QAM)}$$

$$\frac{c^2 E_g}{6} = \frac{E_g}{2} \Rightarrow \boxed{c = \sqrt{3}.}$$

which constellation have lager min. squared Euclidean distance $\min_{ij} D_{ij}^2$?

PSK $\quad D_{ij}^2 = E_g(1 - \cos(\vartheta_i - \vartheta_j))$ [Eq. 2.60 p# 38].

Since distance between neighboring signal points is same, $\min_{ij} D_{ij}^2$ in this case is the distance between any two nearest neighbors on the constellation diagram

$$D_{ij}^2 = 3E_g\left(1 - \cos\left(0 - \frac{\pi}{4}\right)\right) = D_{01}^2$$

$$\boxed{\min_{ij} D_{ij}^2 = D_{01}^2 = .8787\, E_g.}$$

QAM $\quad D_{ij}^2 = \frac{E_g}{2} \cdot ((A_i - A_j)^2 + (B_i - B_j)^2)$ [Eq. 2.95 on P#47

$\min D_{ij}^2$ is the distance between two closest points in the constellation diagram, e.g., between ~~m[0]~~ $l=0$ and ~~m[1]~~ $l=1$.

$$\min \boxed{D_{ij}^2 = E_g}.$$

hence QAM has largest $\min D_{ij}^2$