Lund University
Electrical and Information Technology

Exam in Digital Communications, EITG05

November 1, 2018

- During this exam you are allowed to use a calculator, the compendium, a printout of the lecture slides, and Tefyma (or equivalent).
- Please use a new sheet of paper for each solution. Write your anonymized assessment code + a personal identifier on each paper.
- Solutions should clearly show the line of reasoning and follow the methods presented in the course. If you use results from the compendium or lecture slides, please add a reference in your solution.
- If any data is lacking, make reasonable assumptions.

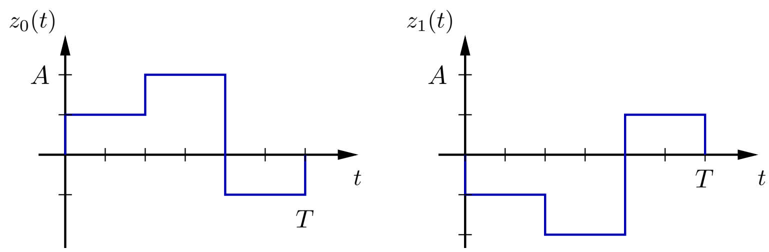
Good luck!

Problem 1

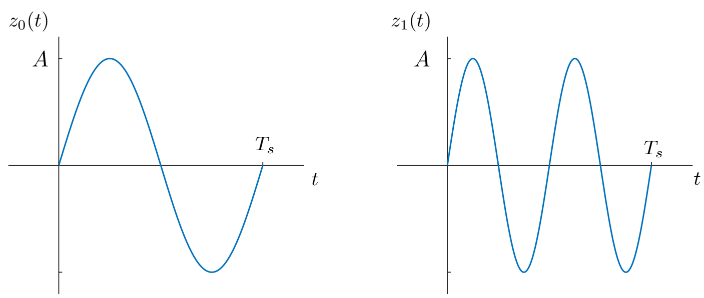
Determine for each of the five statements below if it is true or false.
Give a motivation for each of your answers.

(a) *"Baseband PAM with $M = 4$ and QAM with $M = 16$ have the same bandwidth efficiency and the same energy efficiency."*

(b) *"For the signal constellation given in the figure below, the normalized squared Euclidean distance is equal to $d_{min}^2 = 1.5$"*



(c) Consider a minimum Euclidean distance receiver (as shown in Fig. 4.8 on p. 241) used for binary transmission with the following signal alternatives:



"If the signal $z_0(t)$ is transmitted and the noise is $N(t) = 0$, then the decision variable ξ_1 at time T_s will be equal to zero."

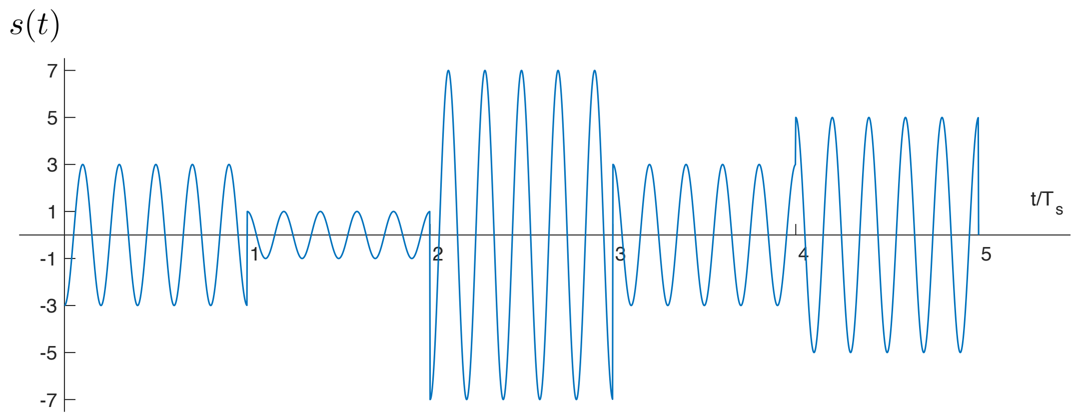
(d) *"For M-ary PSK, the spectrum $|X(f)|$ of the transmitted signal $x(t)$ is always symmetric around the carrier frequency f_c."*

(e) *"For a given symbol rate R_s, a spectral raised cosine pulse with $\beta = 1$ requires twice the bandwidth as a Nyquist pulse."*

(10p)

Problem 2

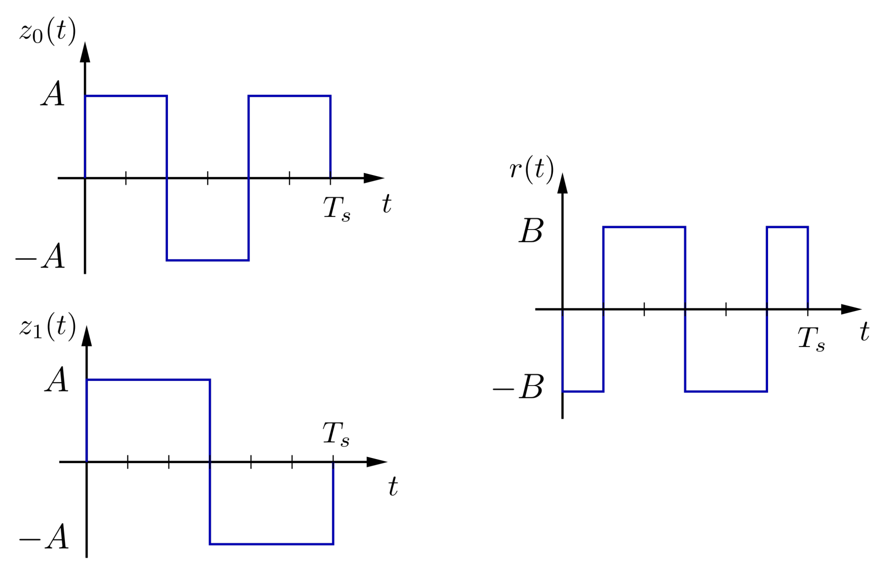
Consider the following transmitted signal $s(t)$, where $T_s = 1\,\mu s$:



(a) Determine the message sequence $\mathbf{m}$, the carrier frequency f_c, and the bit rate R_b.

(b) Draw a constellation diagram for this signaling method, and use some Gray mapping to assign bits to the signal alternatives.
Determine the corresponding sequence of transmitted bits $\mathbf{b}$.

Consider now a receiver for a binary constellation with signal alternatives $z_0(t)$ and $z_1(t)$. The received signal $r(t) = z(t) + N(t)$ is disturbed by an unknown signal $N(t)$.



(c) Determine the decision variables ξ_0 and ξ_1 computed by a minimum Euclidean distance receiver. Which of the signal alternatives is selected by the receiver?

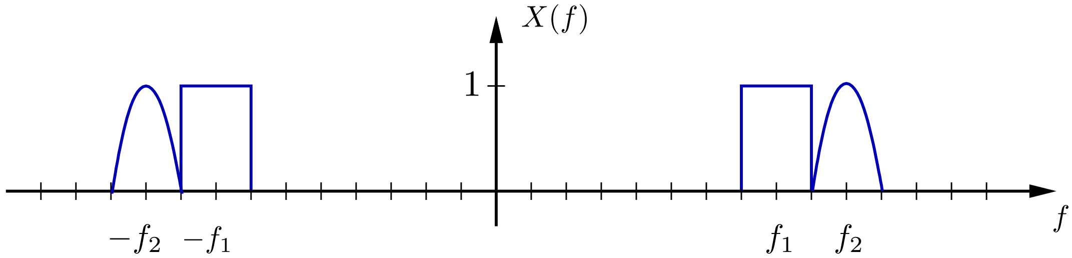
(d) Assuming the decision above was correct, draw the signal $N(t)$ within the interval $0 \leqslant t \leqslant T_s$.

Remark: Parts (c) and (d) can be solved independently from (a) or (b).

(10p)

Problem 3

A communication system can serve two users simultaneously, at carrier frequencies $f_1 = 400\,\text{MHz}$ and $f_2 = 500\,\text{MHz}$, respectively. The frequency spectrum $X(f)$ of the combined bandpass signal $x(t) = x_1(t) + x_2(t)$ is given in the figure below:



A two-stage receiver is used to convert both signals to low-pass domain simultaneously. The second stage has the advantage that it could be implemented with digital signal processing after an analog/digital conversion of the signal.

(a) In the first stage, an analog mixer is used to create the signal

$$x_1(t) = x(t) \cdot \cos(2\pi f_3\, t)\,, \quad f_3 = 350\,\text{MHz}\,.$$

An (ideal) analog low-pass filter with bandwidth $W_{\text{LP},1}$ is applied to this signal, resulting in the signal $x_{1,\text{LP}}(t) = x_1(t) * h_{\text{LP},1}(t)$.

Choose a suitable value for $W_{\text{LP},1}$ and draw the spectrum $X_1(f)$ of the unfiltered signal, the frequency response $H_{\text{LP},1}(f)$ of the filter, and the spectrum $X_{1,\text{LP}}(f)$ after filtering.

(b) In the second stage, the receiver creates the signal

$$x_2(t) = x_{1,\text{LP}}(t) \cdot e^{-j\,2\pi f_4\, t}\,, \quad f_4 = 100\,\text{MHz}\,.$$

A second (ideal) low-pass filter with bandwidth $W_{\text{LP},2} = 100\,\text{MHz}$ is then applied, resulting in the signal $x_{2,\text{LP}}(t) = x_2(t) * h_{\text{LP},2}(t)$.

Draw the spectra $X_2(f)$ and $X_{2,\text{LP}}(f)$ of the signals before and after the filtering.

(c) Which of the five signals $x(t)$, $x_1(t)$, $x_{1,\text{LP}}(t)$, $x_2(t)$, and $x_{2,\text{LP}}(t)$ are real-valued / complex-valued? Explain.

(d) Describe the difference between a coherent receiver and a non-coherent receiver.

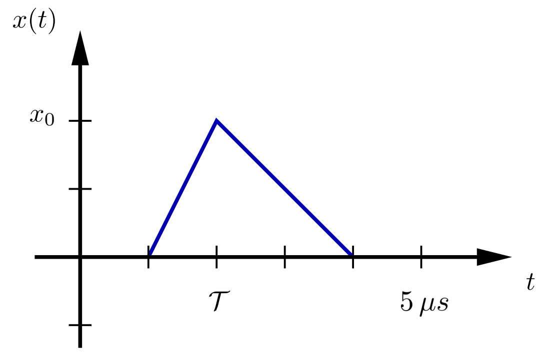
Remark: Part (d) can be solved independently from (a), (b), or (c).

(10p)

Problem 4

Consider a communication system employing 4 PAM modulation with equally likely signal alternatives. The combination of the transmit pulse $g(t)$, channel filter $h(t)$, and receiver filter $v(t)$ can be written as $x(t) = g(t) * h(t) * v(t)$. The signal is sampled in the receiver at time instants $\mathcal{T} + i\,T_s$, $i = 0, 1, 2, \ldots$.

(a) We want to transmit the amplitude sequence $A[0] = 1$, $A[1] = 3$, $A[2] = -1$, $A[3] = -1$. Assume that $T_s = 2\,\mu s$ and the signal $x(t)$ is given in the figure below:



Draw the signal $y(t)$ at the output of the receiver filter in the interval $0 < t < 10\,\mu s$ in the absence of noise, i.e., $w(t) = 0$. Does ISI occur?

Assume now transmission with a rectangular pulse $g(t) = g_{rec}(t)$ of duration $T = 1\,\mu s$ and a multipath channel with $h(t) = \delta(t) + 0.5 \cdot \delta(t - 2T)$.

(b) Let the impulse response of the receiver filter be chosen as $v(t) = g(T - t)$.

Draw the corresponding signal $x(t)$. For the case $T_s = 2\,\mu s$ draw the discrete impulse response $x[i]$. Does ISI occur? Determine the largest symbol rate R_s that can be achieved without ISI.

(c) Let us instead use a receiver filter that is matched to the signal at the output of the channel filter. Draw the corresponding impulse response $v(t)$. What is now the largest symbol rate R_s that can be achieved without ISI?

(10p)

Problem 5

Consider a communication system with binary antipodal baseband PAM signaling. The pulse shape $g(t)$ and pulse duration $T = T_s/2$ are chosen such that a bandwidth efficiency of $\rho = 1/4$ is achieved with a bit rate of $R_b = 100\,\text{kbps}$. The system bandwidth W is measured by the main-lobe.

(a) Assume that BPSK modulation is used instead.
What is the bandwidth efficiency ρ in this case?

(b) Consider again the original signaling method. Is it possible to change the pulse shape $g(t)$ in such a way that $\rho = 1$ can be achieved without changing the bit rate R_b and without introducing ISI? Motivate your answer.

(c) Assume now that a rectangular pulse $g(t) = g_{rec}(t)$ with amplitude A and duration $T = T_s/2$ is used. The parameter of the additive white Gaussian noise is equal to $N_0 = 2 \cdot 10^{-20}$ [W/Hz] and the propagation attenuation is $\alpha = 0.0002$.

What is the smallest amplitude A to ensure that the symbol error probability P_s satisfies $P_s \leqslant 10^{-9}$?

(d) In order to increase the bit rate R_b, you are considering M-ary baseband PAM signaling with equally likely signal alternatives. To be able to fulfill the same requirements on P_s as in (c) you are allowed to reduce the energy efficiency of the system by $15\,\text{dB}$.

How much can you increase R_b by only changing the modulation order M?

(10p)